Circuits I

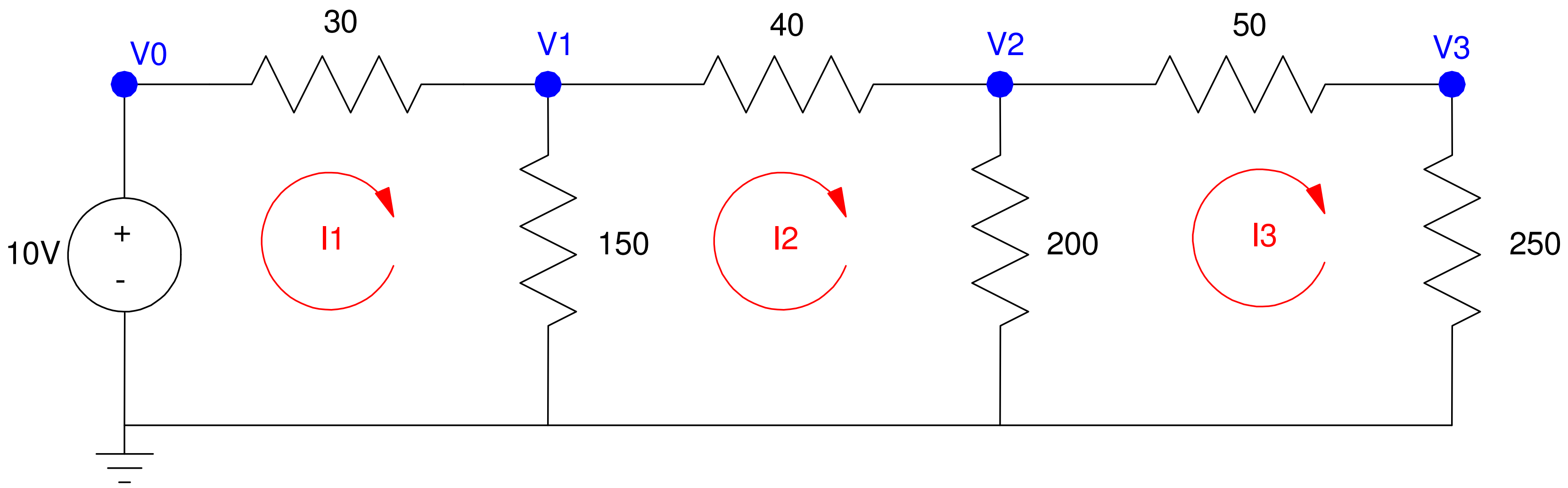
ECE 111 Introduction to ECE

Jake Glower - Week #9

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Circuits I Concepts

- Kirchoff's Current Loops (KCL)
- Kirchoff's Voltage Nodes (KVN)
- Writing N equations for N unknowns
- Solving N equations for N unknowns
- Solving circuits using CircuitLab (www.CircuitLab.com)



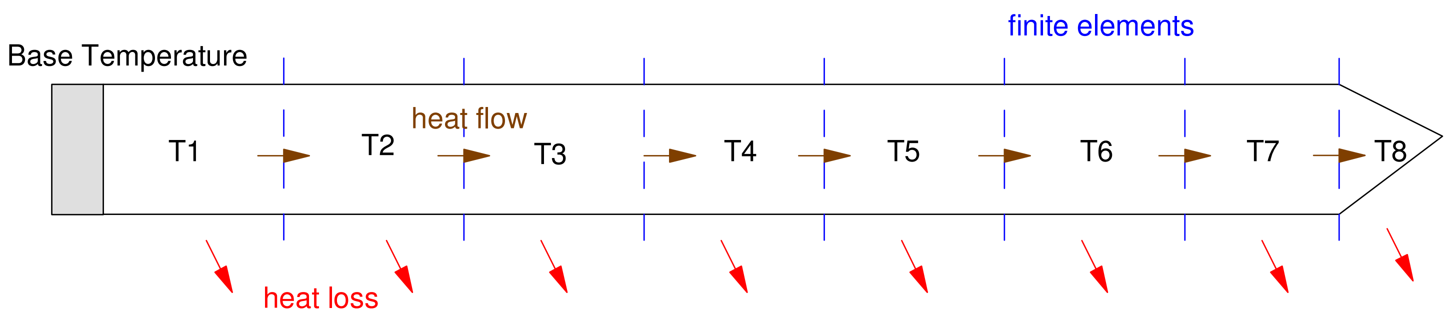
Resistor Networks & Steady-State Heat Flow

- Resistor circuits satisfy the same equations as heat flow
- Both problems can be solve by writing N equations to solve for N unknowns

Example: Temperature along the length or a metal rod

- Split into N finite elements
- Find the temperature at each element so that heat flow balances

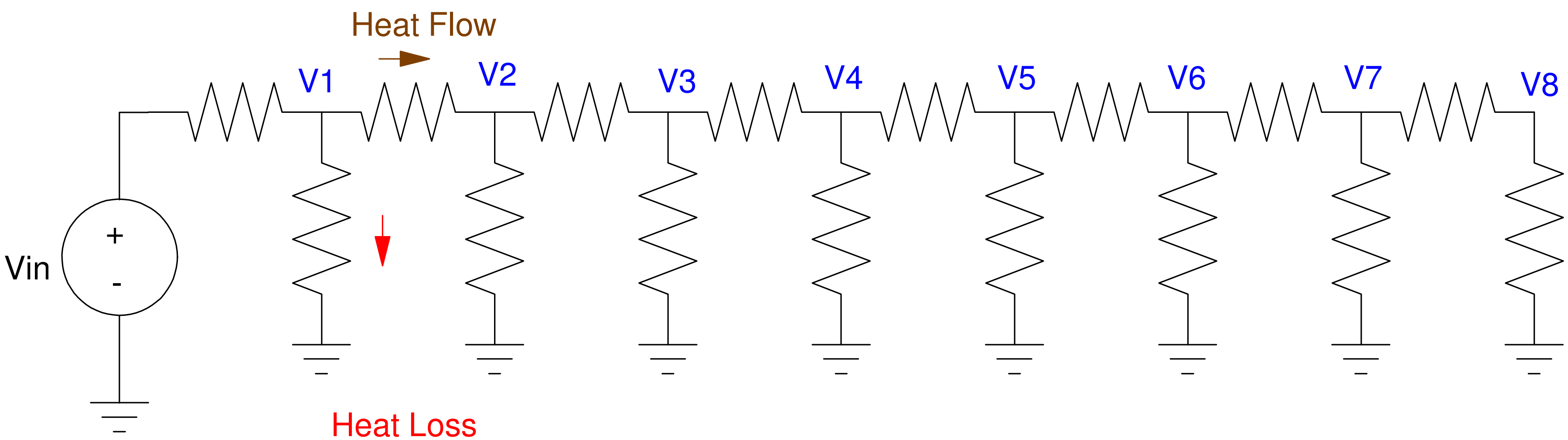
  Heat In = Heat Out



One-dimensional heat flow. Heat flows left to right - with heat loss at each element

Circuit Equivalent:

- Voltage = Temperature
- Current = Heat Flow
- Resistors in Series: Thermal conductivity between nodes
- Resistor to Ground: Heat loss at each node.



