15: Biomedical Engineering

Modeling the cardiovascular System

Objective

- Model the cardiovascular system as an RLC circuit
- From the model, predict what happens if different parameters change

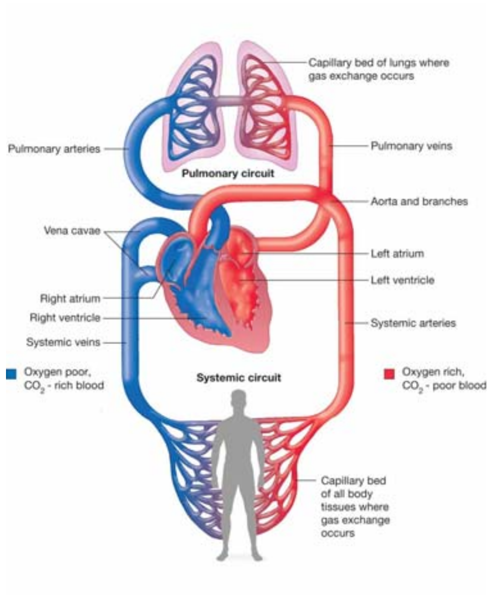
Matlab Functions

- Matlab Scripts
- Nonlinear simulations
- clf
- subplot

Electrical Model of the cardiovascular System

The cardiovascular system consists of to circulatory systems:

- The right ventricle drives the low-pressure system, pumping blood to the lungs, and
- The left ventricle drives the high-pressure system, pumping blood to the rest of the body



http://www.lookfordiagnosis.com/

In essence, the heart acts as a 5W source. Most of the work and medical problems are associated with the high-pressure side (which requires most of the energy). One problem, likewise, is how to model the left ventricle and the high-pressure cardiovascular system. If you can model it, then

- You have a better understanding of how the system works,
- You can build more accurate models of the body for testing of artificial hearts
- You can ask 'what if' questions, such as what happens if the capacitance increases, and predict the result.

- This in turn opens new directions for drug treatment: finding drugs which affect the different parameters of the model

In this lecture, we will look at two models of the high-pressure side of the cardiovascular system:

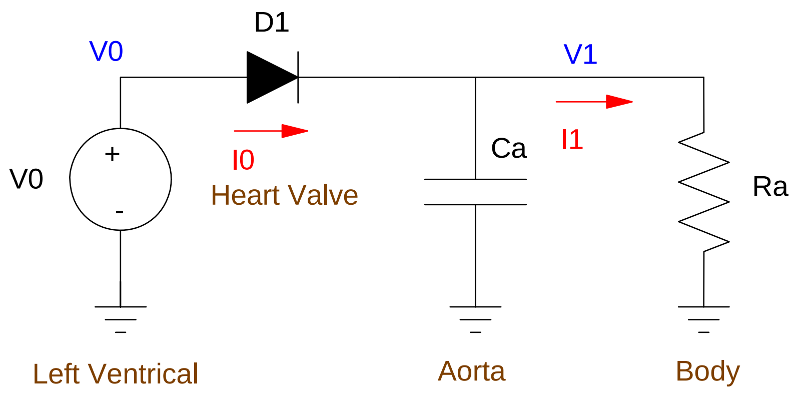
- A 2-element Winkessel model (i.e. an RC circuit), and
- A more complex 5-element model (an RLC circuit)

Heart Parameters:

To develop a circuit equivalent, we need to convert fluid parameters to electrical parameters. What really matters is the differential equation which describes the system: if a circuit satisfies the same differential equation, it behaves the same way. It makes no difference if that differential equation derives from a fluid system, a thermal system, or an electrical network.

Parameter	English Units	Metric Units	Circuit Dual
Pressure	89mm Hg	10.67 N/m2	10.67V
Volume (LV)		70ml	
Volume (Aorta)		500ml	
Inertia (Aorta)			
Normal Flow		70 ml/s	70mA
Blood Inertia		0.07kg	0.07H

2-element Winkessel Model



2-Element Winkessel Model: The left ventricle is modeled as a voltage source (pressure source). The body is modeled as a capacitor (for the Aorta) and a resistor (the capillaries the arteries feed).

First, determine reasonable parameters for the model.

