

ECE 111 - Homework #1

Week #1: Algebra.

Please submit as a Word or pdf file and email to Jacob_Glower@yahoo.com with header ECE 111 HW#1

functions *poly* and *roots*:

1) Use MATLAB, find the roots the the following polynomials:

a) $x^3 + 3x^2 + 6x + 12 = 0$

```
>> roots([1,3,6,12])  
  
-2.5127  
-0.2436 + 2.1717i  
-0.2436 - 2.1717i
```

b) $x^4 + 3x^3 + 6x^2 + 12x + 24 = 0$

```
>> roots([1,3,6,12,24])  
  
-1.9862 + 1.2456i  
-1.9862 - 1.2456i  
0.4862 + 2.0322i  
0.4862 - 2.0322i
```

c) $x^5 + 3x^4 + 6x^3 + 12x^2 + 24x + 48 = 0$

```
>> roots([1,3,6,12,24,48])  
  
-2.3473  
0.9216 + 1.8307i  
0.9216 - 1.8307i  
-1.2480 + 1.8194i  
-1.2480 - 1.8194i
```

2) Use Matlab to multiply our the following polynomials.

a) $(x + 2)(x + 3)(x + 5)(x + 7) = 0$

```
>> poly([-2, -3, -5, -7])
```

```
1 17 101 247 210
```

$$x^4 + 17x^3 + 101x^2 + 247x + 210 = 0$$

b) $(x + 3)(x + 6)(x + 9)(x + 12)(x + 15)(x + 18) = 0$

```
>> poly([-3, -6, -9, -12, -15, -18])
```

```
1 63 1575 19845 131544 428652 524880
```

$$x^6 + 63x^5 + 1575x^4 + 19845x^3 + 131544x^2 + 428652x + 524880 = 0$$

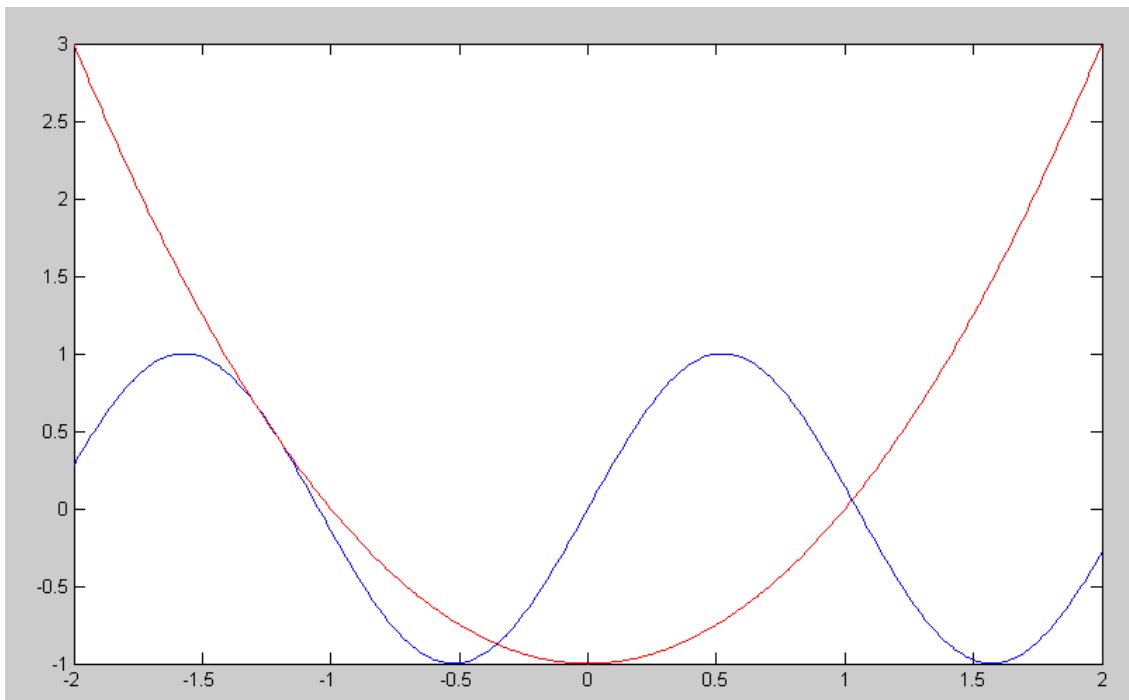
Graphing in Matlab

3) Plot the two functions in Matlab and determine all solutions in the range of $-2 < x < +2$

$$y = \sin(3x)$$

$$y = x^2 - 1$$

```
>> x = [-2:0.01:2]';  
>> y1 = sin(3*x);  
>> y2 = x.^2 - 1;  
>> plot(x,y1,'b',x,y2,'r');
```



