Biomedical Engineering

ECE 111: Intro to ECE

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Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Biomedical Engineering

Objective

- Model the cardiovascular system as an RLC circuit
- From the model, predict what happens if different parameters change

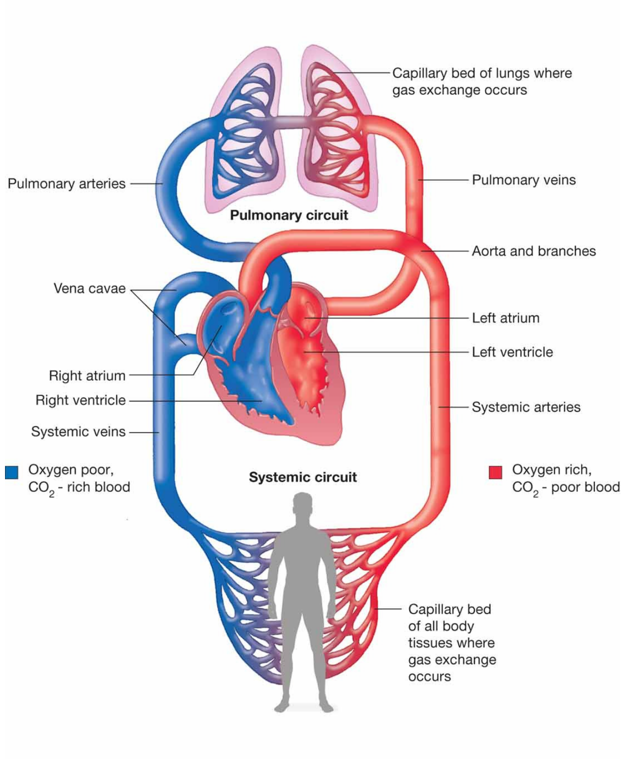
Matlab Functions

- Matlab Scripts
- Nonlinear simulations
- clf
- subplot

Electrical Model of the cardiovascular System

The cardiovascular system consists of to circulatory systems:

- The right ventricle drives the low-pressure system (lungs), and
- The left ventricle drives the high-pressure system, (rest of body)



Modeling as an electrical circuit

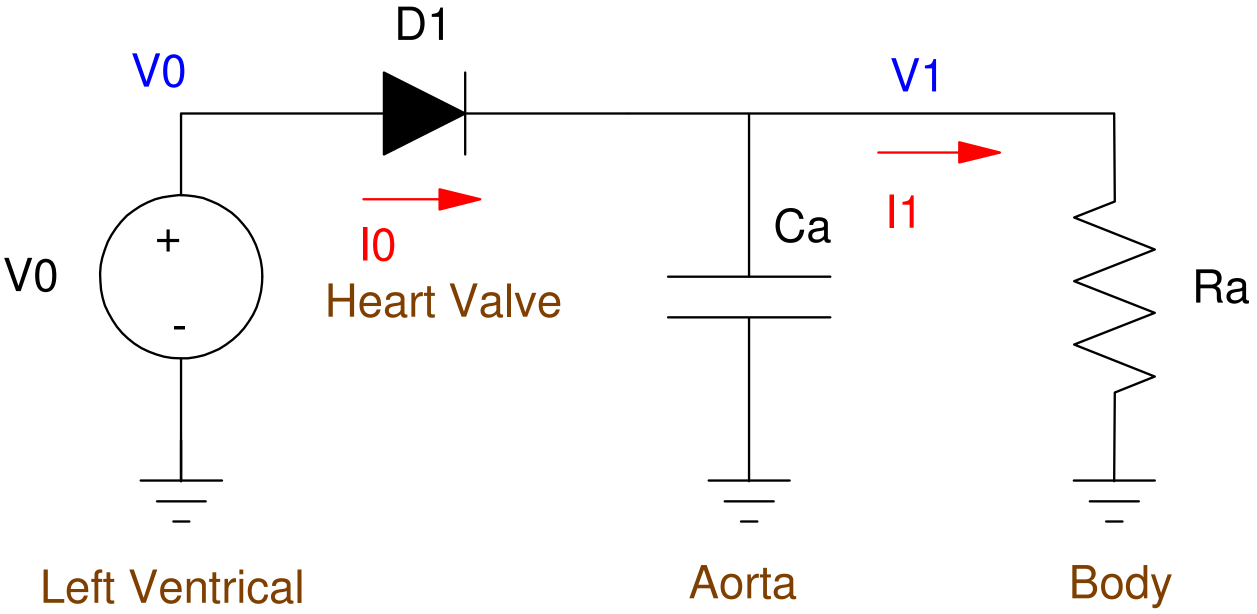
- In essence, the heart acts as a 5W source.
- Most of the work and medical problems are associated with the high-pressure side (which requires most of the energy).

