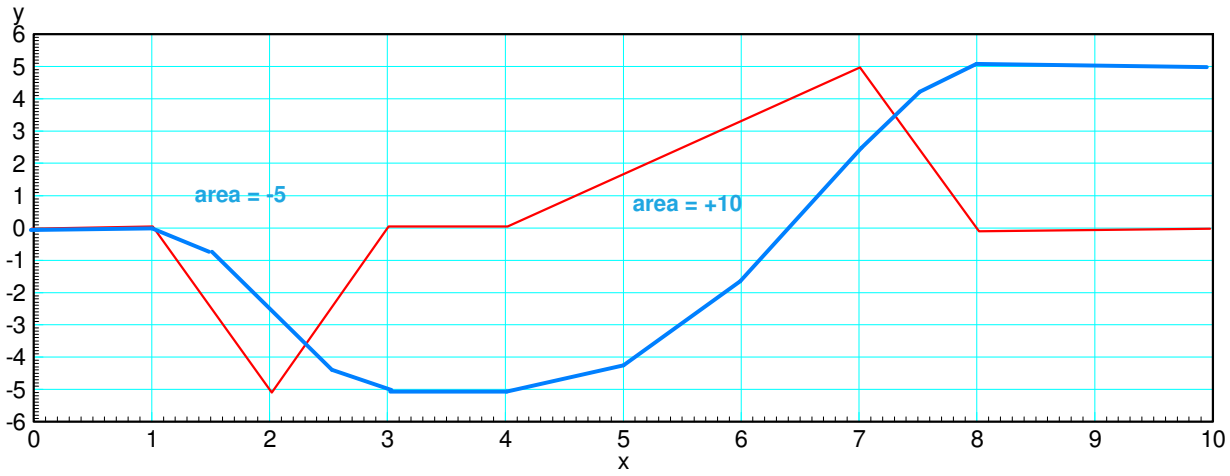
ECE 111 - Homework #7

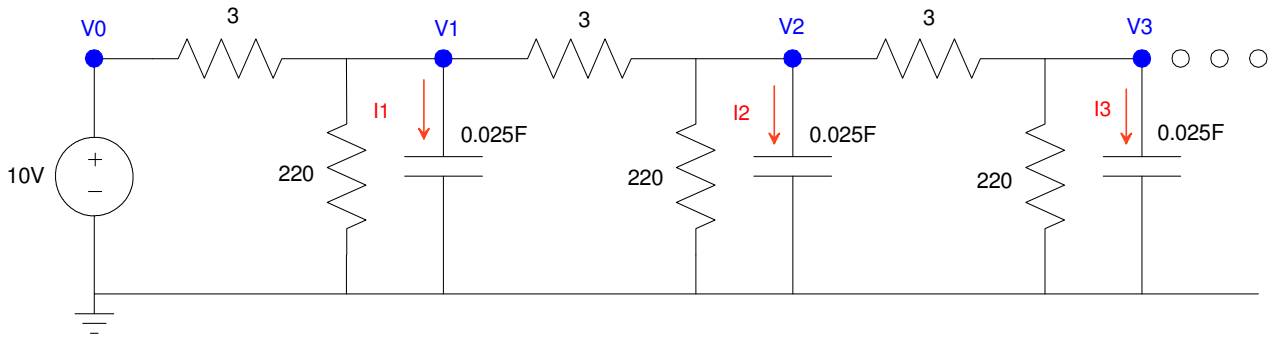
Week #7: ECE 311 Circuits II

1) Assume the current flowing through a one Farad capacitor is shown below. Sketch the voltage. Assume V(0) = 0. The voltage is the integral of the current (capacitors are integrators)

$$V = \frac{1}{C}\int I \cdot dt$$



Problem 2-5: Assume a 10-stage RC filter (V0 .. V10)



