EE 206: Homework #10
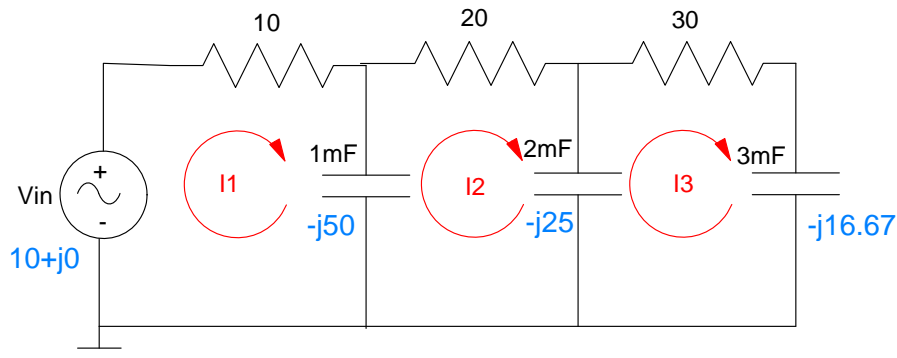
Current Loops, Op-Amps, Superposition with Phasors
Due Monday, April 8th

Problem 1) Assume

$$V_{in} = 10\cos(20t)$$

1a) Write the current-loop equations for the following circuit.

First, convert to phasor form (shown in blue)

$$10\cos(20t) \rightarrow 10 + j0$$

$$1mF \rightarrow \frac{1}{j\omega C} = \frac{1}{j \cdot 20 \cdot 0.001} = -j50$$

Now write the current loop equations

$$-10 + 10I_1 - j50(I_1 - I_2) = 0$$

$$-j50(I_2 - I_1) + 20I_2 - j25(I_2 - I_3) = 0$$

$$-j25(I_3 - I_2) + 30I_3 - j16.67I_3 = 0$$

1b) Solve for I1, I2, and I3

Group terms and place in matrix form

$$\begin{bmatrix} (10 - j50) & j50 & 0 \\ j50 & (20 - j75) & j25 \\ 0 & j25 & (30 - j41.67) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
A = [10-j*50,j*50,0 ; j*50,20-j*75,j*25 ; 0,j*25,30-j*41.67]

   10.0000 -50.0000i        0 +50.0000i        0
        0 +50.0000i  20.0000 -75.0000i        0 +25.0000i
        0                    0 +25.0000i  30.0000 -41.6700i

B = [10;0;0];
I = inv(A)*B

I1    0.2758 + 0.2017i
I2    0.2354 + 0.0569i
I3    0.1092 - 0.0445i
```

meaning...
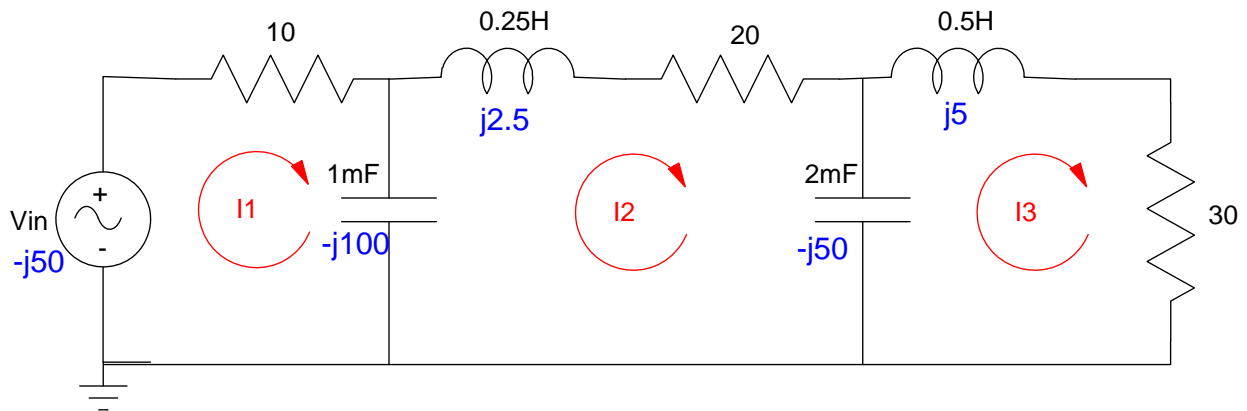
$$i_1(t) = 0.2758\cos(20t) - 0.2017\sin(20t)$$
$$i_2(t) = 0.2354\cos(20t) - 0.0569\sin(20t)$$
$$i_3(t) = 0.1092\cos(20t) + 0.0445\sin(20t)$$

Problem 2) Assume

$$V_{in} = 50\sin(10t)$$

2a) Write the current loop equations for the following circuit

First convert to phasor notation

$$50\sin(10t) \rightarrow 0 - j50$$

$$0.25H \rightarrow j\omega L = j \cdot 10 \cdot 0.25 = j2.5$$

$$1mF \rightarrow \left(\frac{1}{j\omega C}\right) = \left(\frac{1}{j \cdot 10 \cdot 0.001}\right) = -j100$$

Write the loop equations

$$-(-j50) + 10I_1 - j100(I_1 - I_2) = 0$$

$$-j100(I_1 - I_2) + (20 + j2.5)I_2 - j50(I_2 - I_3) = 0$$

$$-j50(I_3 - I_2) + j5I_3 + 30I_3 = 0$$

2b) Solve for I1, I2, and I3

Group terms and place in matrix form

$$\begin{bmatrix} (10 - j100) & j100 & 0 \\ j100 & (20 - j147.5) & j50 \\ 0 & j50 & (30 - j45) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -j50 \\ 0 \\ 0 \end{bmatrix}$$