Circuit model for 1-dimensional heat flow: voltages represent temperature, current represents heat flow

Problems:

- How do you solve for the voltages in the above circuit?

  How do you solve for the temperatures at each point along a 1-dimensional metal rod?

- How do you solve for the currents in the above circuit

  How do you solve for the heat flow along the length of a metal rod?

If you can solve for the voltages and/or currents, you can also solve for temperatures and heat flow

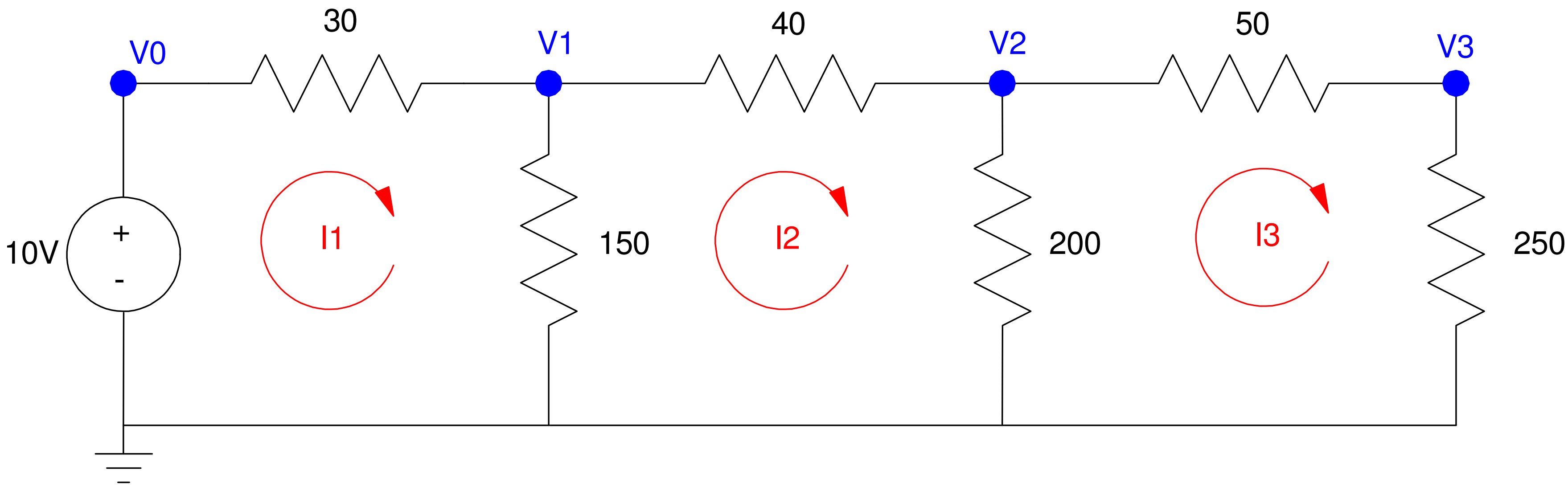
Methods Covered in Circuits I

- Analysis: You can solve N equations for N unknowns where N is the number of voltages or currents you're trying to find,
- Simulation: You can input this circuit into program like CircuitLab and have it compute the voltages and currents
- Experiment: You can go into the lab, build the circuit, and take measurements.

In this class, we'll just cover the first two methods.

Circuit Example:

- Finding the voltages, and
- Finding the currents



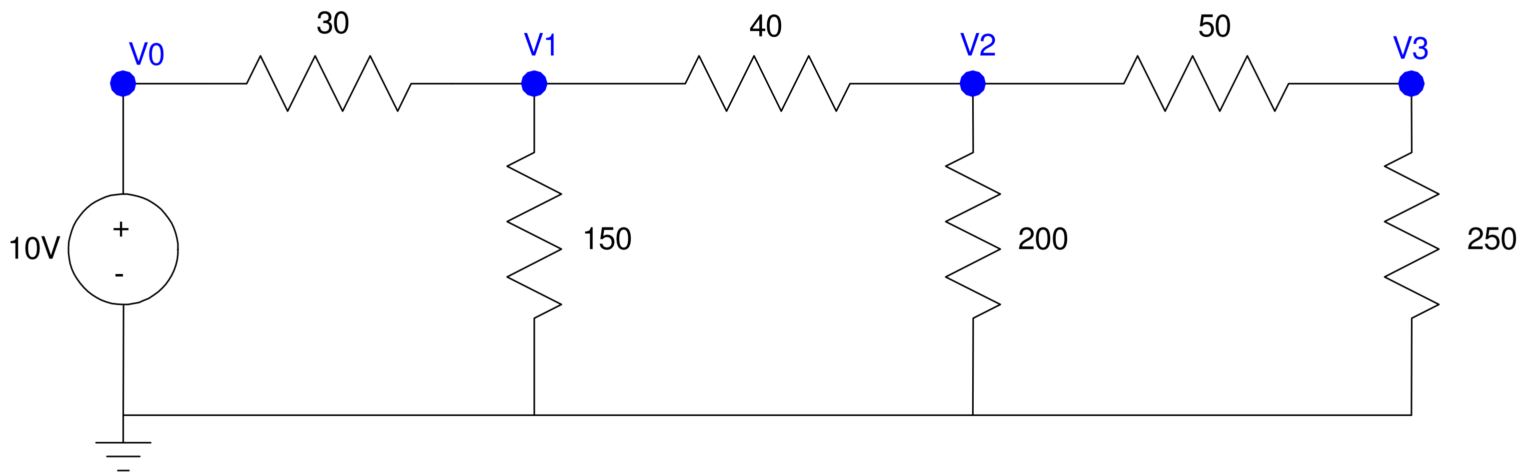
Circuit Problem: Determine the voltages and/or currents

Method #1: Voltage Nodes (KVN)

- At equilibrium, the current going into each node must match the current leaving the node.
- The voltages at each node are whatever it takes to produce equilibrium (current in = current out)

Step 1: Define one node to be the ground reference.

- Then, define and label the nodes of the circuit.



Step #1 for KVN: Define circuit ground and label each voltage node.

Step 2: Write N equations to solve for N unknown voltages.