Problem 2) Write the dynamics for this system as a set of ten coupled differential equations:

$$I_1 = C\frac{dV_1}{dt} = \sum(\text{current to node}V_1)$$

$$I_1 = 0.025V_1' = \left(\frac{V_0 - V_1}{3}\right) + \left(\frac{V_2 - V_1}{3}\right) + \left(\frac{0 - V_1}{220}\right)$$

$$V_1' = 13.33V_0 - 26.85V_1 + 13.33V_2$$

ditto for nodes 2..9

$$V_2' = 13.33V_1 - 26.85V_2 + 13.33V_3$$

$$\vdots$$

$$V_9' = 13.33V_8 - 26.85V_9 + 13.33V_{10}$$

Node 10 is slightly different since there is only one 3 ohm resistor connected

$$I_{10} = 0.025V_{10}' = \left(\frac{V_9 - V_{10}}{3}\right) + \left(\frac{0 - V_{10}}{220}\right)$$

$$V_{10}' = 13.33V_9 - 13.52V_{10}$$

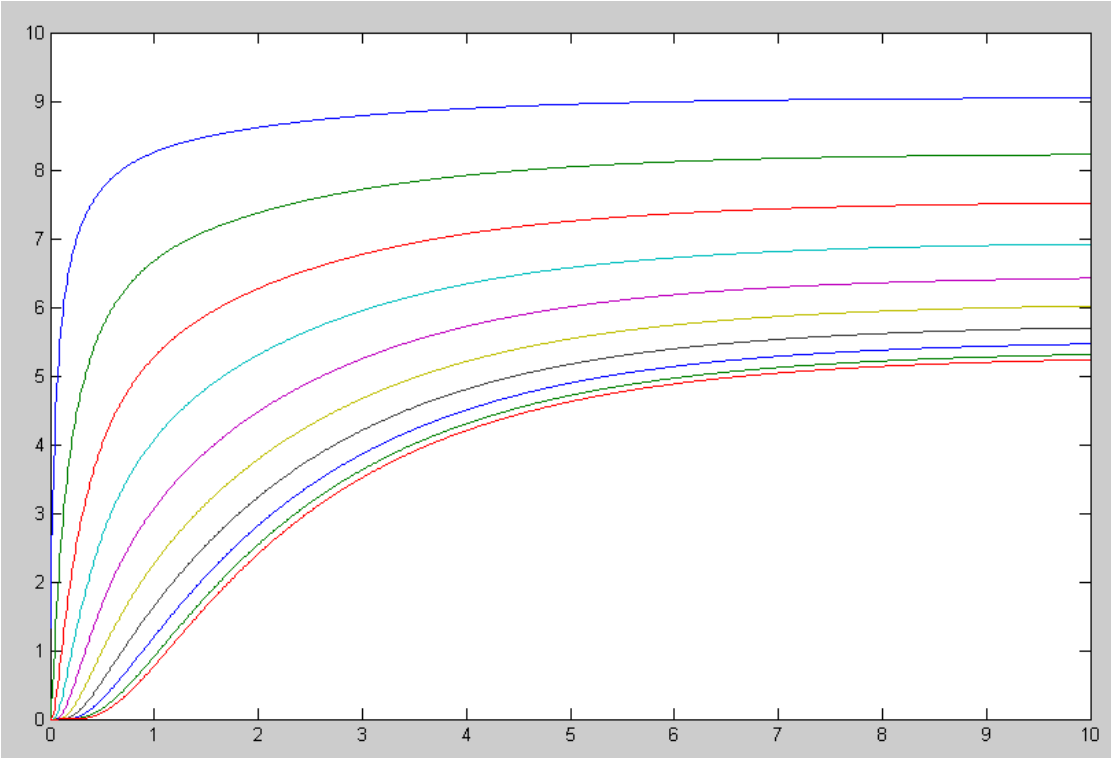
Forced Response for a 10-Node RC Filter (heat.m):

Problem 3) Using Matlab, solve these ten differential equations for $0 < t < 20$ s assuming

- The initial voltages are zero, and
- V0 = 10V.