```
>> A = [10-j*100,j*100,0 ; j*100,20-j*147.5,j*50 ; 0,j*50,30-j*45]

   1.0e+002 *

   0.1000 - 1.0000i        0 + 1.0000i        0
        0 + 1.0000i   0.2000 - 1.4750i        0 + 0.5000i
        0                   0 + 0.5000i   0.3000 - 0.4500i

>> B = [-j*50;0;0]

        0 -50.0000i
        0
        0

>> I = inv(A)*B

   0.4769 - 0.9264i
   0.0695 - 0.8788i
  -0.3972 - 0.7116i
```

meaning

$$i_1(t) = 0.4769\cos(10t) + 0.9264\sin(10t)$$
$$i_2(t) = 0.0695\cos(10t) + 0.8788\sin(10t)$$
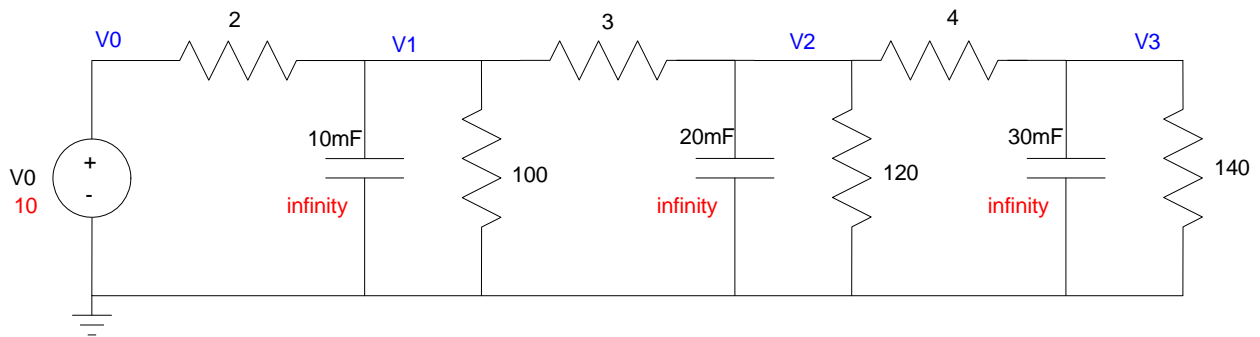$$i_3(t) = -0.3972\cos(10t) + 0.7116\sin(10t)$$

Problem 3) Find V1, V2, and V3 assuming
- a) $V_{in} = 10$
- b) $V_{in} = 5\sin(2t)$
- c) $V_{in} = 10 + 5\sin(2t)$

a) Vin = 10

$$V_0 = 10$$

$$\frac{1}{j\omega C} = \infty$$

Write the voltage node equtions

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{2}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1}{\infty}\right) + \left(\frac{V_1 - V_2}{3}\right) = 0$$

$$\left(\frac{V_2 - V_1}{3}\right) + \left(\frac{V_2}{120}\right) + \left(\frac{V_2}{\infty}\right) + \left(\frac{V_2 - V_3}{4}\right) = 0$$

$$\left(\frac{V_3 - V_2}{4}\right) + \left(\frac{V_3}{140}\right) + \left(\frac{V_3}{\infty}\right) = 0$$

Group terms and place in matrix form

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
\left(\frac{-1}{2}\right) & \left(\frac{1}{2} + \frac{1}{100} + \frac{1}{3}\right) & \left(\frac{-1}{3}\right) & 0 \\
0 & \left(\frac{-1}{3}\right) & \left(\frac{1}{3} + \frac{1}{120} + \frac{1}{4}\right) & \left(\frac{-1}{4}\right) \\
0 & 0 & \left(\frac{-1}{4}\right) & \left(\frac{1}{4} + \frac{1}{140}\right)
\end{bmatrix}
\begin{bmatrix}
V_0 \\
V_1 \\
V_2 \\
V_3
\end{bmatrix}
=
\begin{bmatrix}
10 \\
0 \\
0 \\
0
\end{bmatrix}
$$

Solve in Matlab

