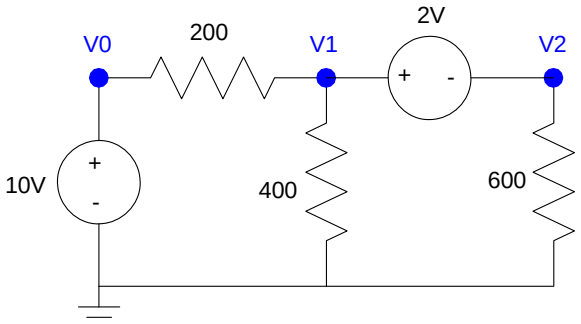
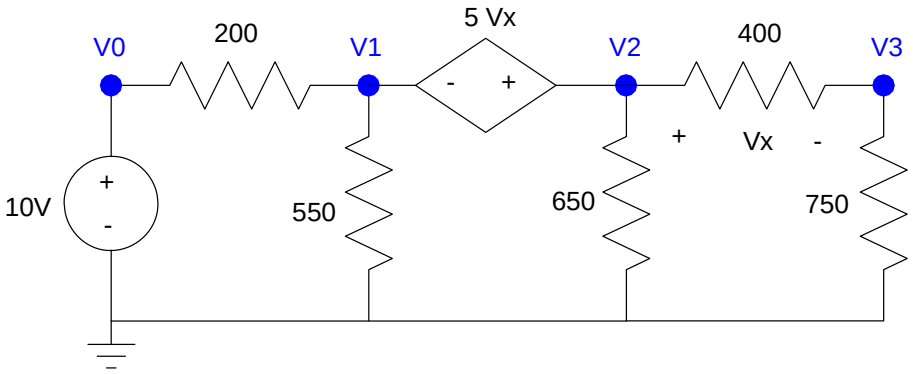
Voltage Super-Nodes

EE 206 Practice Problems

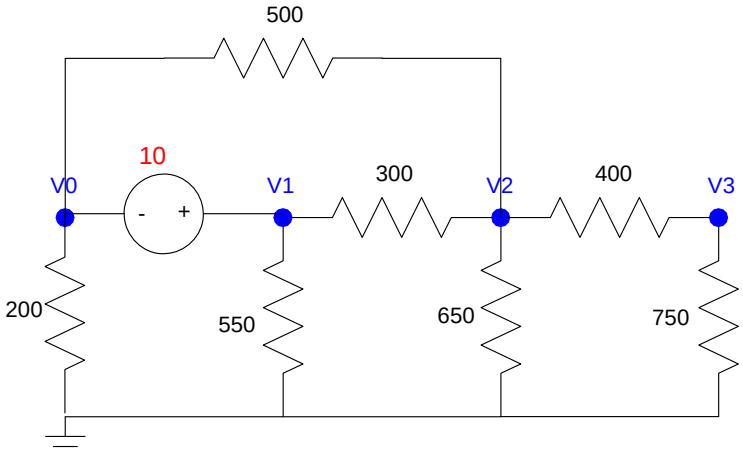
Write the voltage node equations for the following circuits