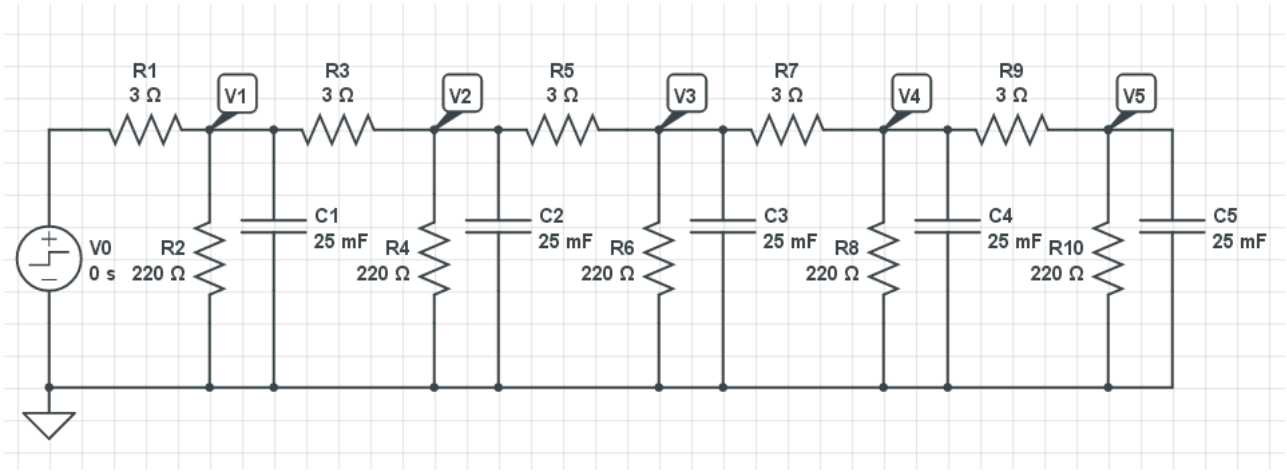
Code:

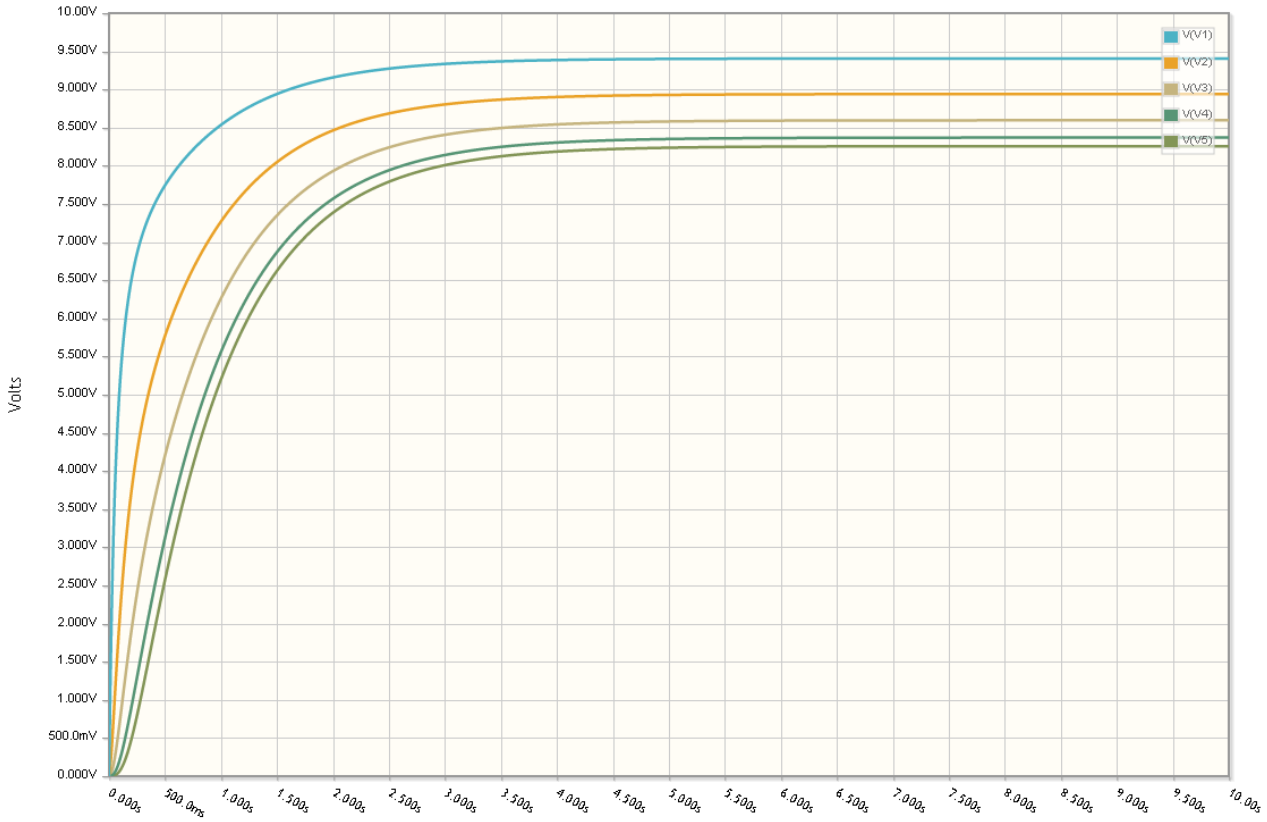
```
% 10-stage RC Filter

V = zeros(10,1);
dV = zeros(10,1);
V0 = 10;
dt = 0.01;
t = 0;

y = [];

while(t < 10)

   dV(1)  = 13.33*V0   - 26.85*V(1) + 13.33*V(2);
   dV(2)  = 13.33*V(1) - 26.85*V(2) + 13.33*V(3);
   dV(3)  = 13.33*V(2) - 26.85*V(3) + 13.33*V(4);
   dV(4)  = 13.33*V(3) - 26.85*V(4) + 13.33*V(5);
   dV(5)  = 13.33*V(4) - 26.85*V(5) + 13.33*V(6);
   dV(6)  = 13.33*V(5) - 26.85*V(6) + 13.33*V(7);
   dV(7)  = 13.33*V(6) - 26.85*V(7) + 13.33*V(8);
   dV(8)  = 13.33*V(7) - 26.85*V(8) + 13.33*V(9);
   dV(9)  = 13.33*V(8) - 26.85*V(9) + 13.33*V(10);
   dV(10) = 13.33*V(9) - 13.52*V(10);

   V = V +  dV*dt;
   t = t + dt;

   y = [y ; V'];

   plot([0:10], [V0;V], '.-');
   ylim([0,10]);
   clc
   disp(t)
   pause(0.01);

   end

hold off
t = [0:length(y)-1]' * dt;
plot(t,y);
```