```
>> A = [1,0,0,0 ; -1/2,1/2+1/100+1/3,-1/3,0;0,-1/3,1/3+1/120+1/4,-1/4 ;
0,0,-1/4,1/4+1/140]

    1.0000         0         0         0
   -0.5000    0.8433   -0.3333         0
        0   -0.3333    0.5917   -0.2500
        0         0   -0.2500    0.2571

>> B = [10;0;0;0]

   10
    0
    0
    0

>> V = inv(A)*B

V1   10.0000
V2    9.5309
V3    9.1132
V4    8.8601
```
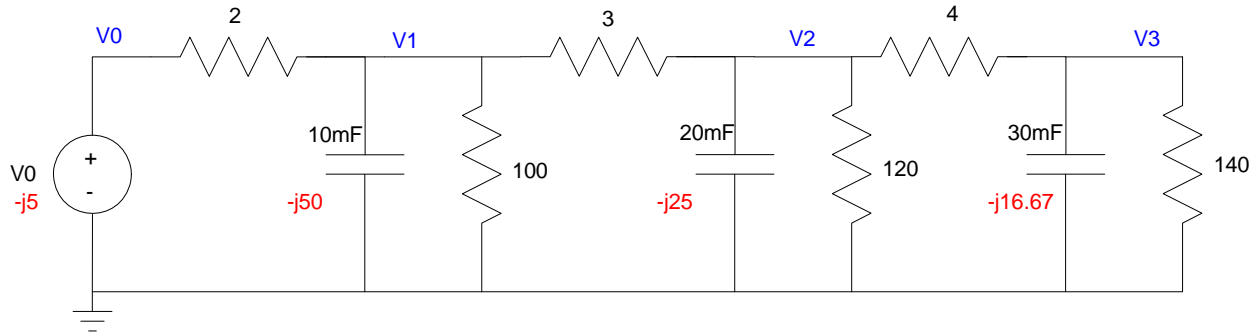
3b) Assume

$$V_{in} = 5\sin(2t)$$

Comvert to phasor form

$$5\sin(2t) \rightarrow 0 - j5$$

$$10mF \rightarrow \left(\frac{1}{j\omega C}\right) = -j50$$

Write the voltage node equations

$$V_0 = -j5$$

$$\left(\frac{V_1 - V_0}{2}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1}{-j50}\right) + \left(\frac{V_1 - V_2}{3}\right) = 0$$

$$\left(\frac{V_2 - V_1}{3}\right) + \left(\frac{V_2}{120}\right) + \left(\frac{V_2}{-j25}\right) + \left(\frac{V_2 - V_3}{4}\right) = 0$$

$$\left(\frac{V_3 - V_2}{4}\right) + \left(\frac{V_3}{140}\right) + \left(\frac{V_3}{-j16.67}\right) = 0$$

Group terms and place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{2}\right) & \left(\frac{1}{2} + \frac{1}{100} + \frac{1}{3} + \frac{1}{-j50}\right) & \left(\frac{-1}{3}\right) & 0 \\ 0 & \left(\frac{-1}{3}\right) & \left(\frac{1}{3} + \frac{1}{120} + \frac{1}{4} + \frac{1}{-j25}\right) & \left(\frac{-1}{4}\right) \\ 0 & 0 & \left(\frac{-1}{4}\right) & \left(\frac{1}{4} + \frac{1}{140} + \frac{1}{-j16.67}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -j5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> >> A = [1,0,0,0 ;
-1/2,1/2+1/100+1/3-1/(j*50),-1/3,0;0,-1/3,1/3+1/120+1/4-1/(j*25),-1/4 ;
0,0,-1/4,1/4+1/140-1/(j*16.67)]

    1.0000                    0                    0                    0
   -0.5000          0.8433 + 0.0200i  -0.3333                    0
         0         -0.3333           0.5917 + 0.0400i  -0.2500
         0                    0         -0.2500           0.2571 + 0.0600i

>> B = [-j*5;0;0;0]

        0 - 5.0000i
        0
        0
        0

>> V = inv(A)*B

V0       0 - 5.0000i
V1  -0.7476 - 4.3759i
V2  -1.6289 - 3.6158i
V3  -2.2796 - 2.9836i
```