Node V0:

$$V_0 = 10$$

At node V1:

$$I_{a1} + I_{b1} + I_{c1} = 0$$

$$\left(\frac{V_1 - V_0}{30}\right) + \left(\frac{V_1}{150}\right) + \left(\frac{V_1 - V_2}{40}\right) = 0$$

At node V2:

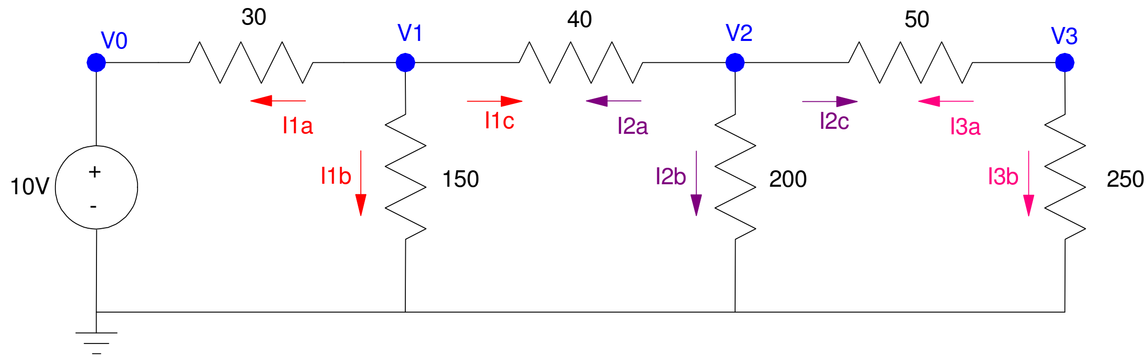
$$I_{a2} + I_{b2} + I_{c2} = 0$$

$$\left(\frac{V_2 - V_1}{40}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{50}\right) = 0$$

At node V3:

$$I_{a3} + I_{b3} = 0$$

$$\left(\frac{V_3 - V_2}{50}\right) + \left(\frac{V_3}{250}\right) = 0$$



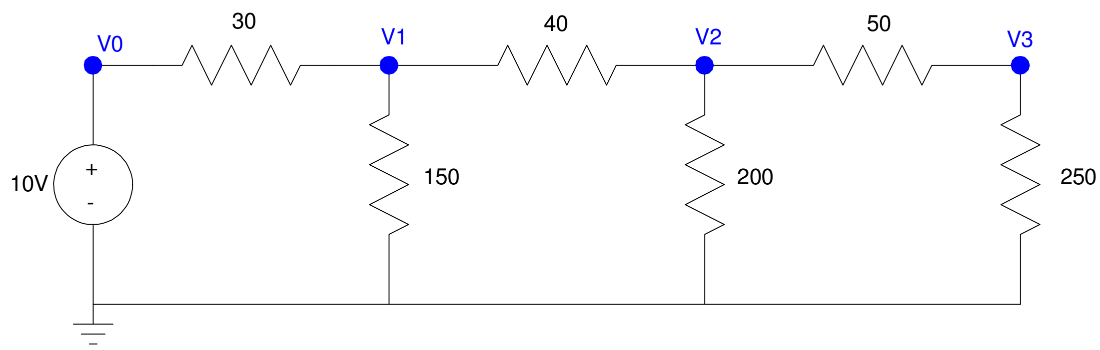
So, the 4 equations for 4 unknowns are:

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{30}\right) + \left(\frac{V_1}{150}\right) + \left(\frac{V_1 - V_2}{40}\right) = 0$$

$$\left(\frac{V_2 - V_1}{40}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{50}\right) = 0$$

$$\left(\frac{V_3 - V_2}{50}\right) + \left(\frac{V_3}{250}\right) = 0$$



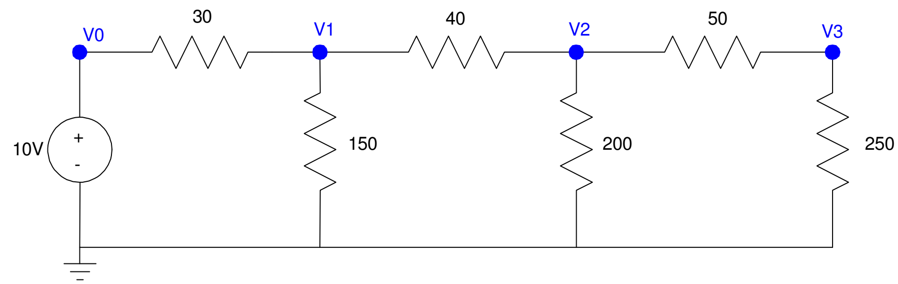
Sidelight: One way to check for sign errors is as follows:

- At node V1, all the coefficients of V1 are positive whereas all other terms are negative
- Likewise at node V2 and V3

This pattern (always?) holds with voltage nodes.

Step 3: Solve N equations for N unknowns.

Group terms



$$V_0 = 10$$

$$\left(\frac{-1}{30}\right) V_0 + \left(\frac{1}{30} + \frac{1}{150} + \frac{1}{40}\right) V_1 + \left(\frac{-1}{40}\right) V_2 = 0$$

$$\left(\frac{-1}{40}\right) V_1 + \left(\frac{1}{40} + \frac{1}{200} + \frac{1}{50}\right) V_2 + \left(\frac{-1}{50}\right) V_3 = 0$$

$$\left(\frac{-1}{50}\right) V_2 + \left(\frac{1}{50} + \frac{1}{250}\right) V_3 = 0$$