Step Response (voltage vs. time for V1 .. V10)

Problem 4) Using CircuitLab, find the response of this circuit to a 10V step input. *note: It's OK if you only build this circuit to 3 nodes...*





Natural Response

Problem 5) Assume V0 = 0V. Determine the initial conditions of V1..V10 so that

- The maximum voltage is 10V and
- 5a) The voltages go to zero as slow as possible
- 5b) The voltages go to zero as fast as possible.

Simulate the response for these initial conditions in Matlab.

This is an eigenvector problem. To find the eigenvectors, input the dynamics in matrix form

$$V' = AV$$

and find the eigenvalues and eigenvectors. In Matlab:

```
>> A = zeros(10,10);
>> for i=1:9
  A(i,i) = -26.85;
  A(i,i+1) = 13.33;
  A(i+1,i) = 13.33;
end
>> A(10,10) = -13.52;
>> A
```

```
  -26.8500   13.3300         0         0         0         0         0         0         0         0
   13.3300  -26.8500   13.3300         0         0         0         0         0         0         0
         0   13.3300  -26.8500   13.3300         0         0         0         0         0         0
         0         0   13.3300  -26.8500   13.3300         0         0         0         0         0
         0         0         0   13.3300  -26.8500   13.3300         0         0         0         0
         0         0         0         0   13.3300  -26.8500   13.3300         0         0         0
         0         0         0         0         0   13.3300  -26.8500   13.3300         0         0
         0         0         0         0         0         0   13.3300  -26.8500   13.3300         0
         0         0         0         0         0         0         0   13.3300  -26.8500   13.3300
         0         0         0         0         0         0         0         0   13.3300  -13.5200
```

```
>> [M,V] = eig(A)
```