V0: A typical blood pressure reading is 120/80. This means the left ventricle should output a peak pressure of approximately 120mmHg (16V - 7.5mmHg = 1 N/m2 = 1V). Model V0 as a half-rectified sine wave with 60 bpm (one beat per second):

$$V_0 = \max(0, 16\sin(2\pi t))$$

D1: The heart valve allows blood to flow from V0 to V1 but not backwards. Model this as a variable resistor:

$$R_{d1} = \begin{cases} 1\Omega & V_0 > V_1 \\ 1M\Omega & V_0 < V_1 \end{cases}$$

Ra: Nominal flow is 70ml/s on average (70mA) for an average voltage of 13.3V

$$R_a = \frac{13.3V}{70mA} = 190\Omega$$

The capacitance required to achieve a pressure range of 120/80 mmHg (26V .. 10.6V) is

$$I = C_a \frac{dV}{dt}$$

$$0.07A = C_a \frac{26V - 10.6V}{1 \text{ sec}}$$

$$C_a = 4.54mF$$

This results in the following dynamic equations for the heart:

$$I_c = C_a \frac{dV_1}{dt} = I_{d1} - \frac{V_1}{R_a}$$

Matlab Code: The following code runs for 3 seconds with data collected every 1ms. Because of the rapid changes when the valve opens, a simulation step-size of 0.01ms is used (dt).

- The outer loop (i) counts how many milliseconds have elapsed.
- The inner loop (j) runs at a smaller step size (0.01ms)

Data is collected in the outer loop to keep the number of data points reasonable.

To speed up the simulation, the data arrays are set up initially. This reserves space in memory, which is filled in as the simulation runs. This executes faster than the method of saying

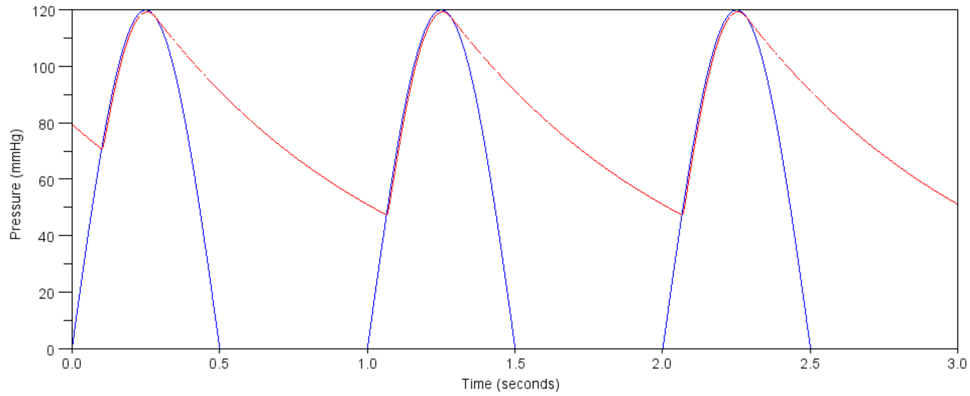
```
Data = [Data ; [V0, V1] ];
```

What this does in Matlab, is each iteration it creates a new matrix (taking time) and overwrites the old one. As written, it takes about five seconds for this simulation to run.

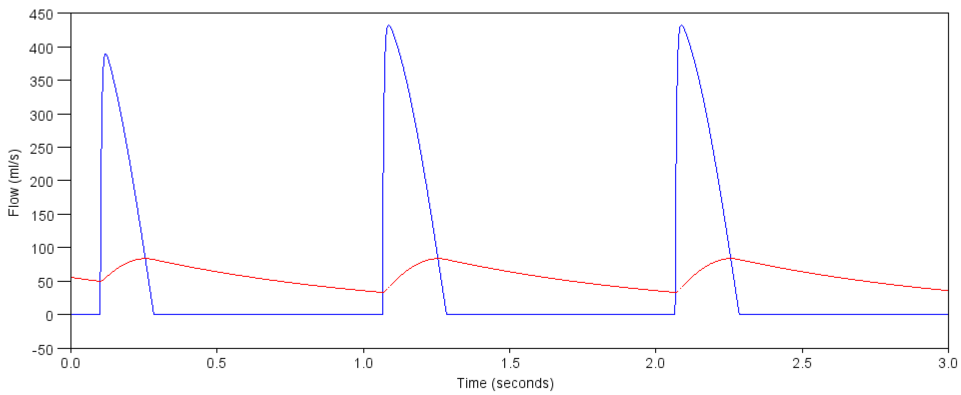
Matlab Code:

```
// 2-element Winkessel Model

V1 = 10.6;                       // initial conditions
Ca = 4.54e-3;
Ra = 190;

Data = zeros(3000,4);
t0 = zeros(3000,1);

t = 0;
dt = 1e-5;
npt = 0.001/dt;

for i=1:3000

   for j=1:npt

      V0 = max(0, 16*sin(6.28*t));

      if (V0 > V1)  Rd1 = 1;
               else Rd1 = 1e5;
               end

      I0 = (V0 - V1)/Rd1;
      I1 = V1 / Ra;

      dV1 = (I0 - I1)/Ca;

      t = t + dt;
      V1 = V1 + dV1*dt;

      end

    t0(i) = t;
    Data(i,:) = [I0*1000, I1*1000, V0, V1];


   end

clf

subplot(211)
plot(t0, Data(:,[3,4])*7.5);
xlabel('Time (seconds)');
ylabel('Pressure (mmHg)');

subplot(212)
plot(t0, Data(:,[1,2]));
xlabel('Time (seconds)');
ylabel('Flow (ml/s)');
```