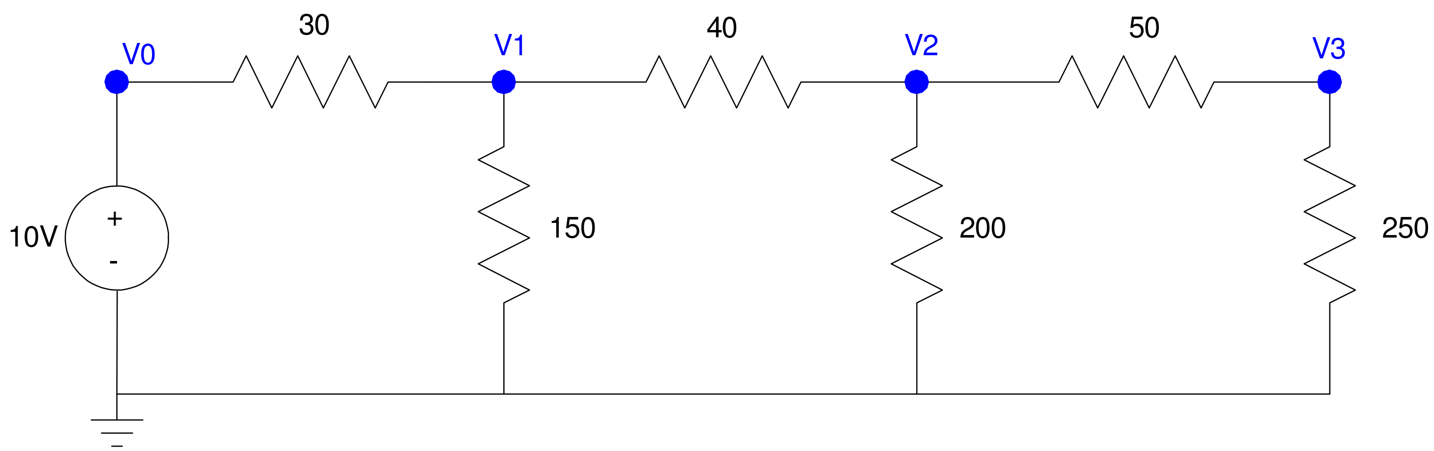
Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{30}\right) & \left(\frac{1}{30} + \frac{1}{150} + \frac{1}{40}\right) & \left(\frac{-1}{40}\right) & 0 \\ 0 & \left(\frac{-1}{40}\right) & \left(\frac{1}{40} + \frac{1}{200} + \frac{1}{50}\right) & \left(\frac{-1}{50}\right) \\ 0 & 0 & \left(\frac{-1}{50}\right) & \left(\frac{1}{50} + \frac{1}{250}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve (Matlab helps here)

```
A = [1,0,0,0 ; -1/30,1/30+1/150+1/40,-1/40,0];
A = [A ; 0,-1/40,1/40+1/200+1/50,-1/50 ; 0,0,-1/50,1/50+1/250]

    1.0000         0         0         0
   -0.0333    0.0650   -0.0250         0
         0   -0.0250    0.0500   -0.0200
         0         0   -0.0200    0.0240

B = [10;0;0;0]

    10
     0
     0
     0

V = inv(A)*B
```

V0 10.0000
V1 7.2072
V2 5.4054
V3 4.5045

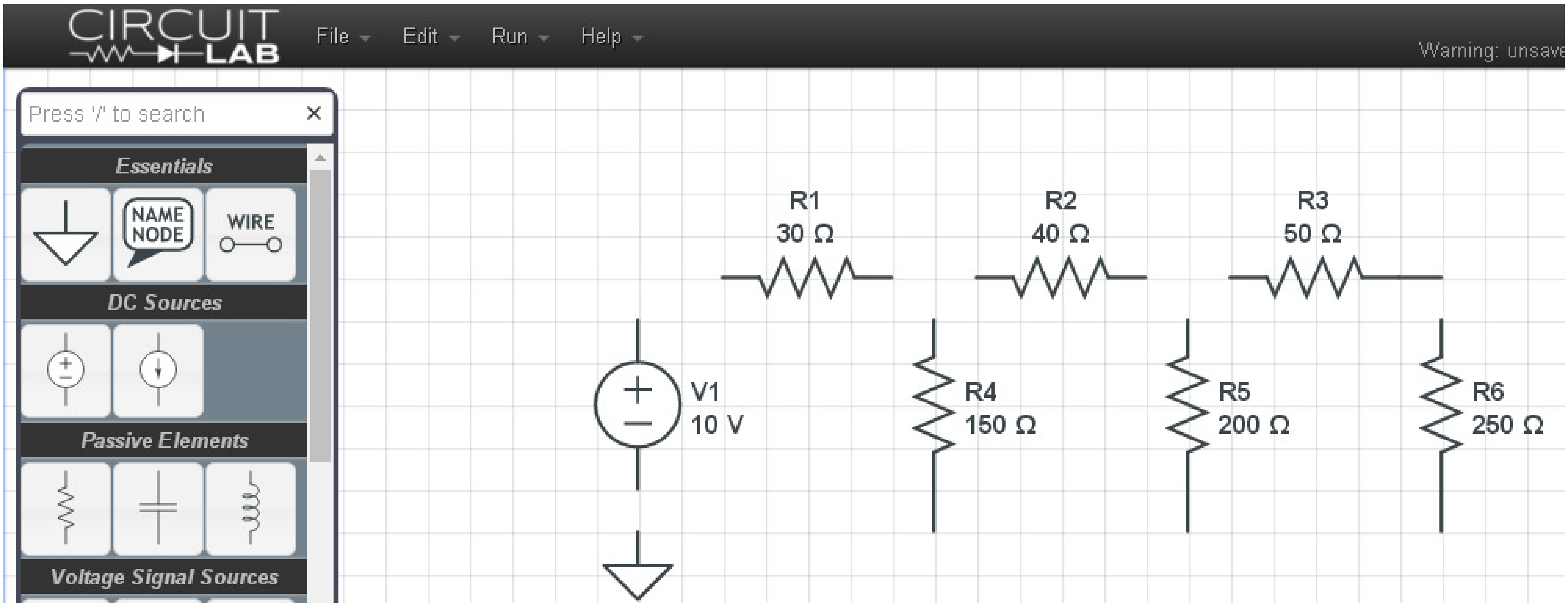


Validation in CircuitLab (www.CircuitLab.com):

There are many circuit simulation packages. We are currently using Circuitlab in our department since

- It works
- It's fairly easy to use,
- The answers are correct, and
- It's free to use for NDSU students (use your NDSU email when registering)
- It's $24/year otherwise

First, go the menu items on the left and add the voltage source and resistors.