If you can model the high-pressure cardiovascular system

- You have a better understanding of how the system works,
- You can build more accurate models of the body for testing of artificial hearts
- You can ask 'what if' questions, such as what happens if the capacitance increases, and predict the result.
- This in turn opens new directions for drug treatment: finding drugs which affect the different parameters of the model

Electrical Models of the Heart

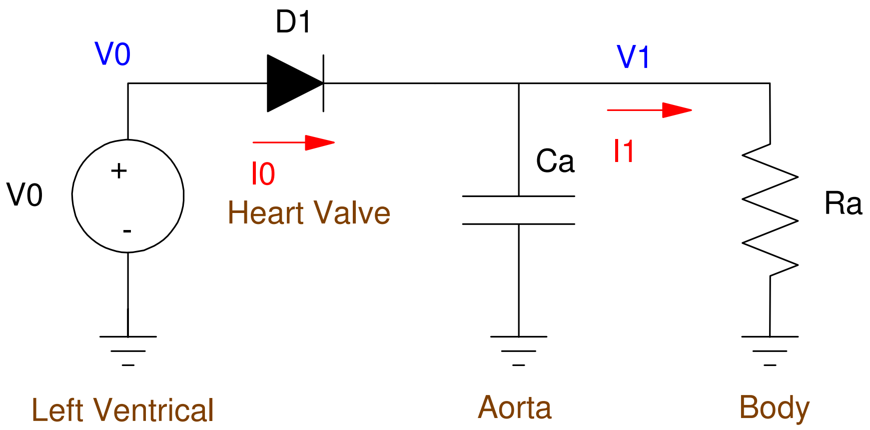
- A 2-element Winkessel model (i.e. an RC circuit), and
- A more complex 5-element model (an RLC circuit)



Heart Parameters:

Parameter	English Units	Metric Units	Circuit Dual
Pressure	89mm Hg	10.67 N/m2	10.67V
Volume (LV)		70ml	
Volume (Aorta)		500ml	
Inertia (Aorta)			
Normal Flow		70 ml/s	70mA
Blood Inertia		0.07kg	0.07H

2-element Winkessel Model



First, determine reasonable parameters for the model.

V0: A typical blood pressure reading is 120/80.

$$V_0 = \max(0, 16\sin(2\pi t))$$

D1: Heart Valve:

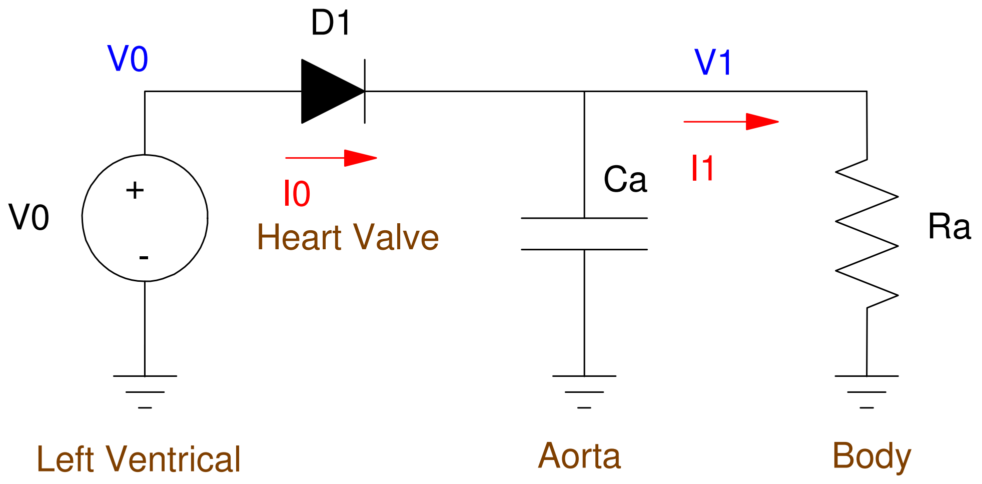
$$R_{d1} = \begin{cases} 1\Omega & V_0 > V_1 \\ 1M\Omega & V_0 < V_1 \end{cases}$$