meaning

$$
\begin{array}{l}
v_0 = 5\sin(2t) \\
v_1 = -0.7476\cos(2t) + 4.3759\sin(2t) \\
v_2 = -1.6289\cos(2t) + 3.6158\sin(2t) \\
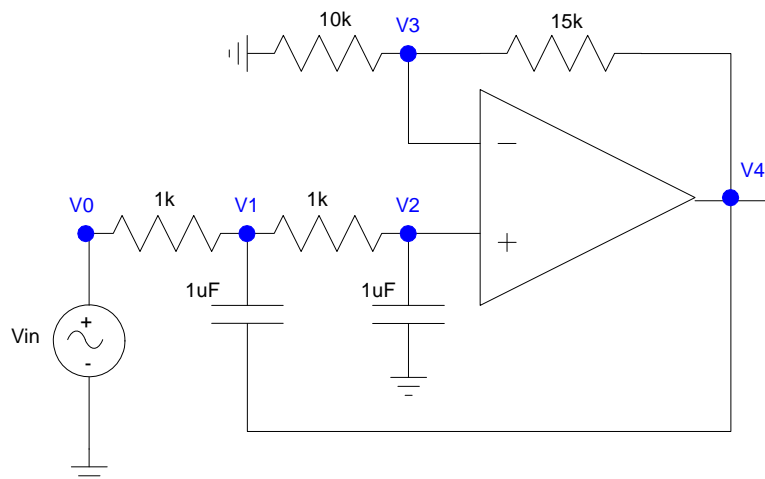v_3 = -2.2796\cos(2t) + 2.9836\sin(2t)
\end{array}
$$

3c) Find the votlages for

$$V_{in} = 10 + 5\sin(2t)$$

Using superposition, it's the sum of part a) and part b)

$$
\begin{array}{l}
v_0 = 10 + 5\sin(2t) \\
v_1 = 9.5309 - 0.7476\cos(2t) + 4.3759\sin(2t) \\
v_2 = 9.1132 - 1.6289\cos(2t) + 3.6158\sin(2t) \\
v_3 = 8.8601 - 2.2796\cos(2t) + 2.9836\sin(2t)
\end{array}
$$