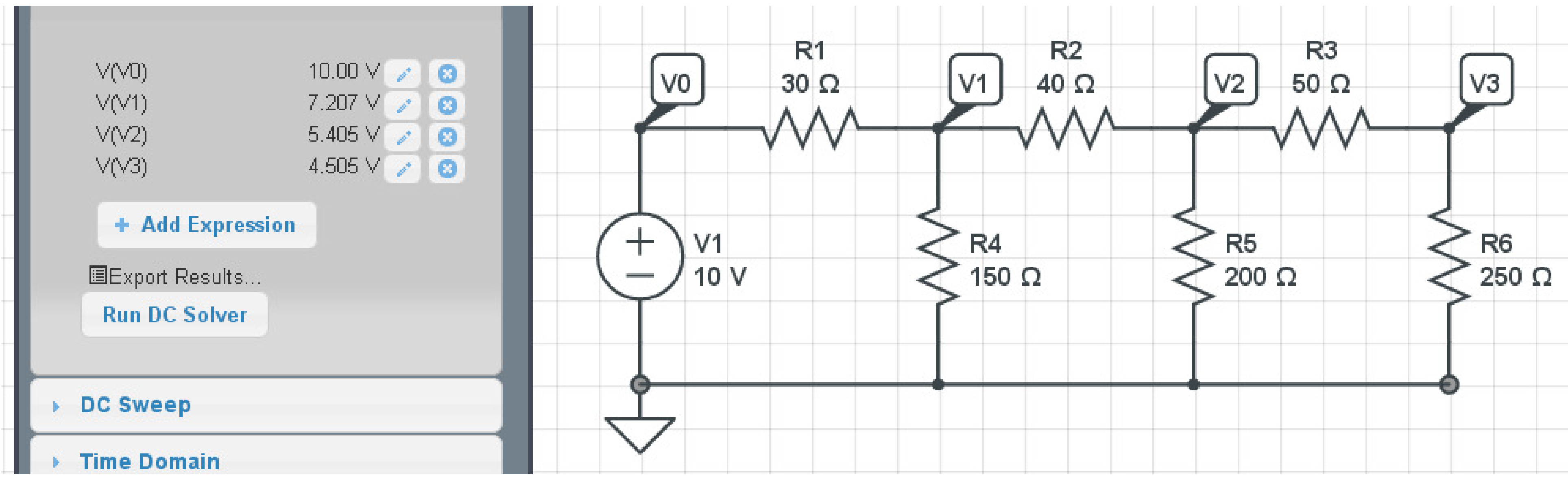


CircuitLab: Drag and drop components. Double click to change values

Connect the components by clicking on one end of an element and dragging the wire to another element. Once done, change the values to match the previous circuit

Once connected, you can solve for the voltages by clicking

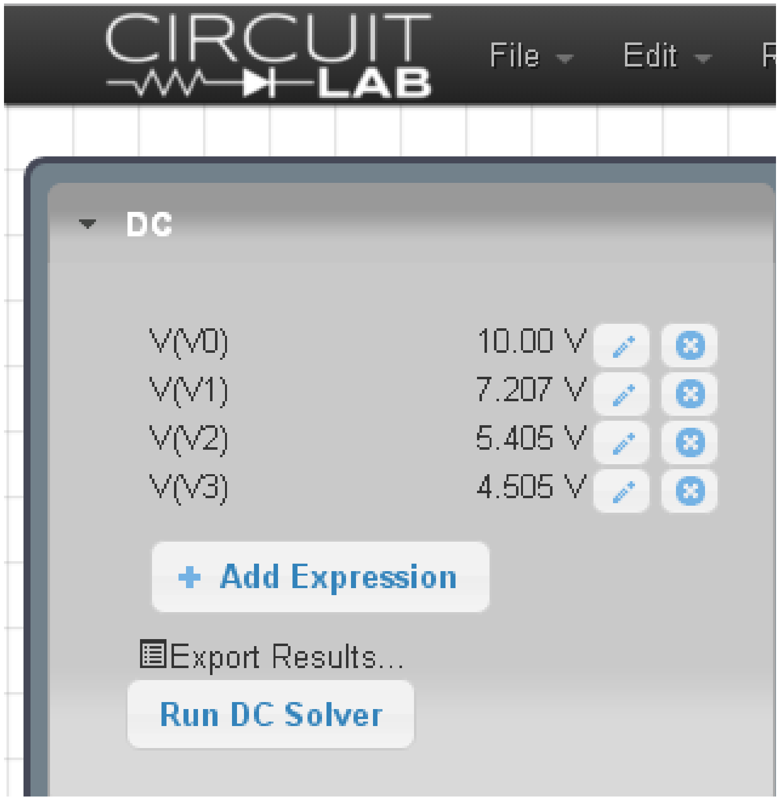
- Add Expression: click on V0, V1, V2, V3
- Simulate
- DC Solver



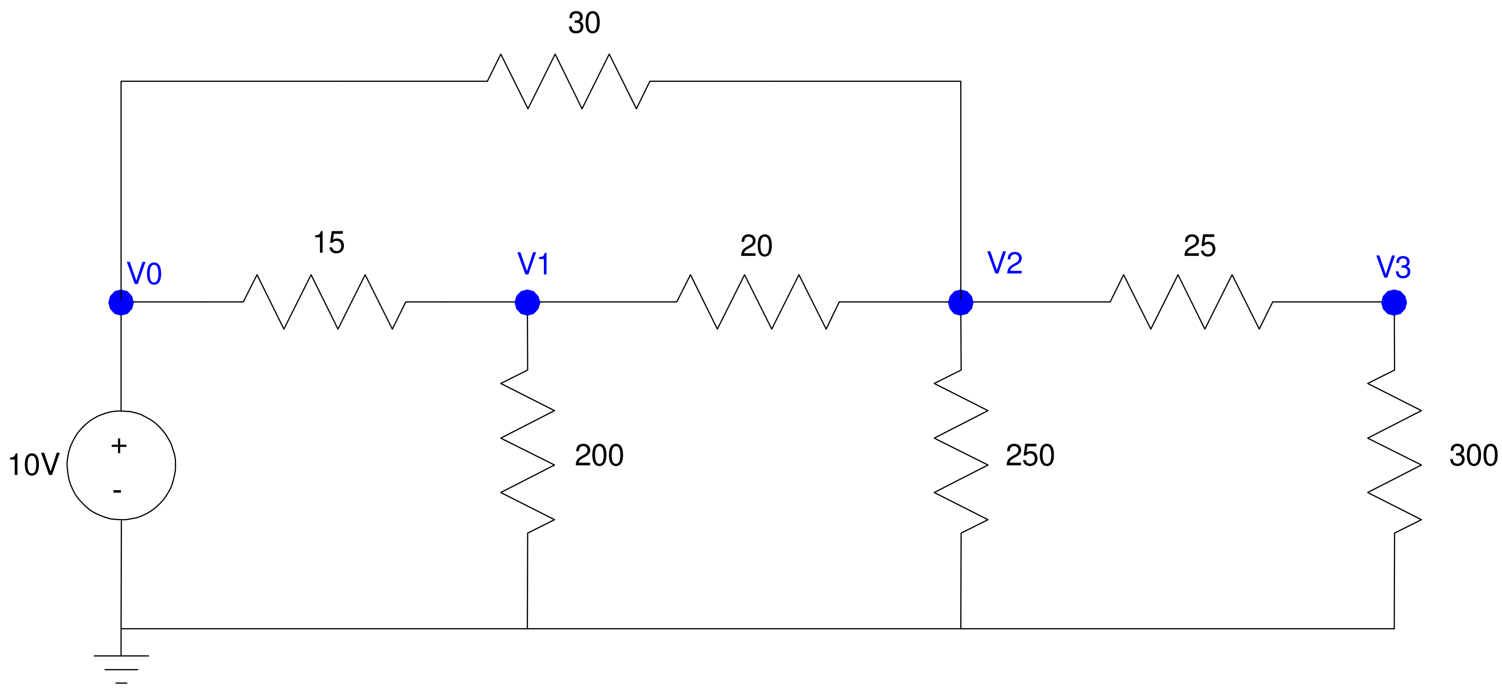
CircuitLab: DC Solver for node voltages

Note that the solution matches our calculations

```
V = inv(A)*B

V0    10.0000
V1     7.2072
V2     5.4054
V3     4.5045
```