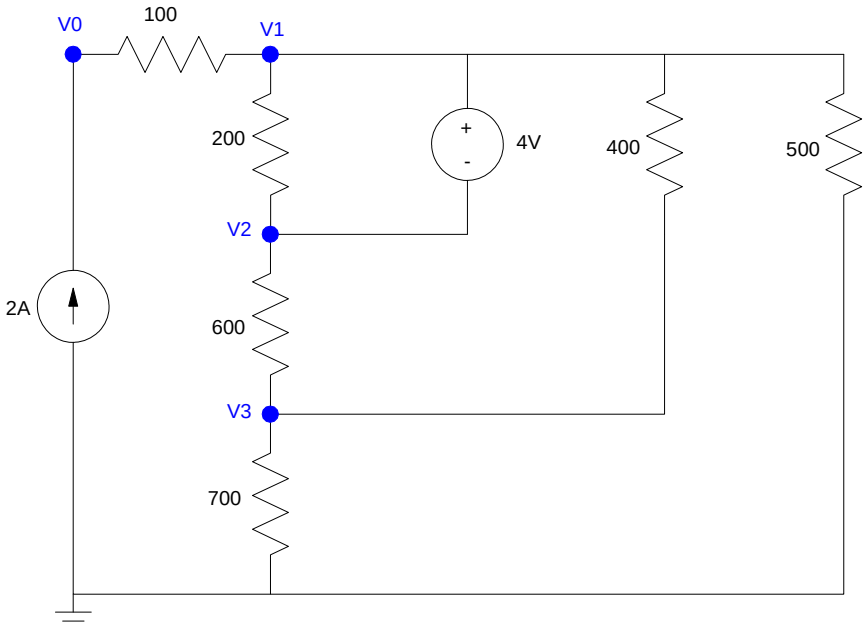
Problem 1



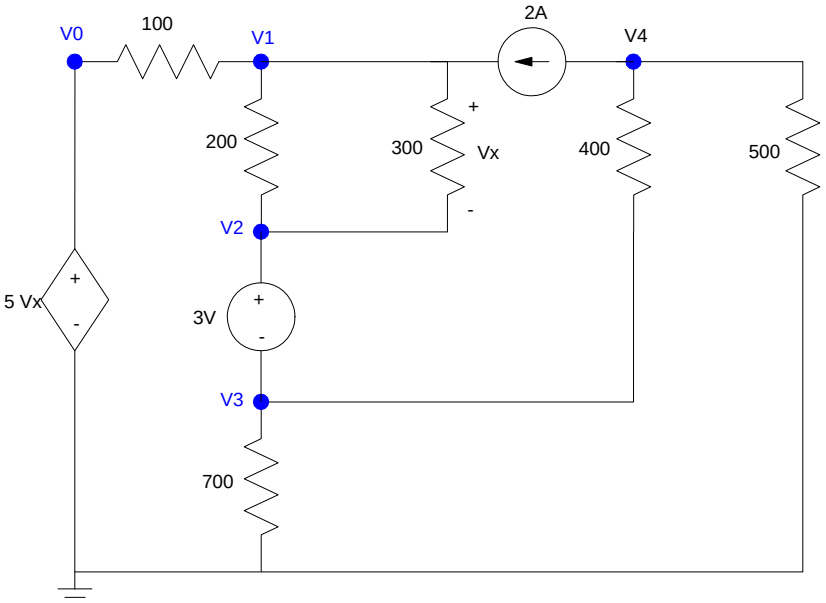
Problem 2



Problem 3



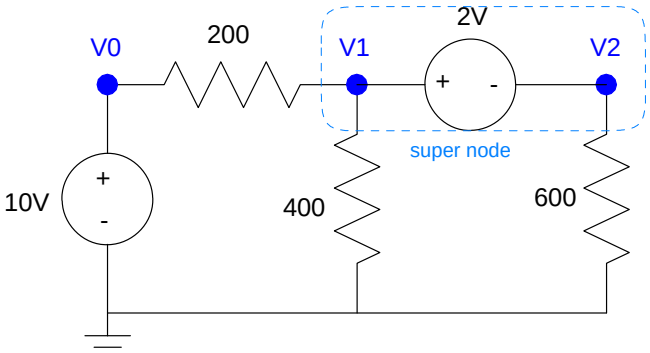
Problem 4



Problem 5

Solutions

Problem 1) There are 3 voltage nodes. You need 3 equations to solve for 3 unknowns



Start with the easy equations (the voltage sources)