There are four solutions:

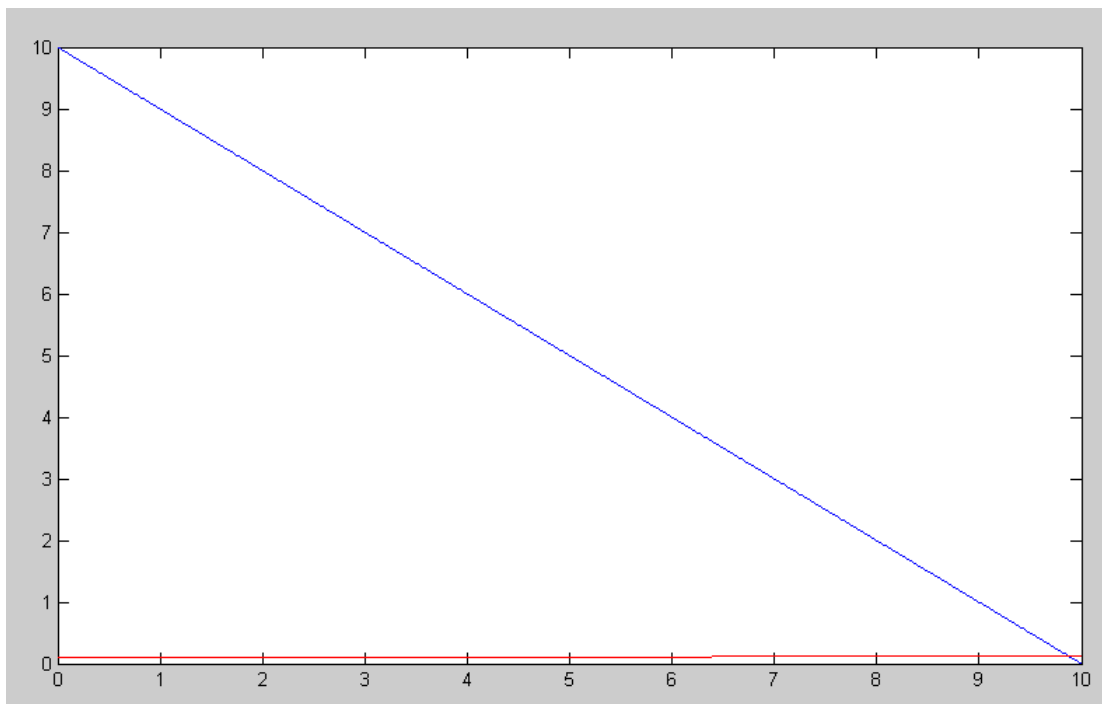
$(-1.2942, 0.6749)$, $(-1.1905, 0.4168)$, $(-0.3546, -0.8742)$, $(1.0283, 0.0567)$

4) Plot the two functions in Matlab and determine all solutions in the range of $0 < x < 10$

$$y = 10 - x$$

$$y = 0.1 e^{0.02x}$$

```
>> x = [-2:0.01:2]';  
>> y1 = 10 - x;  
>> y2 = 0.1*exp(0.02*x);  
>> plot(x,y1,'b',x,y2,'r');
```



There is only one solution

(9.8782, 0.1218)

Monte-Carlo Simulations:

Two teams, A and B, are playing a game. Team A has a 65% chance of winning any given game.

5) For Loops: Suppose the two teams play a 7-game match. The match winner is whoever has 4 wins or more. Determine the probability that team A will win the match.

hint: use a for-loop (for i=1:7) and count how many times team A wins during the 7-game match).

```
Matches = 0;

for n=1:1e6
    A = 0;
    for i=1:7
        if(rand < 0.65)
            A = A + 1;
        end
    end
    if(A >= 4)
        Matches = Matches + 1;
    end
end

Matches / 1e6

    0.800344
```

Team A has an 80.03% chance of winning the match.

6) While Loops: Suppose the two teams play until one team is up by 4 games. Determine the probability that team A will win the match.

hint: use a while-loop and keep looping until one team is up by 4 games.

```
>> Matches = 0;
Games = 0;
for n=1:1e6
    A = 0;
    N = 0;
    while(abs(A) < 4)
        N = N + 1;
        if(rand < 0.65) A = A + 1;
            else A = A - 1;
        end
    end

    if(A > 0)
        Matches = Matches + 1;
    end

    Games = max(Games, N);
end

Games
    112

Matches/1e6
    0.9224
```

Team A has a 92.24% chance of winning the match with this format.