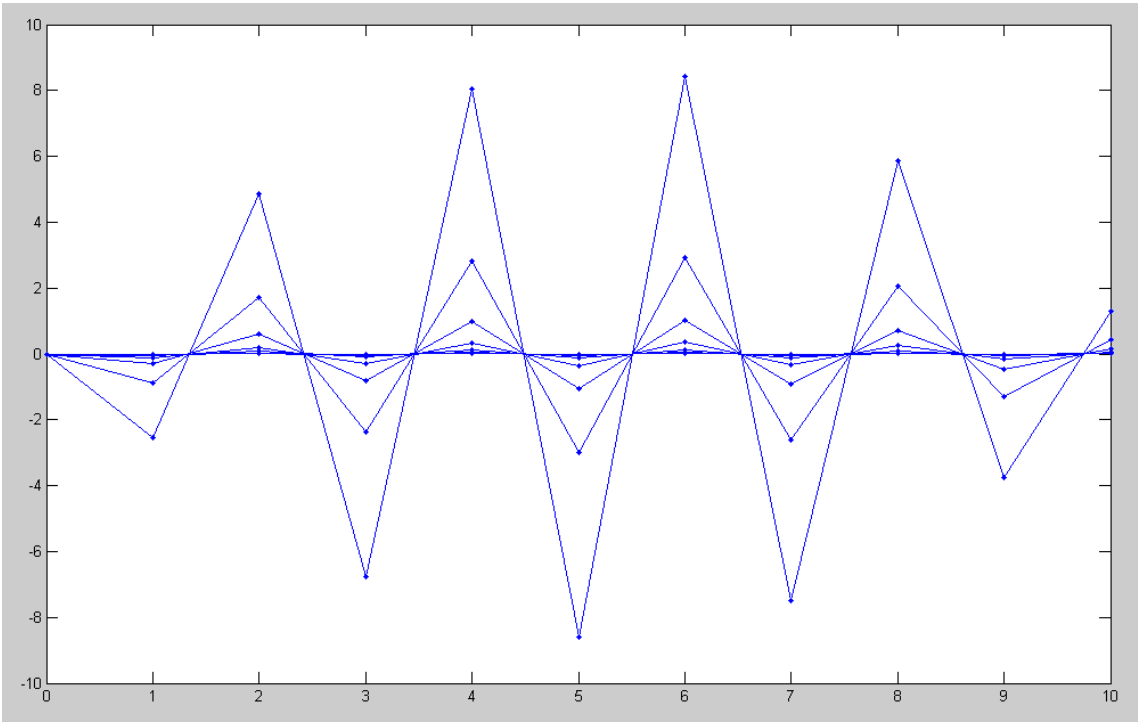
fast									slow
-0.1286	-0.2459	0.3412	0.4063	0.4352	0.4255	0.3780	0.2969	-0.1894	**0.0650**
0.2459	0.4063	-0.4255	-0.2969	-0.0650	0.1894	0.3780	0.4352	-0.3412	**0.1286**
-0.3412	-0.4255	0.1894	-0.1894	-0.4255	-0.3412	0.0000	0.3412	-0.4255	**0.1894**
0.4063	0.2969	0.1894	0.4352	0.1286	-0.3412	-0.3780	0.0650	-0.4255	**0.2459**
-0.4352	-0.0650	-0.4255	-0.1286	0.4063	0.1894	-0.3780	-0.2459	-0.3412	**0.2969**
0.4255	-0.1894	0.3412	-0.3412	-0.1894	0.4255	-0.0000	-0.4255	-0.1894	**0.3412**
-0.3780	0.3780	-0.0000	0.3780	-0.3780	0.0000	0.3780	-0.3780	-0.0000	**0.3780**
0.2969	-0.4352	-0.3412	0.0650	0.2459	-0.4255	0.3780	-0.1286	0.1894	**0.4063**
-0.1894	0.3412	0.4255	-0.4255	0.3412	-0.1894	0.0000	0.1894	0.3412	**0.4255**
0.0650	-0.1286	-0.1894	0.2459	-0.2969	0.3412	-0.3780	0.4063	0.4255	**0.4352**

-52.3256	-48.8775	-43.4722	-36.5900	-28.8423	-20.9176	-13.5200	-7.3068	-2.8302	**-0.4878**