Ra: Nominal flow is 70ml/s on average

$$R_a = \frac{13.3V}{70mA} = 190\Omega$$

C: Pressure range of 120/80 mmHg (26V .. 10.6V) is

$$C_a = 4.54mF$$



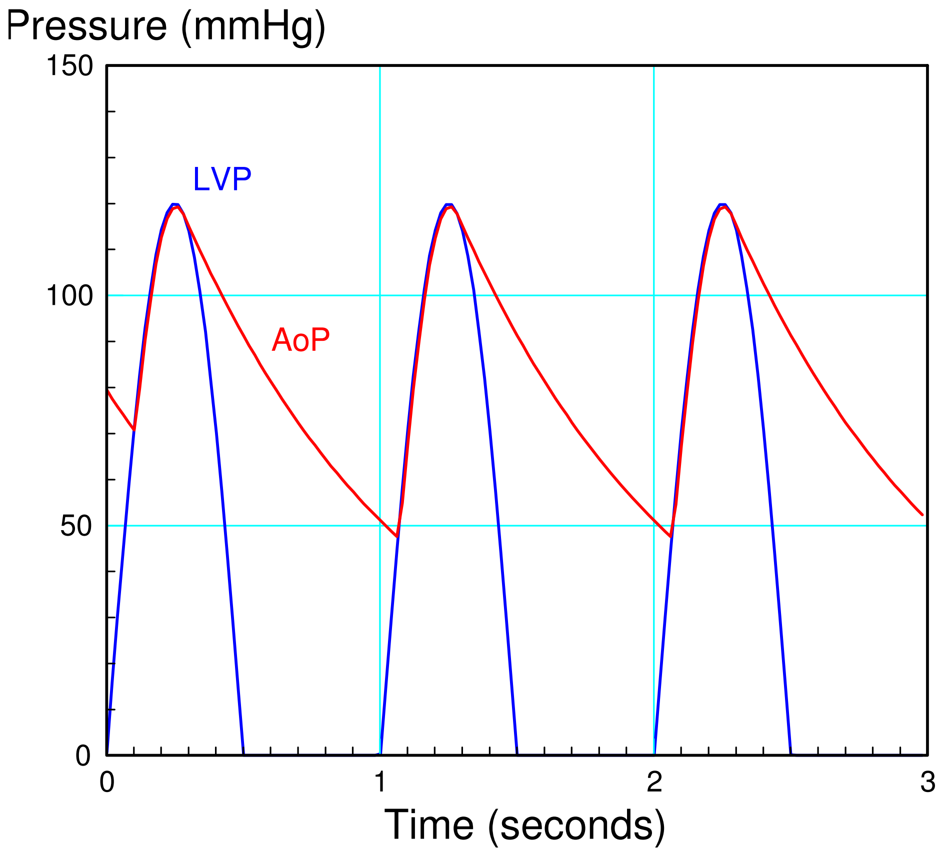
This results in the following dynamic equations for the heart:

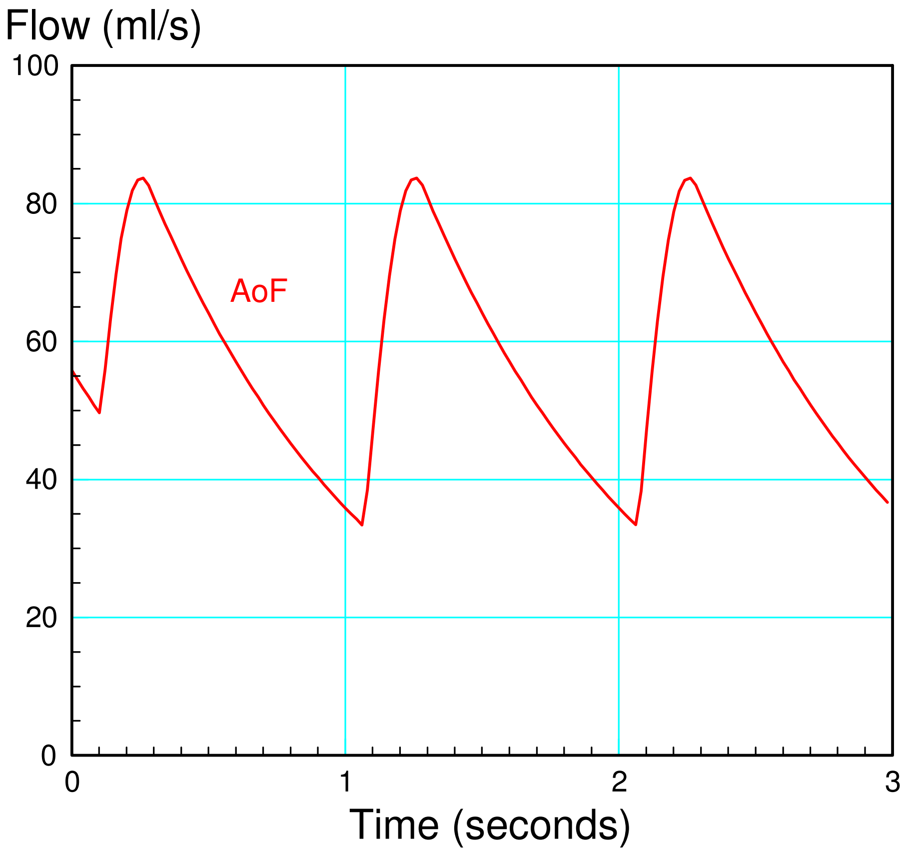
$$I_c = C_a \frac{dV_1}{dt} = I_{d1} - \frac{V_1}{R_a}$$

Matlab Code:

```
for j=1:npt

   V0 = max(0, 16*sin(6.28*t));

   if (V0 > V1)  Rd1 = 1;
            else Rd1 = 1e6;
            end

   I0 = (V0 - V1)/Rd1;
   I1 = V1 / Ra;

   dV1 = (I0 - I1)/Ca;

   t = t + dt;
   V1 = V1 + dV1*dt;

   end
```

Results for Three Beats:

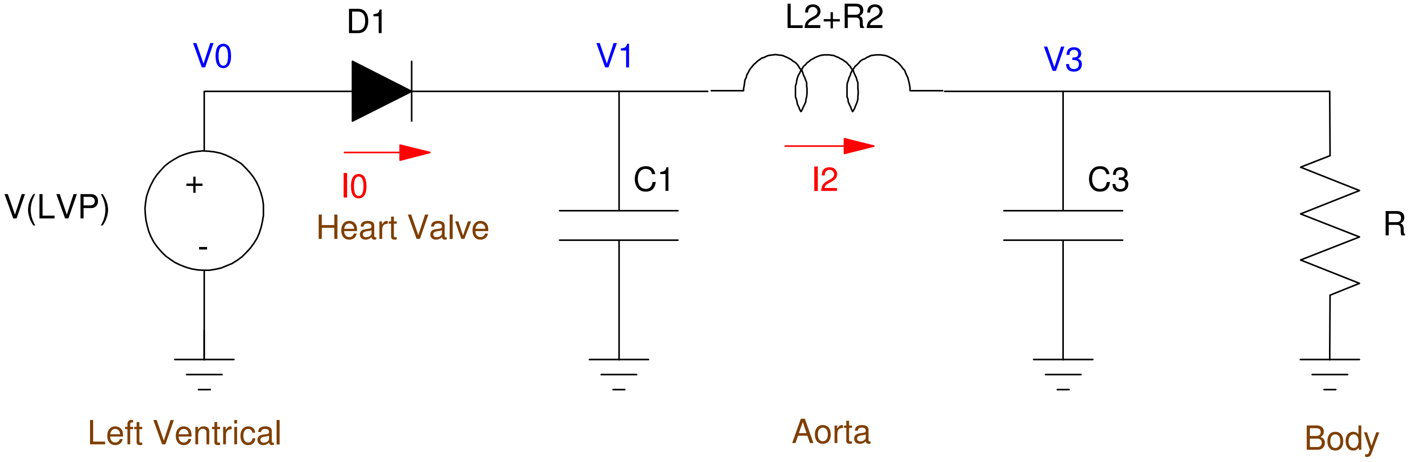




Improved Winkessel Model

A slightly more accurate model splits the Aorta into two sections:

- C1 accepts blood from the heart and charges up
- L2 is the inertia of the blood in this section of the aorta,
- C3 is the rest of the arterial system which pushes blood throughout the body, and
- R models the capillaries, drawing 70ml/s (70mA) on average



Computations for the parameters are as follows:

$$70mA = C_3 \frac{40mmHg}{0.8s} = C_3 \frac{5.33V}{0.8s} \qquad \Rightarrow \qquad C_3 = 10.5mF$$

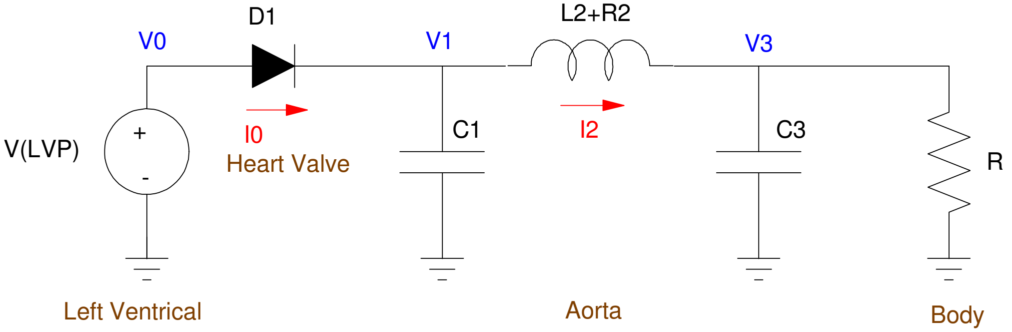
$$C_1 = \left(\frac{70ml}{500ml}\right) \cdot 10.5mF \qquad \Rightarrow \quad C_1 = 1.5mF$$

L2: 70ml = 0.07kg = 0.07H + 6 Ohms.

R: $R = 190\Omega$ (same as before)

V0: Model the left ventricle

$$V_0 = \max(0, 17.6 \sin(2\pi t))$$



D1: Model the valve as a variable resistor

$$R_{d1} = \begin{cases} 1\Omega & V_0 > V_1 \\ 1M\Omega & V_0 < V_1 \end{cases}$$

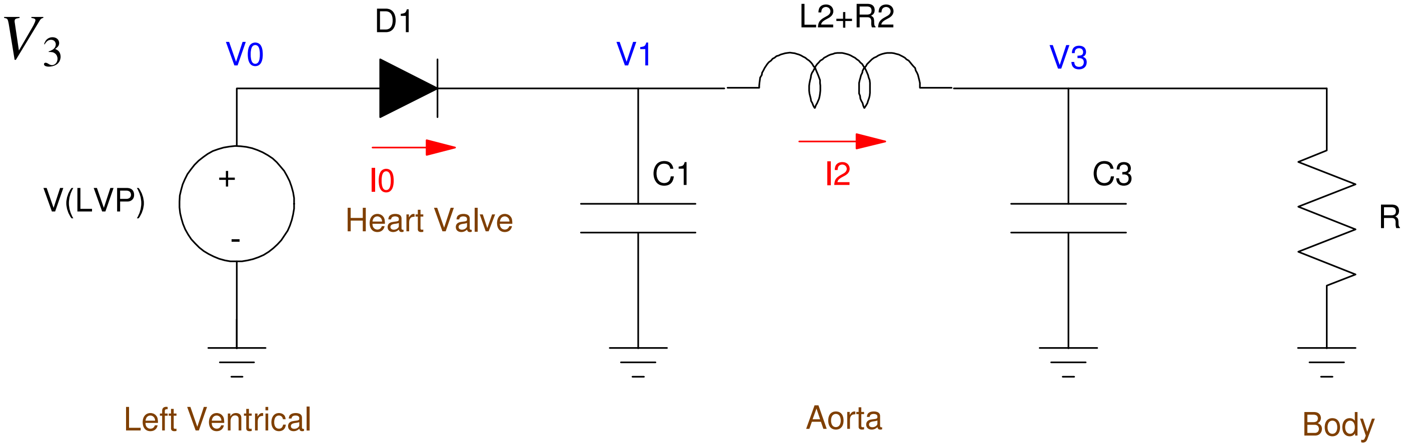
Dynamics equations:

$$I_0 = \frac{V_0 - V_1}{R_{d1}}$$

$$C_1 \frac{dV_1}{dt} = I_0 - I_2$$

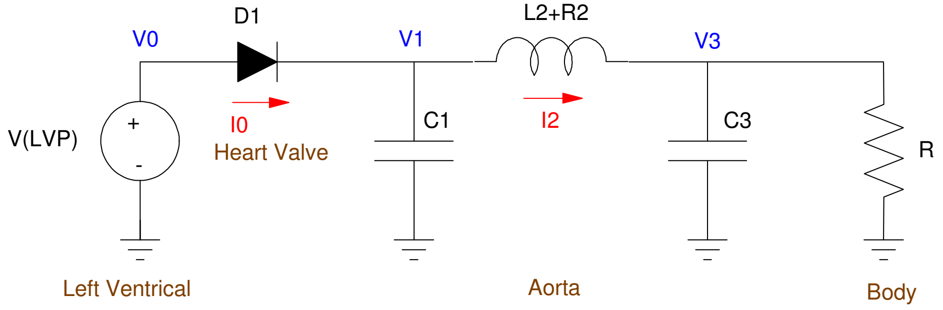
$$L_2 \frac{dI_2}{dt} = V_1 - I_2 R_2 - V_3$$