The longest match in 1 million matches went 112 games

7) Gauss' Dilema: Play the following game 100 times. (i.e. use Matlab and a for loop along with a while loop)

- It costs \$25 to play. The pot starts at \$1.
- Flip a coin. If you get a heads, the pot doubles. If you get a tails, the game is over and you collect the money in the pot.
- Keep flipping until you get a tails.

How much money do you expect to win (or lose) each time you play this game?

```
>> Winnings = 0;
for n=1:100
    Pot = 1;
    while(rand < 0.5)
        Pot = Pot * 2;
    end
    Winnings = Winnings - 25 + Pot;
end
```

Winnings

-2026

After 100 games, I'm down \$2,026

Dice:

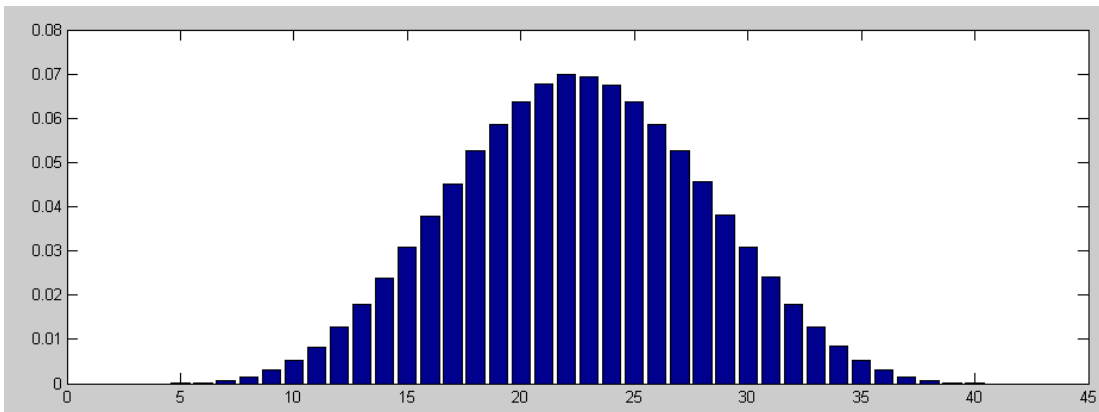
8a) Determine the probability distribution for the following:

- Roll a 4-sided die, a 6-sided die, an 8-sided die, a 10-sided die, and a 12-sided die.
- The total is the sum of all of the dice.

$$Y = d4 + d6 + d8 + d10 + d12$$

```
result = zeros(40,1);
for i=1:1e6
    d4 = ceil(4*rand);
    d6 = ceil(6*rand);
    d8 = ceil(8*rand);
    d10 = ceil(10*rand);
    d12 = ceil(12*rand);

    Y = d4 + d6 + d8 + d10 + d12;
    result(Y) = result(Y) + 1;
end
bar(result/1e6)
```



```
sum(result(30:40)) / 1e6
```

8b) What is the probability of the total being 30?

ans: 3.08% chance

```
result(30) / 1e6
ans = 0.0308
```

8c) What is the probability of the total being 30 or more?

```
sum(result(30:40)) / 1e6
ans = 0.1046
```

ans: 10.46% chance

9) Determine whether you should bet on Player A or Player B:

- Player A rolls five dice and takes the total (d4 + d6 + d8 + d10 + d12)
- Player B rolls two 100-sided dice and takes the lower of the two numbers.
- Whoever has the highest score wins.

Matlab Code:

```
W = 0;
T = 0;
L = 0;

for i=1:1e6
    d4 = ceil(4*rand);
    d6 = ceil(6*rand);
    d8 = ceil(8*rand);
    d10 = ceil(10*rand);
    d12 = ceil(12*rand);

    A = d4 + d6 + d8 + d10 + d12;

    d100 = ceil(100*rand(2,1));
    B = min(d100);

    if(A > B) W = W + 1;
        elseif (A == B) T = T + 1;
            else L = L + 1;
        end

end

[W,T,L] / 1e6

ans =
    0.3802    0.0154    0.6044
```

A wins 38.02% of the time

It's a tie 1.54% of the time

B wins 60.44% of the time

You should bet on B.