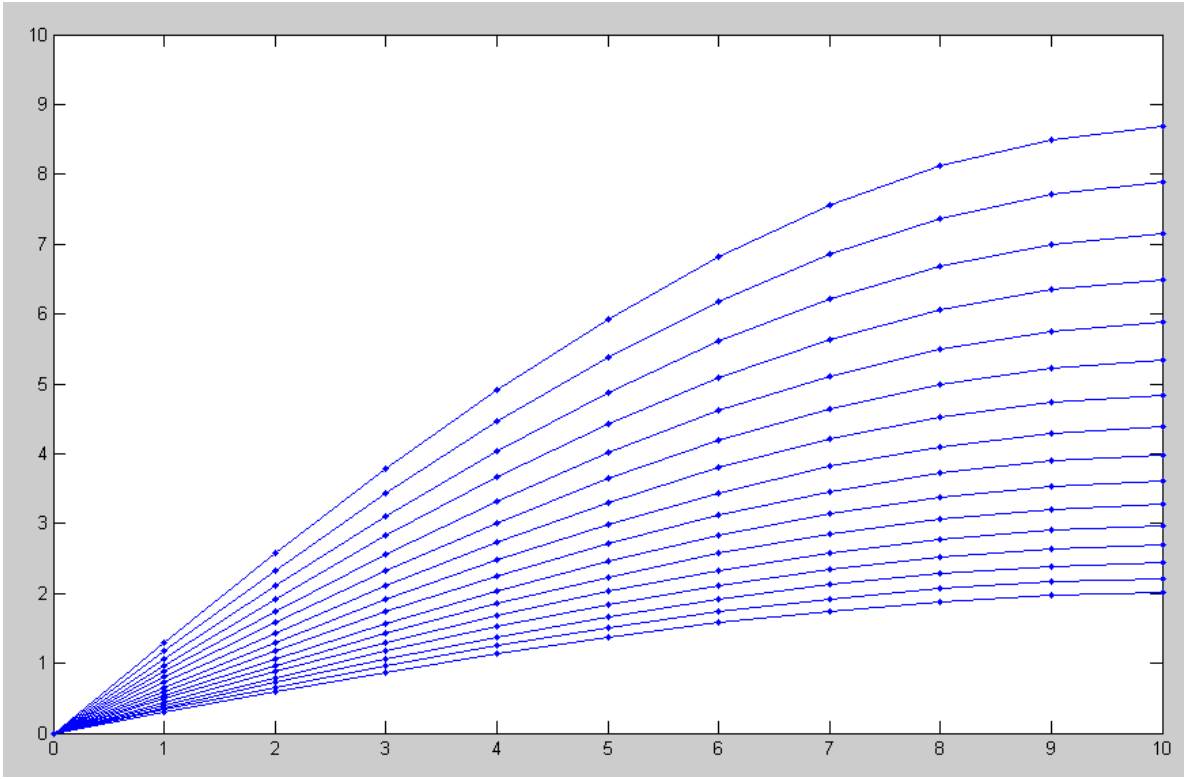
What this means is...

- The fast mode (blue) decays as exp(-52.3256t).
- If you make the initial condition proportional to the fast eigenvector (in blue), the voltages decay quickly.

- The slow mode (red) decays as exp(-0.4878t)
- If you make the initial condition proportional to the slow eigenvector (in red), the voltages decay slowly



