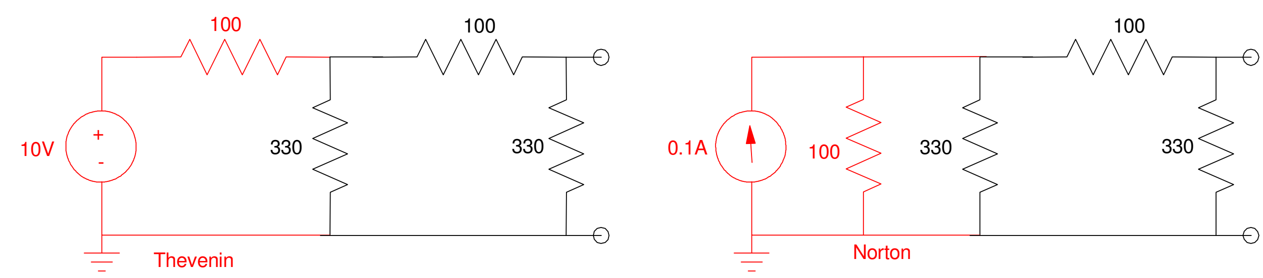
EE 206: Solution #5

Thevenin Equivalents, Maximum Power Transfer. Due Wed, Feb 20th

Thevenin Equivalents

1) Find the Thevenin equivalent for the following circuit:

Convert the 10V and 100 Ohm to a Norton equivalent



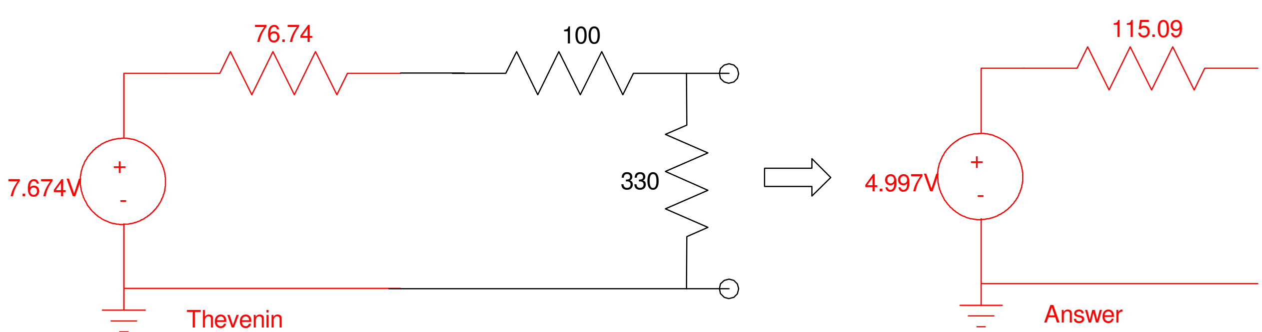
Combine the 100 and 330 Ohms

$$100||330 = 76.74\Omega$$

Convert back to Thevenin

$$R_{th} = R_N = 76.74\Omega$$

$$V_{th} = I_N R_N = 0.1A \cdot 76.74\Omega = 7.646V$$



Now find the Thevenin voltage and resistance

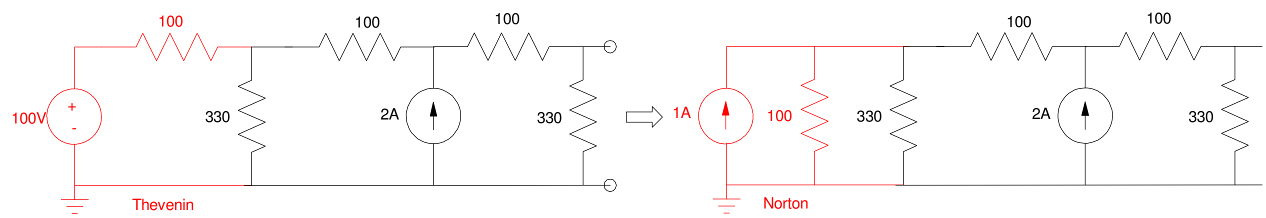
$$V_{th} = V_{open} = \left(\frac{330}{330+100+76.64}\right)7.647V = 4.997V$$

$$R_{th} = 330||176.74 = 115.09\Omega$$

2) Find the Thevenin equivalent for the following circuit:

Convert from Thevenin to Norton

$$I_N = \frac{V_{th}}{R_{th}} = \frac{100V}{100\Omega} = 1A$$



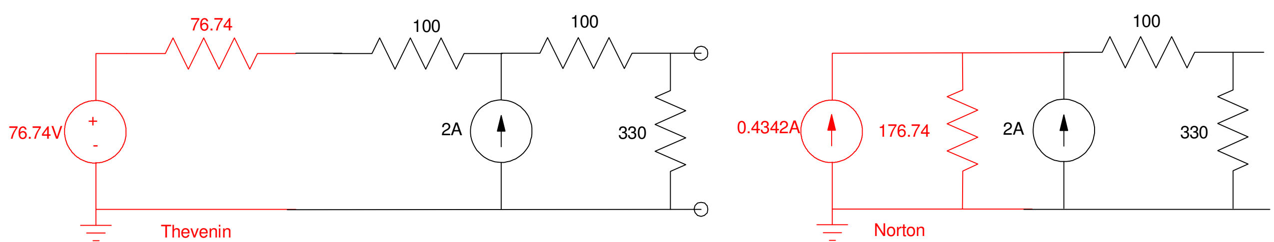
Combine the 100 Ohm and 330 Ohm in parallel

$$100||330 = 76.74\Omega$$

Convert back to Thevenin

$$R_{th} = R_N = 76.74\Omega$$

$$V_{th} = I_N R_N = 1A \cdot 76.74\Omega = 76.74V$$



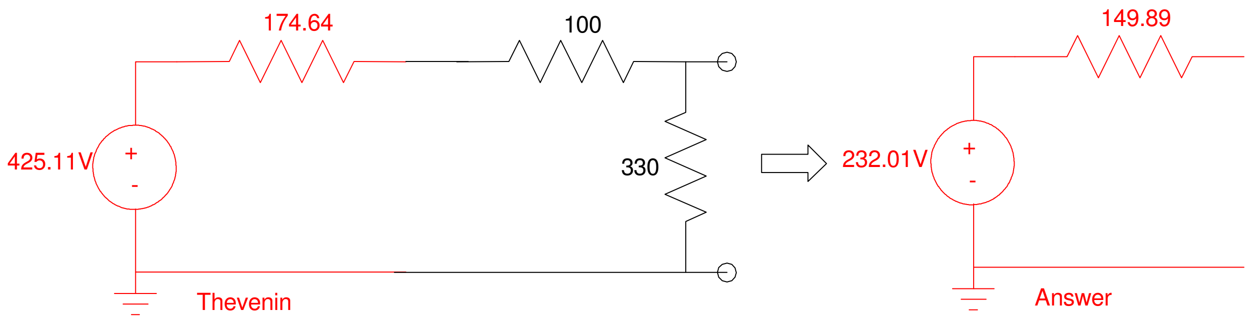
Combine the currents in parallal

$$I_N = 0.4342A + 2A = 2.4342A$$

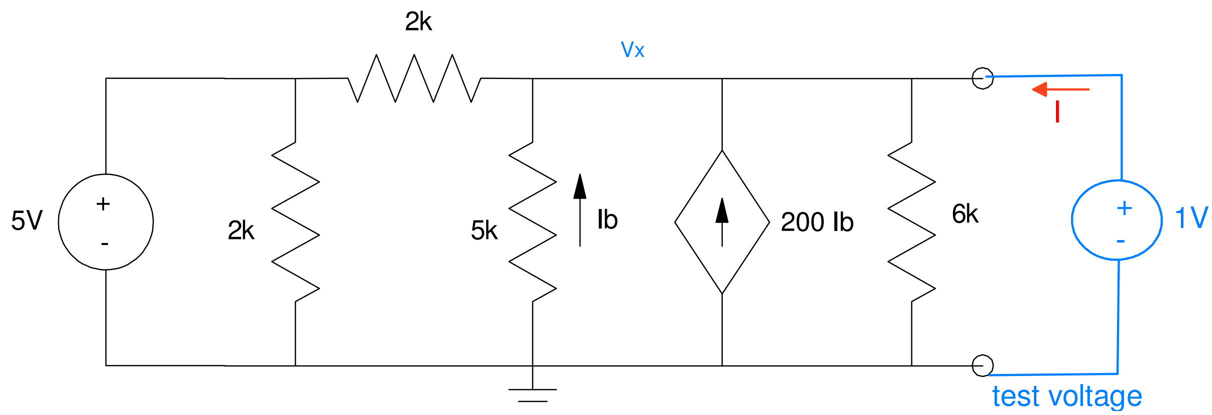
Convert to Thevenin

$$R_{th} = R_N = 174.64\Omega$$

$$V_{th} = I_N R_N = 2.4342A \cdot 174.64\Omega = 425.11V$$


174.64
100
425.11V
+
-
330
Thevenin
149.89
232.01V
+
-
Answer

3) Find the Thevenin equivalent for the following circuit:



Open Circuit Voltage: (Vth)

$$\left(\frac{V_x-5}{2k}\right)+\left(\frac{V_x}{5k}\right)+200\left(\frac{V_x}{5k}\right)+\left(\frac{V_x}{6k}\right)=0$$

$$V_x=0.0612V$$

Resistance: Turn off the 5V supply and measure the resistance.

This isn't obvious so apply a 1V test voltage and compute the current

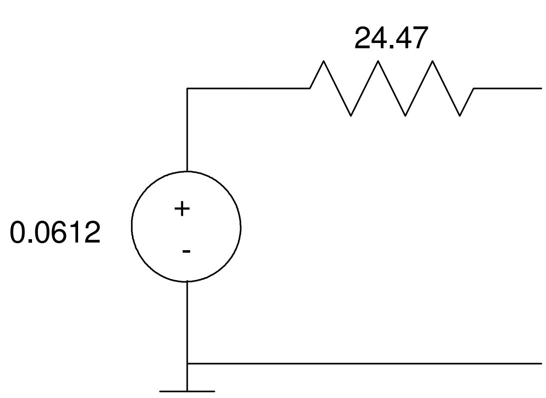
$$I=\left(\frac{1V}{2k}\right)+\left(\frac{1V}{5k}\right)+200\left(\frac{1V}{5k}\right)+\left(\frac{1V}{6k}\right)$$

$$I=40.9mA$$

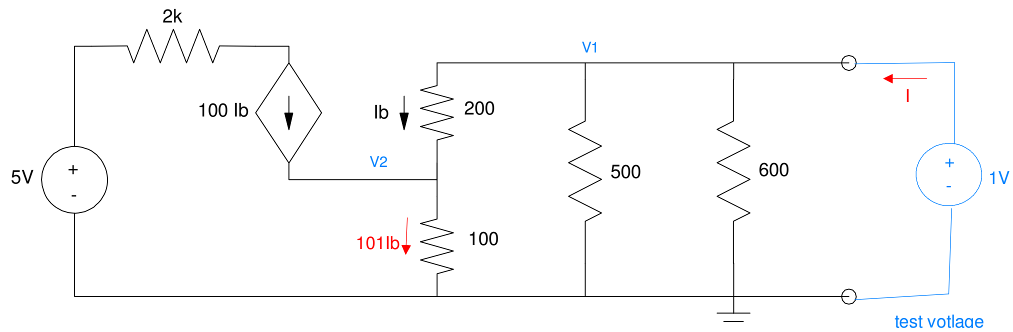
The resistance is then

$$R_{th}=\frac{1V}{40.9mA}=24.47\Omega$$

answer



4) Find the Thevenin equivalent for the following circuit:



Vth: Measure the open-circuit voltage. Write N equations for N unknowns

$$I_b = \left(\frac{V_1 - V_2}{200}\right)$$

$$\left(\frac{V_1 - V_2}{200}\right) + \left(\frac{V_1}{500}\right) + \left(\frac{V_1}{600}\right) = 0$$

$$-100I_b - I_b + \left(\frac{V_2}{100}\right) = 0$$

There are no forcing funcitons so the solutions is Ib = V1 = V2 = 0

Vth = 0

Rth: Turn off the input and solve for the resitance at the output. This isn't obvious so add a 1V test voltage. First solve for Ib

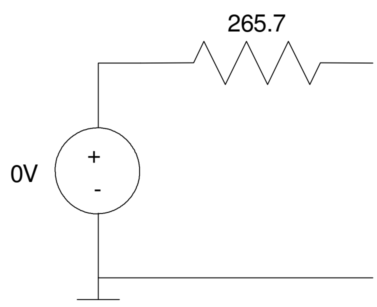
$$200\Omega \cdot I_b + 101I_b \cdot 100\Omega = 1V$$

$$I_b = 97.09\mu A$$

$$I = \left(\frac{1}{500}\right) + \left(\frac{1}{600}\right) + I_b = 3.764mA$$

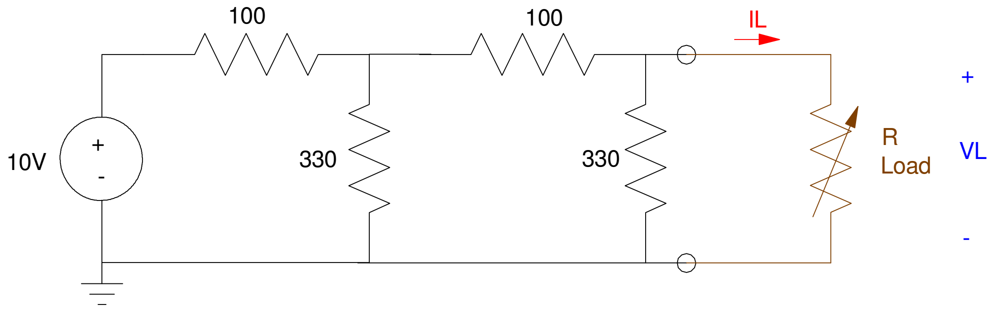
$$R_{th} = \frac{1V}{3.764mA} = 265.7\Omega$$

answer:

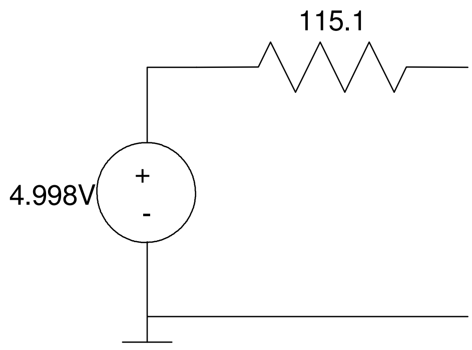


Maximum Power Transfer

5) Determine R so that the maximum power is delivered to the load (R)



From problem #2, the Thevenin equivalent is



The maximum power to the load is when R = Rth

answer: **R = 115.1 Ohms**

The maximum power is then

$$I = \left(\frac{4.998V}{115.1+115.1}\right) = 21.71mA$$

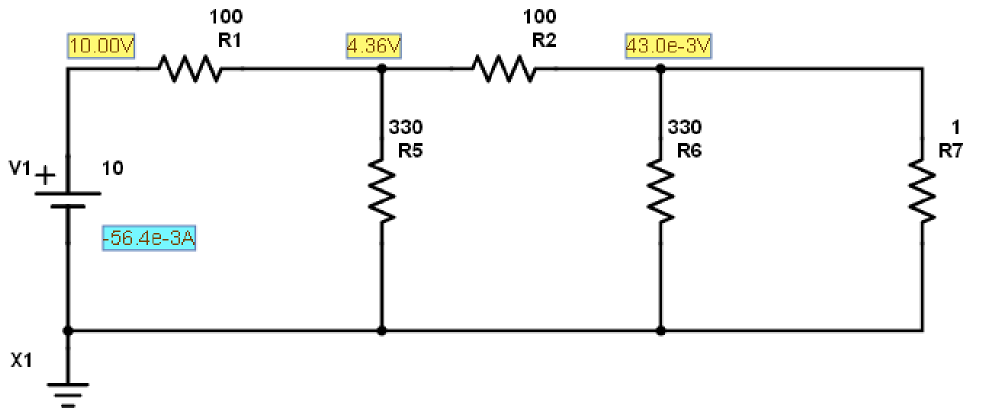
$$P = I^2R = (21.71mA)^2 \cdot 115.1\Omega = 54.26mW$$