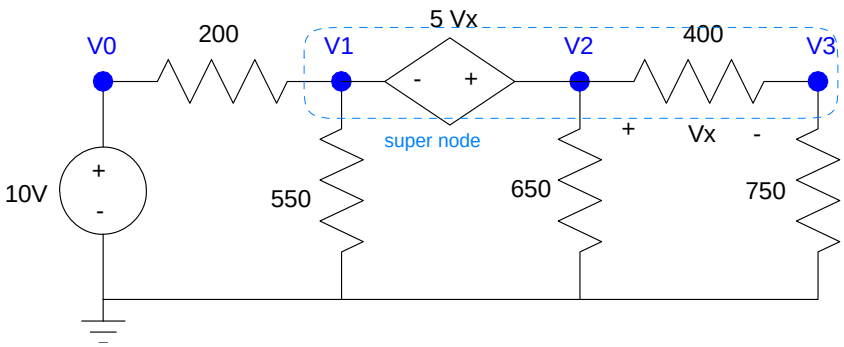
$$V_0 = 10$$

$$V_1 - V_2 = 2$$

The third equation is a super-node (shown in blue dotted line). Other super nodes are possible

$$\left(\frac{V_1 - V_0}{200}\right) + \left(\frac{V_1}{400}\right) + \left(\frac{V_2}{600}\right) = 0$$

Problem 2) There are 4 voltage nodes plus a dependent source. You need 5 equations to solve for 5 unknowns



$$V_x = V_3 - V_2$$

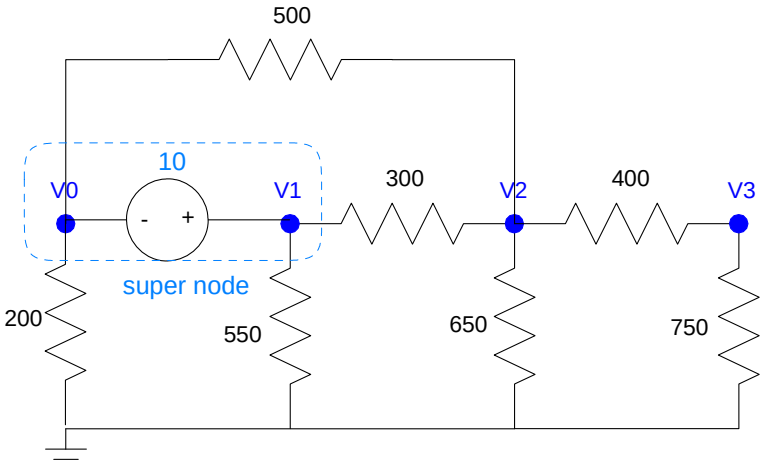
$$V_2 - V_1 = 5V_x$$

$$V_0 = 10$$

$$\left(\frac{V_3 - V_2}{400}\right) + \left(\frac{V_3}{750}\right) = 0$$

$$\left(\frac{V_1 - V_0}{200}\right) + \left(\frac{V_1}{550}\right) + \left(\frac{V_2}{650}\right) + \left(\frac{V_3}{750}\right) = 0 \qquad \textit{super node}$$

Problem 3) There are 4 voltage nodes. You need 4 equations to solve for 4 unknowns



Start with the easy equation: the voltage souce

$$V_1 - V_0 = 10$$

Sum the currents to zero

$$\left(\frac{V_2 - V_1}{300}\right) + \left(\frac{V_2}{650}\right) + \left(\frac{V_2 - V_3}{400}\right) + \left(\frac{V_2 - V_0}{500}\right) = 0 \qquad \textit{node V2}$$

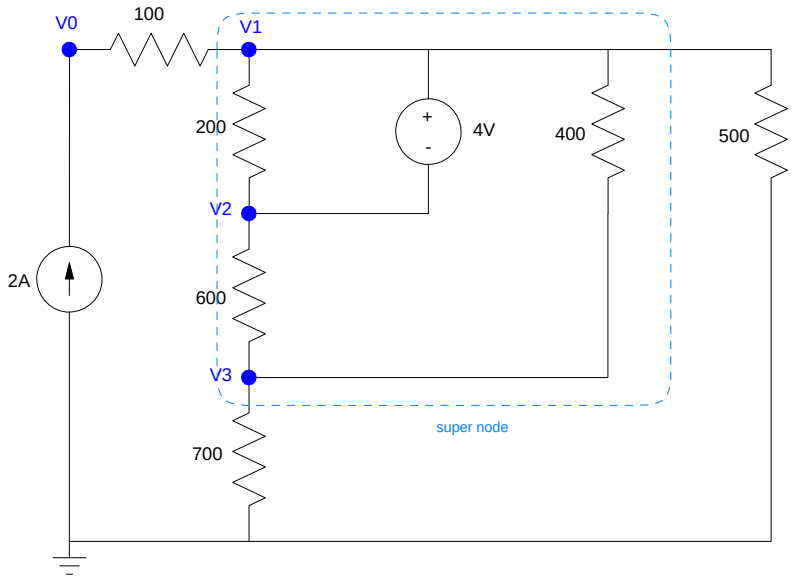
$$\left(\frac{V_3 - V_2}{400}\right) + \left(\frac{V_3}{750}\right) = 0 \qquad \textit{node V3}$$