Problem 4) Determine V1 .. V4 assuming

a) $V_{in} = 10$

Capacitors are open circuits

The node equations are

$$V_3 = V_2$$

$$V_2 = V_0$$

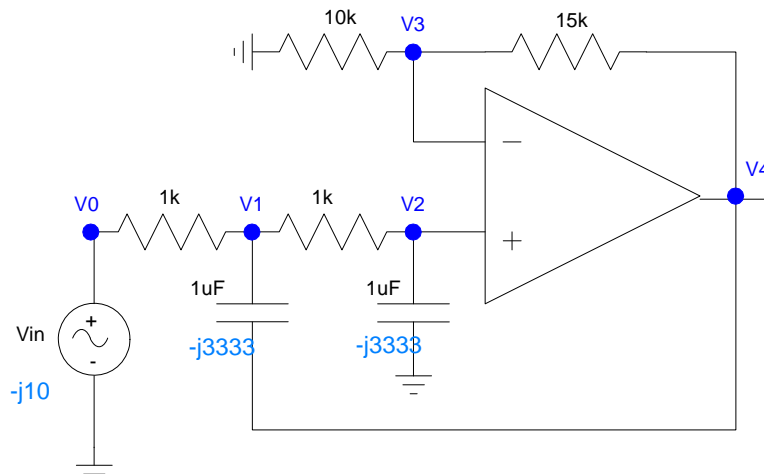$$\left(\frac{V_3}{10k}\right) + \left(\frac{V_3 - V_4}{15k}\right) = 0$$

This results in

$$V_4 = 2.5 V_{in}$$

$$V_4 = 25$$

- V0 = 10
- V1 = 10
- V2 = 10
- V3 = 10
- V4 = 25

b) $V_{in} = 10\sin(300t)$

Convert to phasors

$$V_{in} \rightarrow 0 - j10$$

$$1\mu F \rightarrow \left(\frac{1}{j\omega C}\right) = -j3333\Omega$$

Write the votlage node equations

$$V_0 = -j10$$

$$\left(\frac{V_1 - V_0}{1000}\right) + \left(\frac{V_1 - V_4}{-j3333}\right) + \left(\frac{V_1 - V_2}{1000}\right) = 0 \qquad * 1000$$

$$\left(\frac{V_2 - V_1}{1000}\right) + \left(\frac{V_2}{-j3333}\right) = 0 \qquad * 1000$$

$$\left(\frac{V_3}{10000}\right) + \left(\frac{V_3 - V_4}{15000}\right) = 0 \qquad * 10,000$$

$$V_2 = V_3 \qquad node\ equation\ at\ V4$$

Group terms and place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & \left(2+\frac{1}{-j3.33}\right) & -1 & 0 & \left(\frac{-1}{-j3.33}\right) \\ 0 & -1 & \left(1+\frac{1}{-j3.33}\right) & 0 & 0 \\ 0 & 0 & 0 & \left(1+\frac{1}{1.5}\right) & \frac{-1}{1.5} \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -j10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve

```
a1 = [1,0,0,0,0];
a2 = [-1,2-1/(j*3.33),-1,0,1/(j*3.33)]
a3 = [0,-1,1+1/(-j*3.33),0,0]
a4 = [0,0,0,1+1/1.5,-1/1.5]
a5 = [0,0,1,-1,0]

A = [a1;a2;a3;a4;a5]

    1.0000                0                0                0                0
   -1.0000   2.0000 + 0.3003i  -1.0000                     0           0 - 0.3003i
        0       -1.0000         1.0000 + 0.3003i           0                0
        0            0                0              1.6667        -0.6667
        0            0          1.0000              -1.0000             0

B = [-j*10;0;0;0;0]

        0 -10.0000i
        0
        0
        0
        0

V = inv(A)*B

        0 -10.0000i
   1.4473 -11.2300i
  -1.7658 -10.6998i
  -1.7658 -10.6998i
  -4.4145 -26.7494i
```

meaning

$$v_0 = 10\sin(300t)$$
$$v_1 = 1.4473\cos(300t) + 11.2300\sin(300t)$$
$$v_2 = -1.7658\cos(300t) + 10.6998\sin(300t)$$
$$v_3 = -1.7658\cos(300t) + 10.6998\sin(300t)$$
$$v_4 = -4.4145\cos(300t) + 26.7494\sin(300t)$$

c) $V_{in} = 10 + 10 \sin(300t)$

Use superposition: it's the sum of part a) and part b)

$$v_0 = 10 + 10 \sin(300t)$$
$$v_1 = 10 + 1.4473 \cos(300t) + 11.2300 \sin(300t)$$
$$v_2 = 10 - 1.7658 \cos(300t) + 10.6998 \sin(300t)$$
$$v_3 = 10 - 1.7658 \cos(300t) + 10.6998 \sin(300t)$$
$$v_4 = 25 - 4.4145 \cos(300t) + 26.7494 \sin(300t)$$