Practice Problem: Write the voltage node equations for the following circuit

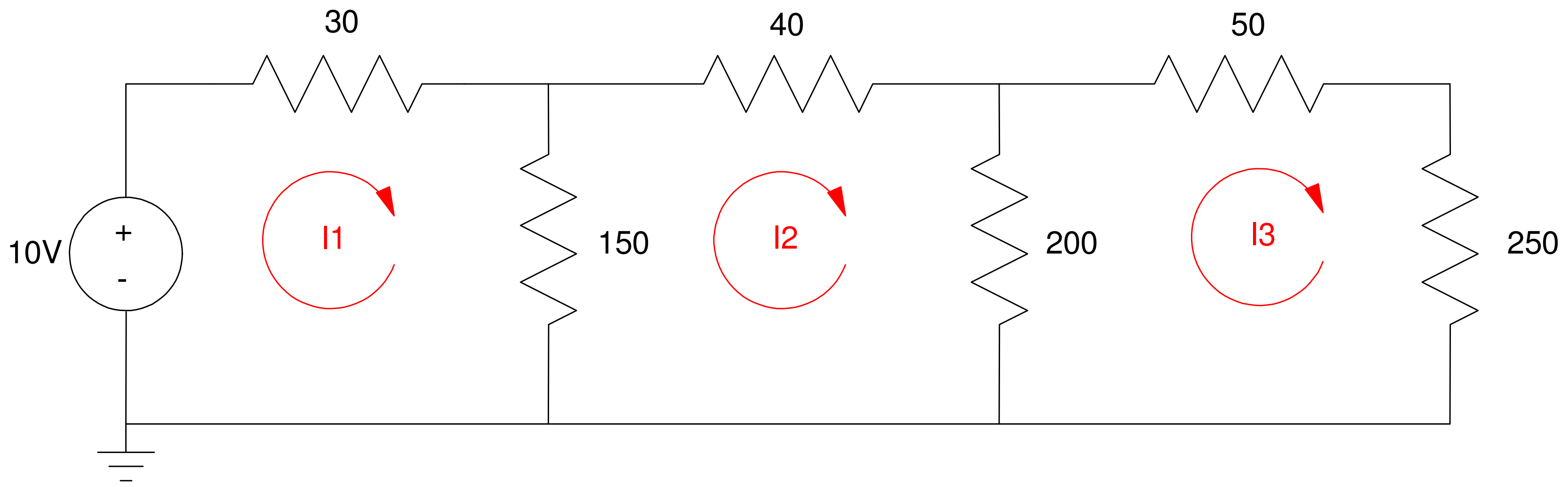


Method #2: Current Loops

- Define the currents in the circuit so that
- The sum of the voltages around any closed path is zero

Step 1) Draw the circuit so that there are N distinct "windows".

- Define the current in each window.



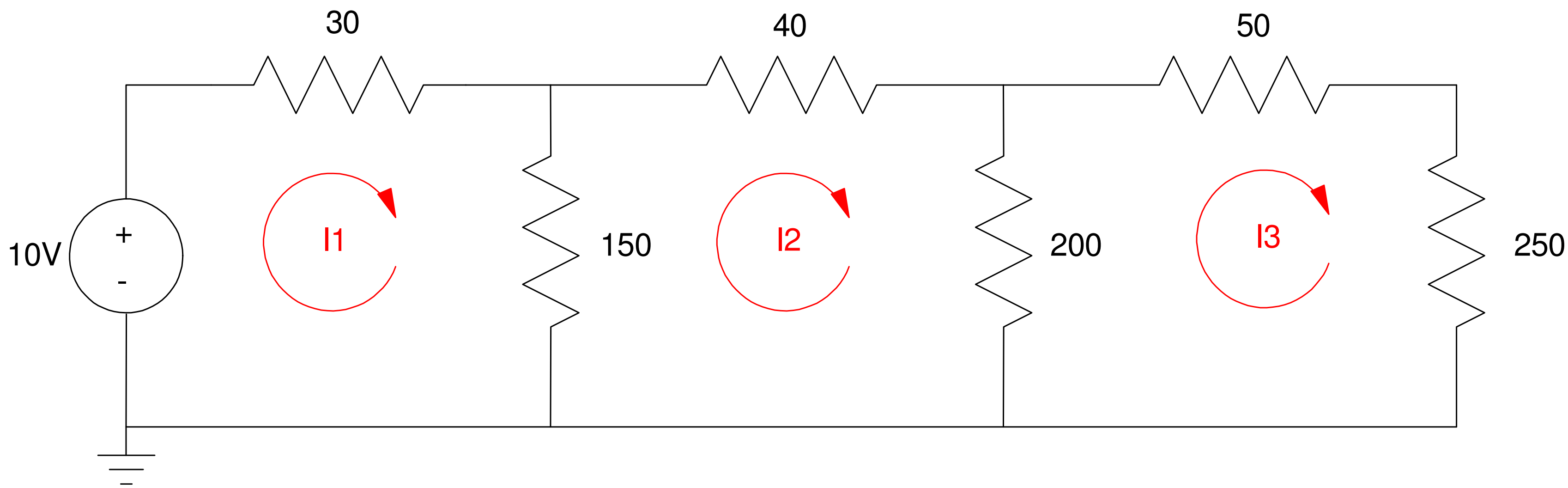
Step 2: Write N equations for N unkowns.

- 3 Loops: Need 3 equations for 3 unknowns
- Note that for loop #1, all the I1 terms are positive and all other terms are negative.

$$-10 + 30I_1 + 150(I_1 - I_2) = 0$$

$$150(I_2 - I_1) + 40I_2 + 200(I_2 - I_3) = 0$$

$$200(I_3 - I_2) + 50I_3 + 250I_3 = 0$$



Step 3: Solve.