Initial Condition = The Fast Eigenvector: Voltages plotted every 20ms



Initial Condition = The Slow Eigenvector: Voltages plotted every 200ms

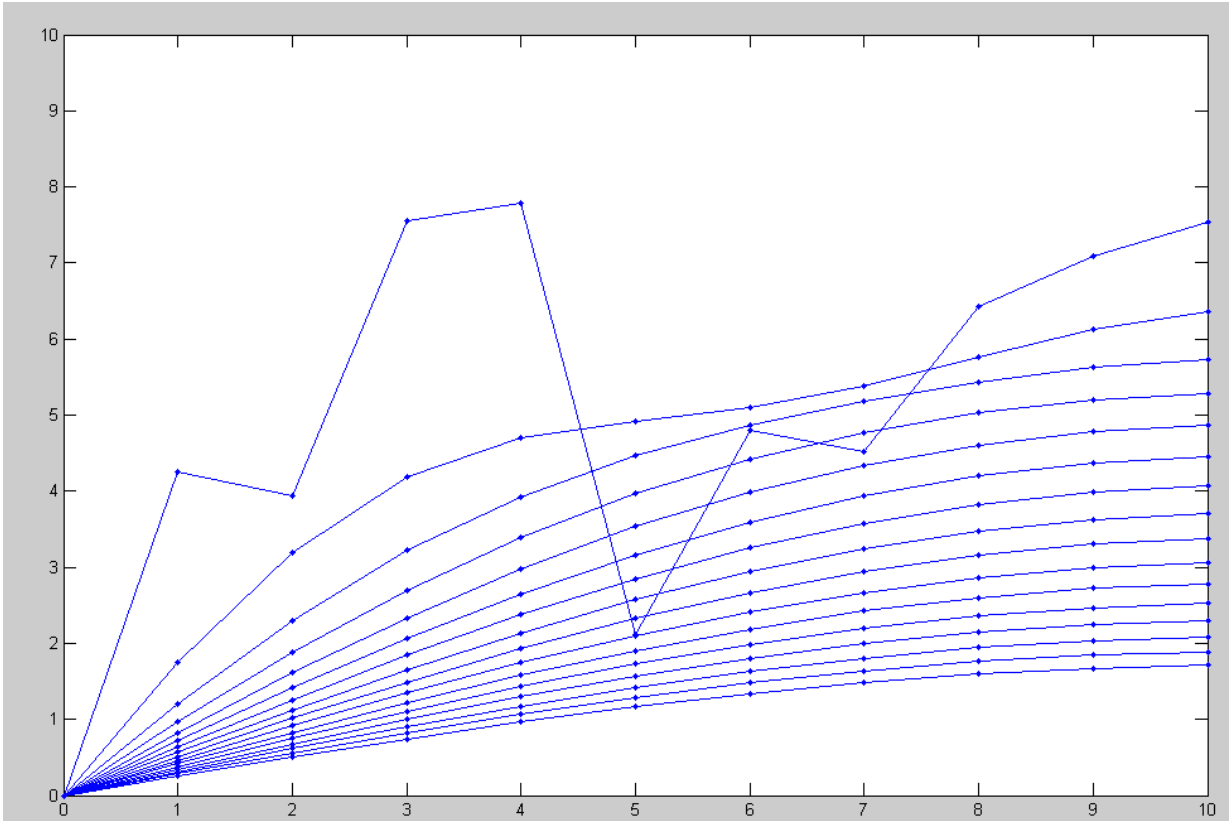
Code:

```
% 10-stage RC Filter

V = [
   -2.5728
    4.9171
   -6.8244
    8.1253
   -8.7043
    8.5099
   -7.5593
    5.9370
   -3.7872
    1.3009];

dV = zeros(10,1);
V0 = 0;
dt = 0.0002;
t = 0;

y = [];
n = 0;


while(t < 0.3)

   dV(1)  = 13.33*V0   - 26.85*V(1) + 13.33*V(2);
   dV(2)  = 13.33*V(1) - 26.85*V(2) + 13.33*V(3);
   dV(3)  = 13.33*V(2) - 26.85*V(3) + 13.33*V(4);
   dV(4)  = 13.33*V(3) - 26.85*V(4) + 13.33*V(5);
   dV(5)  = 13.33*V(4) - 26.85*V(5) + 13.33*V(6);
   dV(6)  = 13.33*V(5) - 26.85*V(6) + 13.33*V(7);
   dV(7)  = 13.33*V(6) - 26.85*V(7) + 13.33*V(8);
   dV(8)  = 13.33*V(7) - 26.85*V(8) + 13.33*V(9);
   dV(9)  = 13.33*V(8) - 26.85*V(9) + 13.33*V(10);
   dV(10) = 13.33*V(9) - 13.52*V(10);

   V = V +  dV*dt;
   t = t + dt;

   y = [y ; V'];

   n = mod(n+1, 100);
   if(n == 1)
      plot([0:10], [V0;V], '.-');
      ylim([-10,10]);
      hold on
      end

   clc
   disp(t)
   pause(0.01);

   end
```

Problem 6) Assume Vin = 0V. Pick random voltages for V1 .. V10 in the range of (0V, 10V):

```
V = 10 * rand(10,1)
```

Plot the votlages at t = 1. Which eigenvector does it look like?



Voltages plotted every 200ms with a random initial condition

Comment:

- A random initial condition contains all 10 eigenvectors
- All 10 modes have a non-zero initial condition
- The fast modes decay quickly
- Leaving the slow (dominant) eigenvector