## ECE 111: Handout \#11

Week \#8: ECE 351 Electromagnetics

1) Assume the current flowing into a 1 H inductor is as shown below. Sketch the voltage.

- $V=L \frac{d I}{d t}$


2) Write the differential equations which describe the following circuit. Assume

- $\mathrm{L}=0.1 \mathrm{H}$
- $\mathrm{C}=0.1 \mathrm{~F}$
- $\mathrm{R}=10$ Ohms



## Solutions

1) Inductors are differentiators:

$$
V=L \frac{d I}{d t}
$$


2) Write the differential equations which describe the following circuit. Assume

- $\mathrm{L}=0.1 \mathrm{H}$
- $\mathrm{C}=0.1 \mathrm{~F}$
- $\mathrm{R}=10 \mathrm{Ohms}$


Use the VI relationship for $\mathrm{R}, \mathrm{L}$, and C

$$
\begin{aligned}
& V=I R, \quad V=L \frac{d I}{d t}, \quad I=C \frac{d V}{d t} \\
& V_{1}-V_{2}=L \dot{I}_{a} \\
& V_{3}-V_{2}=L \dot{I}_{c} \\
& I_{b}=-\frac{V_{2}}{R}
\end{aligned}
$$

The node eqution at V2 is

$$
\begin{aligned}
& I_{2}=C \dot{V}_{2}=I_{a}+I_{b}+I_{c} \\
& C \dot{V}_{2}=I_{a}-\left(\frac{V_{2}}{R}\right)+I_{c}
\end{aligned}
$$

Differentiate

$$
\begin{aligned}
& C \ddot{V}_{2}=\dot{I}_{a}-\left(\frac{\dot{V}_{2}}{R}\right)+\dot{I}_{c} \\
& C \ddot{V}_{2}=\left(\frac{V_{1}-V_{2}}{L}\right)-\left(\frac{\dot{V}_{2}}{R}\right)+\left(\frac{V_{3}-V_{2}}{L}\right)
\end{aligned}
$$

Ditto for nodes 1..3. Node 4 is a little different since no node \#5

$$
\begin{aligned}
& C \ddot{V}_{1}=\left(\frac{V_{0}-V_{1}}{L}\right)-\left(\frac{\dot{V}_{1}}{R}\right)+\left(\frac{V_{2}-V_{1}}{L}\right) \\
& C \ddot{V}_{2}=\left(\frac{V_{1}-V_{2}}{L}\right)-\left(\frac{\dot{V}_{2}}{R}\right)+\left(\frac{V_{3}-V_{2}}{L}\right) \\
& C \ddot{V}_{3}=\left(\frac{V_{2}-V_{3}}{L}\right)-\left(\frac{\dot{V}_{3}}{R}\right)+\left(\frac{V_{4}-V_{3}}{L}\right) \\
& C \ddot{V}_{4}=\left(\frac{V_{3}-V_{4}}{L}\right)-\left(\frac{\dot{V}_{4}}{R}\right)
\end{aligned}
$$