The results for three beats are:



Left Ventricle Pressure (V0: blue) and Arterial Pressure (V1: red)



Left Ventricle Flow (I0: blue) and Arterial Flow (I1: red)

Note that the model gives reasonable results:

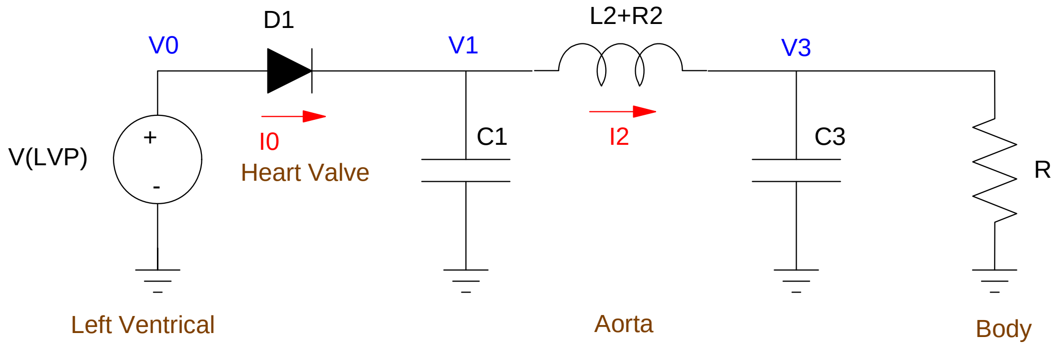
- Aortic pressure peaks at 120mmHg and drops as expected.
- The mean arterial flow is 70ml/s (70mA) as desired,

The lower pressure is a bit low, however (60mmHg). You can tweak the model to fix this by increasing the size of Ca (left as a homework problem).

Improved Winkessel Model

A slightly more accurate model splits the Aorta into two sections:

- C1 accepts blood from the heart and charges up (expands) to accept this blood each heart beat
- L2 is the inertia of the blood in this section of the aorta,
- C3 is the rest of the arterial system which pushes blood throughout the body, and
- R models the capillaries which are fed from the arterial system, drawing 70ml/s (70mA) on average



5-Element Winkessel Model

Computations for the parameters are as follows:

C3: $I = C_3 \frac{dV}{dt}$

$$70mA = C_3 \frac{40mmHg}{0.8s} = C_3 \frac{5.33V}{0.8s}$$

$$C_3 = 10.5mF$$

C1: The heart volume is 70ml, compared to the Aorta (500ml), makes

$$C_1 = \left(\frac{70ml}{500ml}\right) \cdot 10.5mF$$

$$C_1 = 1.5mF$$

L2: 70ml = 0.07kg = 0.07H + 6 Ohms. The 6 Ohms is added to stabilize the simulation. Without it, an LC circuit rings. (the 6 Ohms is somewhat of a guess)

R: $R = 190\Omega$ (same as before)

V0: Model the left ventricle as a voltage source (pressure source) so that the peak of V1 is 120mmHg

$$V_0 = \max(0, 17.6\sin(2\pi t))$$

D1: Model the valve as a variable resistor: low resistance for forward flow, high for reverse flow:

$$R_{d1} = \begin{cases} 1\Omega & V_0 > V_1 \\ 1M\Omega & V_0 < V_1 \end{cases}$$

Dynamics equations:

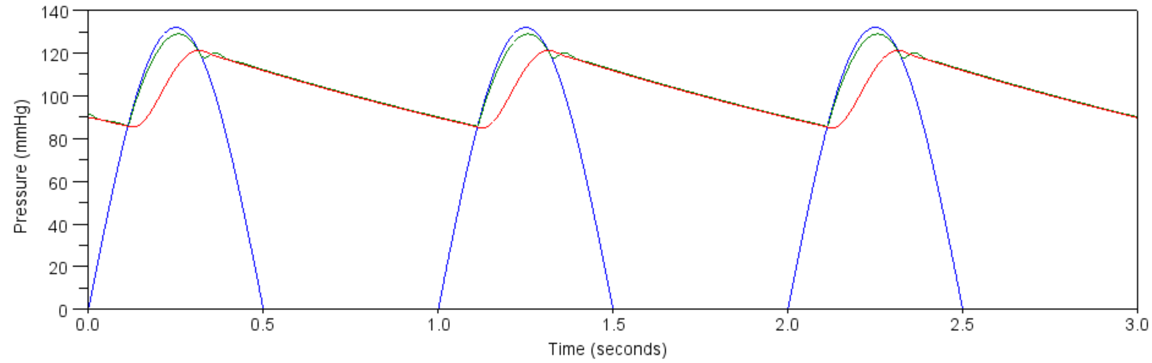
$$I_0 = \frac{V_0 - V_1}{R_{d1}}$$

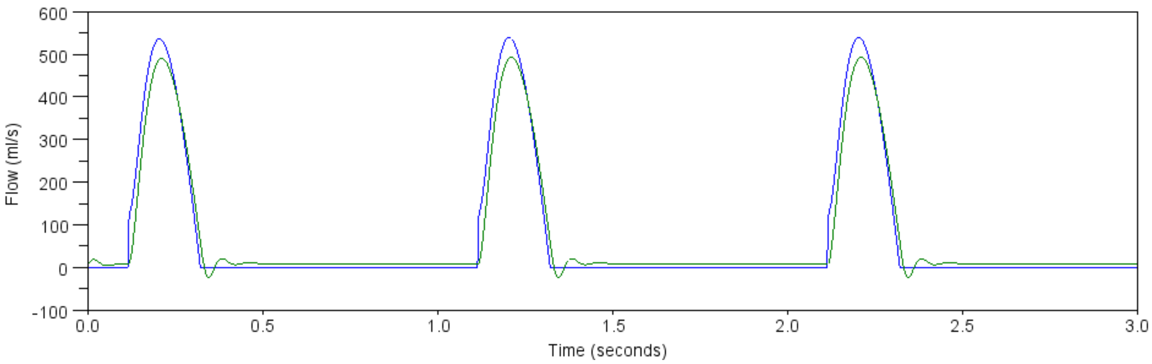
$$C_1 \frac{dV_1}{dt} = I_0 - I_2$$

$$L_2 \frac{dI_2}{dt} = V_1 - I_2 R_2 - V_3$$

$$C_3 \frac{dV_3}{dt} = I_2 - \frac{V_3}{R}$$

Simulation Results: After tweaking the pressure so that V1 peaks at 120mmHg, the following results (code follows)





What this model shows you is:

- The aortic pressure (top graph red) is smoother. This agrees with what you see better.
- At the left ventricle (top green), there is a dip in pressure when the valve closes. This is caused by the pressure wave reflecting back in time to help the valve close (something you also see in a healthy person)
- The arterial flow (I2: bottom green) also has a negative dip when the valve closes. This is the inertia of the blood providing suction, helping the heart to eject more blood.

With this model you can start to answer questions, such as

- What is the effect of Angina (the aorta expands)
- What is the effect of hardening of the arteries?
- What is the effect of obstructed flow?
- How does the body respond to the above to maintain blood flow?

These are left for homework sets and for Cardiovascular Engineering as well as more accurate (and complex) models.

```
// 5-element Winkessel Model

V1 = 12.2;
I2 = 0.0078;
V3 = 12.0;

C1 =   1.5e-3;
L2 =   0.07;
C3 =  10.5e-3;
R2 = 6;

R  = 190;

Data = zeros(3000,5);
t0 = zeros(3000,1);

t = 0;
dt = 1e-5;
npt = 0.001/dt;

for i=1:3000

   for j=1:npt

      V0 = max(0, 17.6*sin(6.28*t));

      if (V0 > V1)  Rd1 = 1;
               else Rd1 = 1e5;
               end

      I0 = (V0 - V1)/Rd1;

      dV1 = (I0 - I2)/C1;
      dI2 = (V1 - R2*I2 - V3)/L2;
      dV3 = (I2 - V3/R)/C3;

      t = t + dt;
      V1 = V1 + dV1*dt;
      I2 = I2 + dI2*dt;
      V3 = V3 + dV3*dt;

      end

    t0(i) = t;
    Data(i,:) = [I0*1000, V0, V1, I2*1000, V3];

   end

clf
subplot(211);

plot(t0, Data(:,[2,3,5])*7.5);
xlabel('Time (seconds)');
ylabel('Pressure (mmHg)');
subplot(212);

plot(t0,Data(:,[1,4]))
xlabel('Time (seconds)');
ylabel('Flow (ml/s)');
```