answer: 54.26mW

PartSim

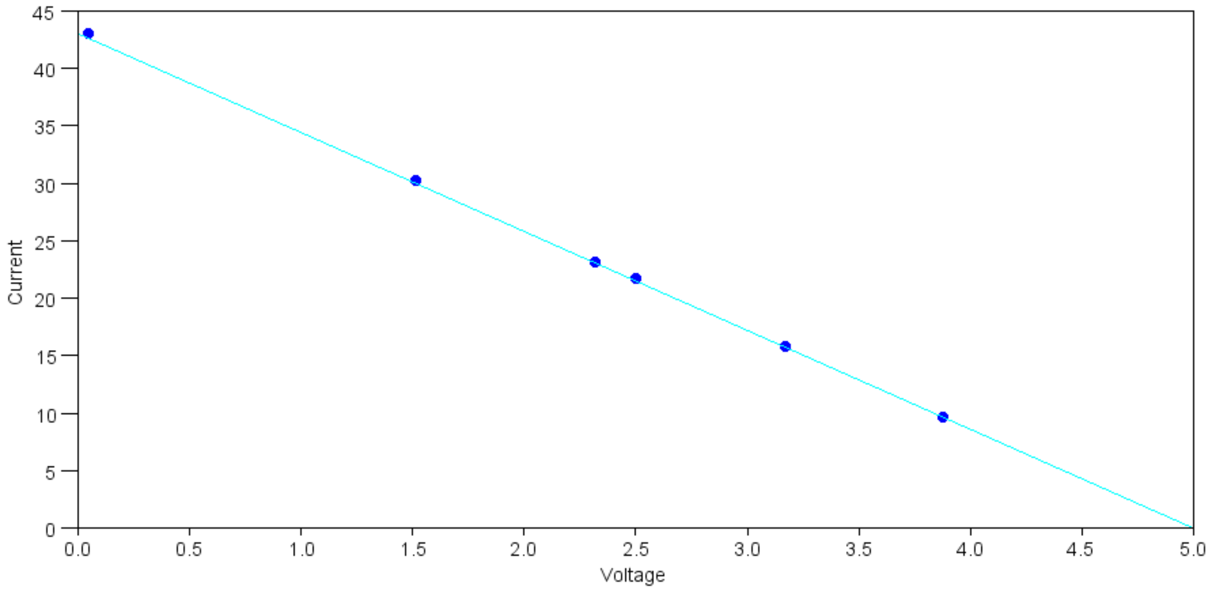
6) Simulate the circuit of problem 5. Determine the voltage and current at the load for

- R = 0
- R = 50 Ohms
- R = max power (from problem #5)
- R = 100 Ohms
- R = 200 Ohms
- R = infinity



	V	I	Power
R = 1	43.0mV	43.0mA	1.849mW
R = 50	1.51V	30.20 mA	45.60 mW
R = 100	2.32V	23.2 mA	53.82 mW
R = 115.1	2.50V	21.72 mA	54.30 mW max power
R = 200	3.17V	15.85 mA	50.24 mW
R = 400	3.88V	9.70 mA	37.64 mW
R = infinite	5.00V	0 mA	0 mW

7) Plot V vs. I on a graph and draw a line between these points.



How does this line relate to the Thevenin equivalent for circuit #5?

- The open-circuit voltage is Vth
- The short circuit current is Vth / Rth