Group terms

$$180I_1 - 150I_2 = 10$$

$$-150I_1 + 390I_2 - 200I_3 = 0$$

$$-200I_2 + 500I_3 = 0$$

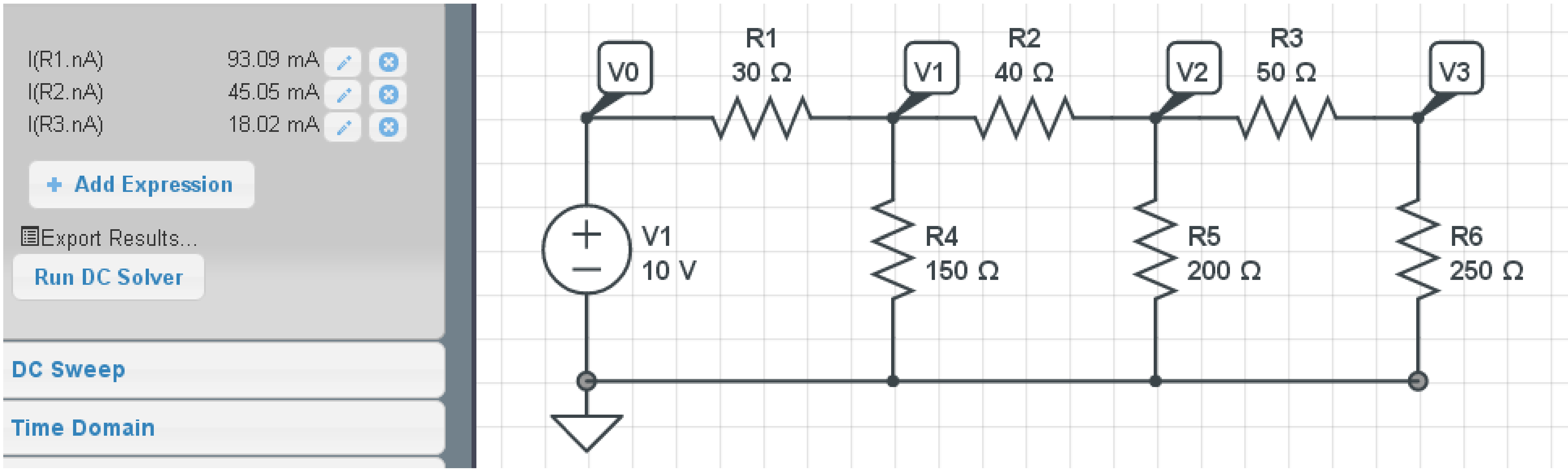
Place in matrix form

$$\begin{bmatrix} 180 & -150 & 0 \\ -150 & 390 & -200 \\ 0 & -200 & 500 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

```
>> A = [180,-150,0 ; -150,390,-200 ; 0,-200,500]

A =

   180  -150     0
  -150   390  -200
     0  -200   500

>> B = [10;0;0]

B =

    10
     0
     0

>> I = inv(A)*B

I =

    0.0931
    0.0450
    0.0180
```

Validation: Using CircuitLab, you can check your answers.

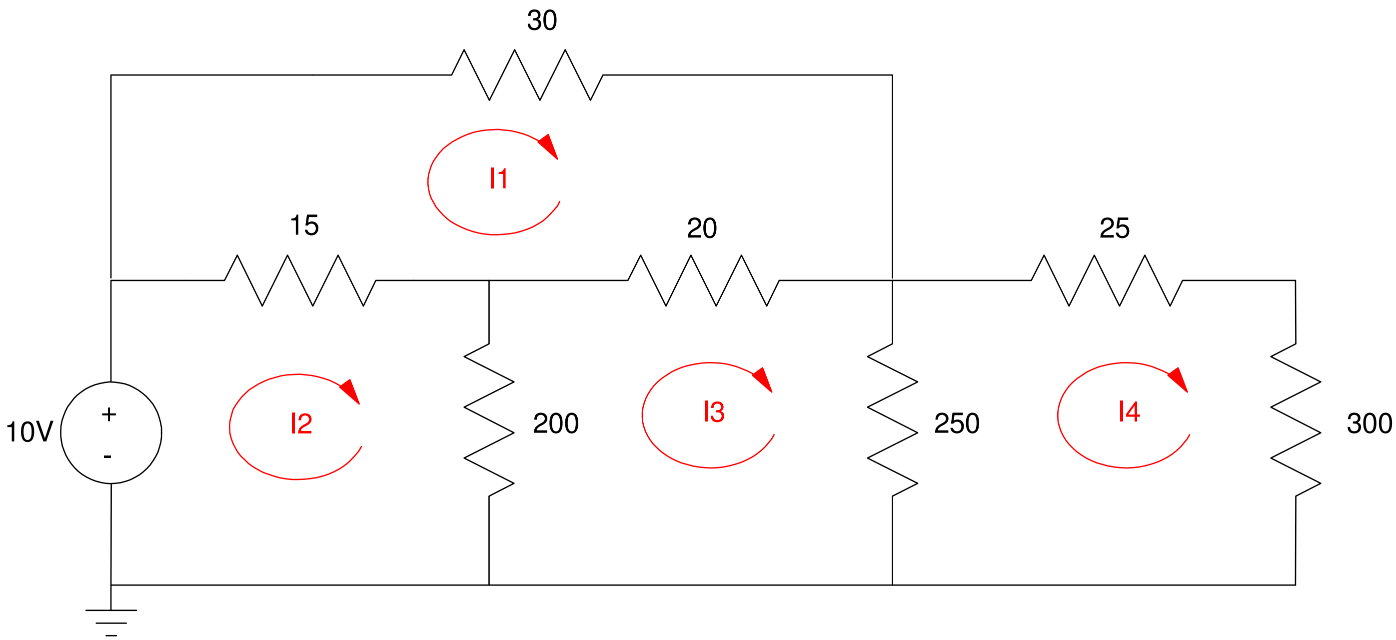
- Click on the left side of R1, R2, R3



Calculations:

I1	0.0931
I2	0.0450
I3	0.0180

Practice Problem: Write the current loop equation for the following circuit



Sidelight

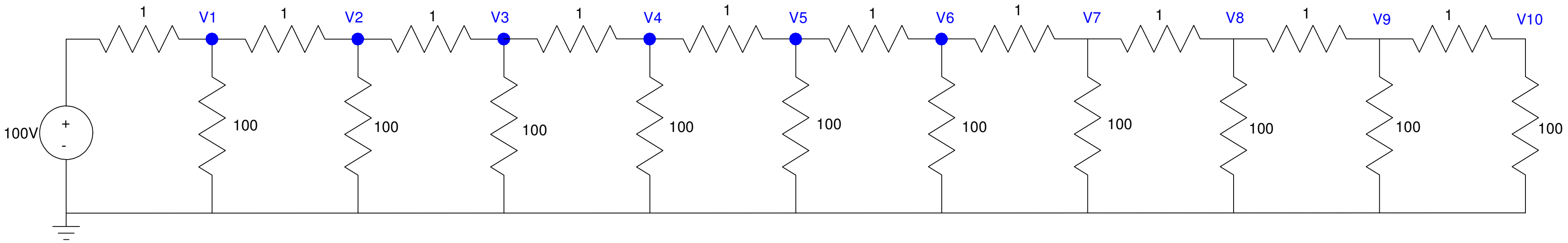
- 1970: Solving 4 equations for 4 unknowns was hard
- Today: Solving 20 equations for 20 unknowns is easy

Matlab doesn't care. Works the same for both cases.