$$C_3 \frac{dV_3}{dt} = I_2 - \frac{V_3}{R}$$

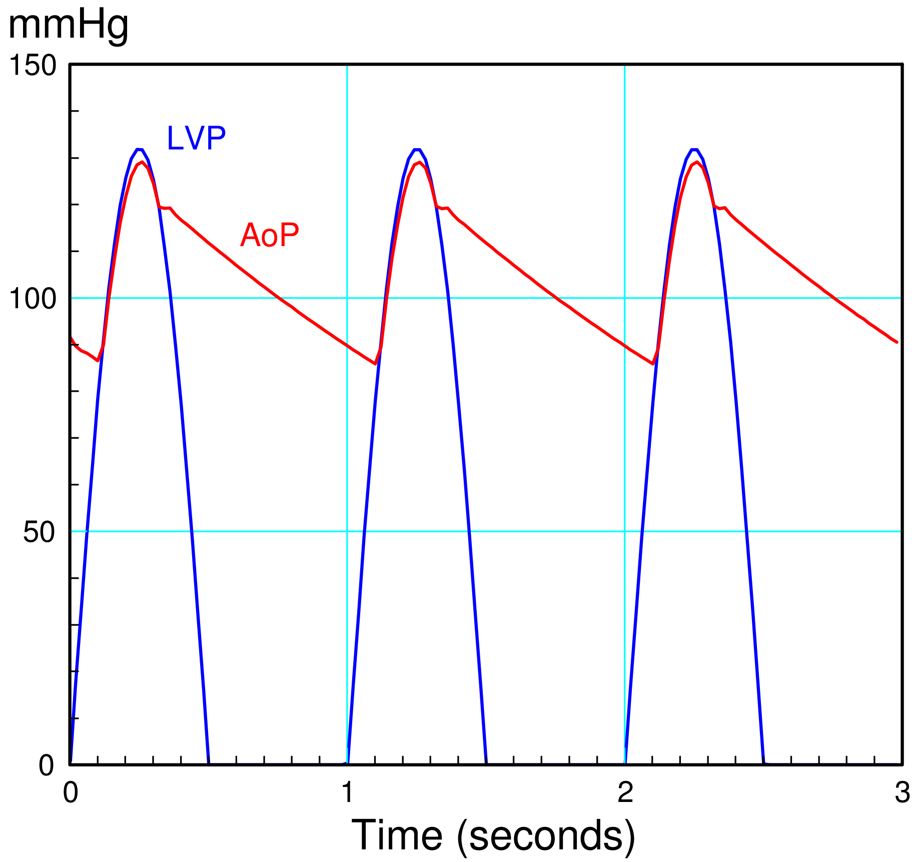


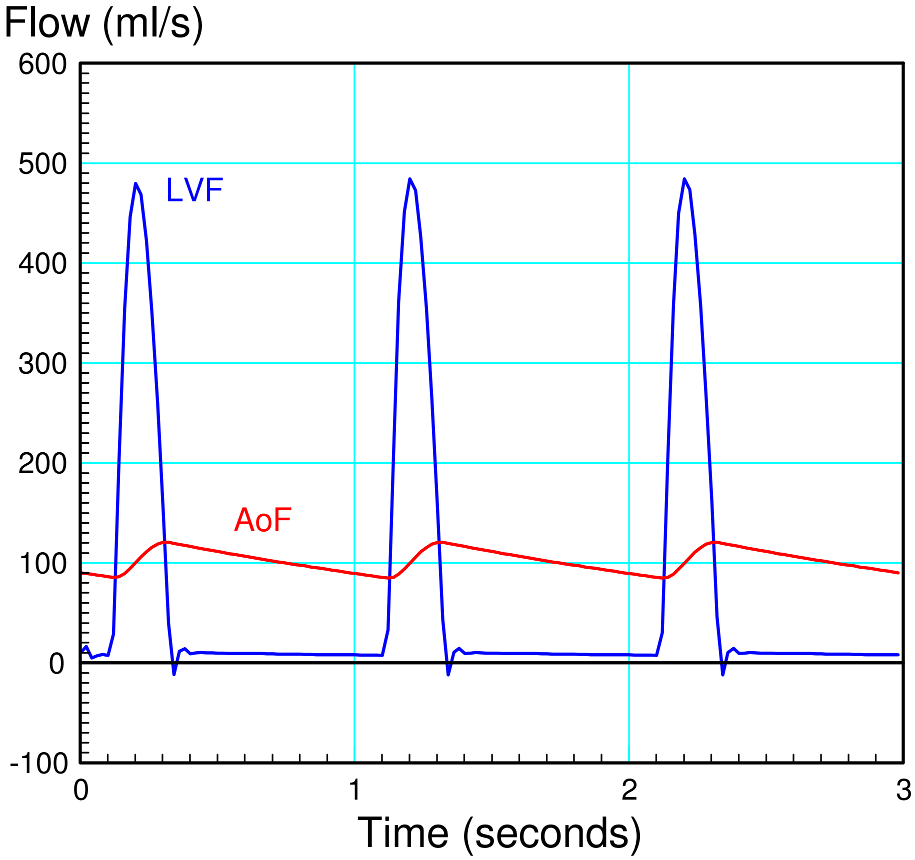
Matlab Code (main loop)

```
for j=1:npt

    V0 = max(0, 17.6*sin(6.28*t));

    if (V0 > V1)  Rd1 = 1;
             else Rd1 = 1e6;
             end

    I0 = (V0 - V1)/Rd1;

    dV1 = (I0 - I2)/C1;
    dI2 = (V1 - R2*I2 - V3)/L2;
    dV3 = (I2 - V3/R)/C3;

    t  = t + dt;
    V1 = V1 + dV1*dt;
    I2 = I2 + dI2*dt;
    V3 = V3 + dV3*dt;

    end
```



Result for 3 beats





What this model shows you is:

- The aortic pressure (top graph red) is smoother. This agrees with what you see better.
- At the left ventricle (right blue), there is a dip in pressure when the valve closes. This is caused by the pressure wave reflecting back in time to help the valve close (something you also see in a healthy person)
- The arterial flow (right red) also has a negative dip when the valve closes. This is the inertia of the blood providing suction, helping the heart to eject more blood.

With this model you can start to answer questions, such as

- What is the effect of Angina (the aorta expands)
- What is the effect of hardening of the arteries?
- What is the effect of obstructed flow?
- How does the body respond to the above to maintain blood flow?