We need one more equation. Define a super-node (blue dotted line)

$$\left(\frac{V_0}{200}\right) + \left(\frac{V_0 - V_2}{550}\right) + \left(\frac{V_1}{550}\right) + \left(\frac{V_1 - V_2}{300}\right) = 0 \qquad \textit{super node}$$

Other super nodes are possible

Problem 4) There are 4 voltage nodes. You need 4 equations to solve for 4 unknowns



Start wit the easy equation (the voltage source)

$$V_1 - V_2 = 4$$

Node V0

$$-2 + \left(\frac{V_0 - V_1}{100}\right) = 0$$

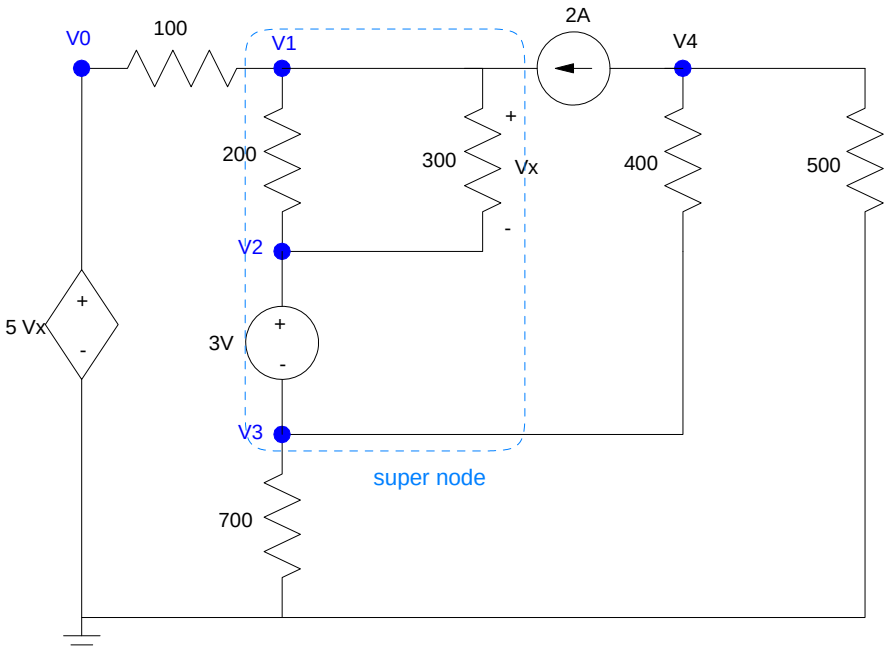
Node V3

$$\left(\frac{V_3 - V_2}{600}\right) + \left(\frac{V_3 - V_1}{400}\right) + \left(\frac{V_3}{700}\right) = 0$$

Super Node

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{500}\right) + \left(\frac{V_3}{700}\right) = 0$$

Problem 5) There are 5 voltage nodes plus a dependent source. You need 6 equations to solve for 6 unknowns



Start with the easy equations:

$$V_x = V_1 - V_2$$

$$V_2 - V_3 = 3$$

$$V_0 = 5V_x$$

Node V1:

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1 - V_2}{200}\right) + \left(\frac{V_1 - V_2}{300}\right) - 2 = 0$$

Node V4

$$2 + \left(\frac{V_4 - V_3}{400}\right) + \left(\frac{V_4}{500}\right) = 0$$

Super Node

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_3}{700}\right) - 2 = 0$$

Other super nodes are also valid.