What's most important today:

- Get the equations right
- The number of equations doesn't really matter

Example: 10-stage resistor circuit



Node #5: (same for nodes 1..9)

$$\left(\frac{V_5 - V_4}{1}\right) + \left(\frac{V_5 - V_6}{1}\right) + \left(\frac{V_5}{100}\right) = 0$$

$$-V_4 + 2.01 V_5 - V_6 = 0$$

Node 10:

$$-V_9 + 1.01 V_{10} = 0$$

In matrix form, the equations will be

$$A\begin{bmatrix} V_1 \\ \vdots \\ V_{10} \end{bmatrix} = \begin{bmatrix} B_1 \\ \vdots \\ B_{10} \end{bmatrix}$$

In MATLAB, you can input this using a for statement

Start with a zero matrix with dimension 10x10

```
-->A = zeros(10,10);
```

The diagonal is -2.01. The off diagonals are +1

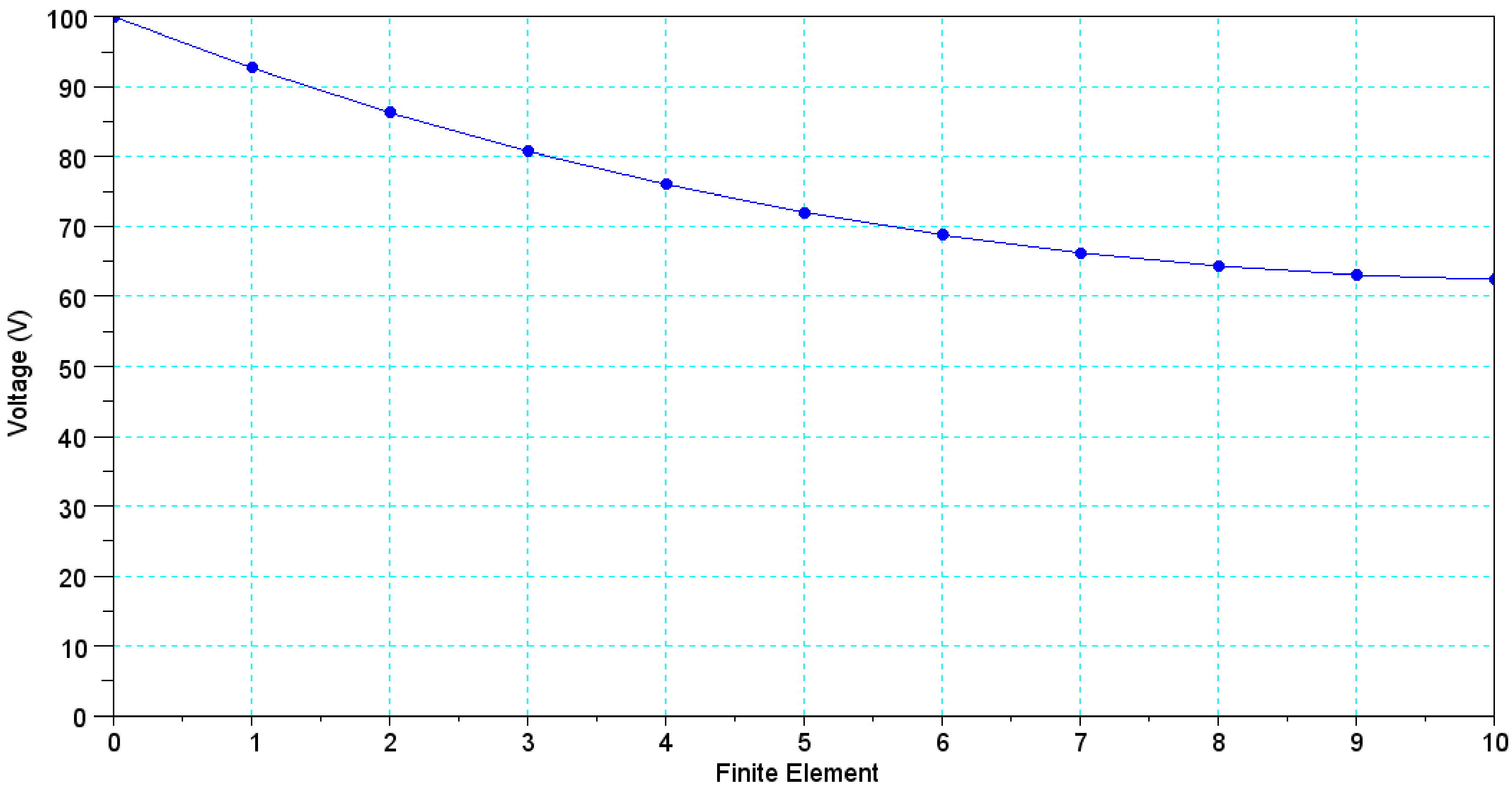
```
-->for i=1:9
-->   A(i,i) = 2.01;
-->   A(i+1,i) = -1;
-->   A(i,i+1) = -1;
-->   end

-->A(10,10) = 1.01;
```

```
A

    2.01  - 1.      0.      0.      0.      0.      0.      0.      0.      0.
  - 1.      2.01  - 1.      0.      0.      0.      0.      0.      0.      0.
    0.    - 1.      2.01  - 1.      0.      0.      0.      0.      0.      0.
    0.      0.    - 1.      2.01  - 1.      0.      0.      0.      0.      0.
    0.      0.      0.    - 1.      2.01  - 1.      0.      0.      0.      0.
    0.      0.      0.      0.    - 1.      2.01  - 1.      0.      0.      0.
    0.      0.      0.      0.      0.    - 1.      2.01  - 1.      0.      0.
    0.      0.      0.      0.      0.      0.    - 1.      2.01  - 1.      0.
    0.      0.      0.      0.      0.      0.      0.    - 1.      2.01  - 1.
    0.      0.      0.      0.      0.      0.      0.      0.    - 1.      1.01

-->V = inv(A)*B

    92.673872
    86.274482
    80.737837
    76.008571
    72.03939
    68.790603
    66.229722
    64.331139
    63.075867
    62.451353
```

If you build this circuit in CircuitLab, you'll get the same result.

A graph shows the cooling of the bar as you go along a little better:

```
-->plot([0:10],[100;V],'.-')
-->xlabel('Finite Element');
-->ylabel('Voltage (V)');
```



Summary

Two determine the voltages and/or currents in a circuit, write N equations for N unknowns.

Current Loops:

- The sum of the voltages around any closed loop must sum to zero
- Write N such equations

Voltage Nodes:

- The net current flowing from a node must sum to zero
- Write N such equations

Once you have N equations and N unknowns, solve

- Place in matrix form
- Solve in Matlab