EE 206: Solution to Homework #11

Fourier Transform and Superposition with Phasors
Due Monday, April 15th

Let Vin be a 100Hz half-rectified sine wave

$$V_{in} = \begin{cases} 10\sin(628t) & \sin(628t) > 0 \\ 0 & otherwise \end{cases}$$

1) Find y(t) by approximating Vin as

$$V_{in} = a + b\sin(628t)$$

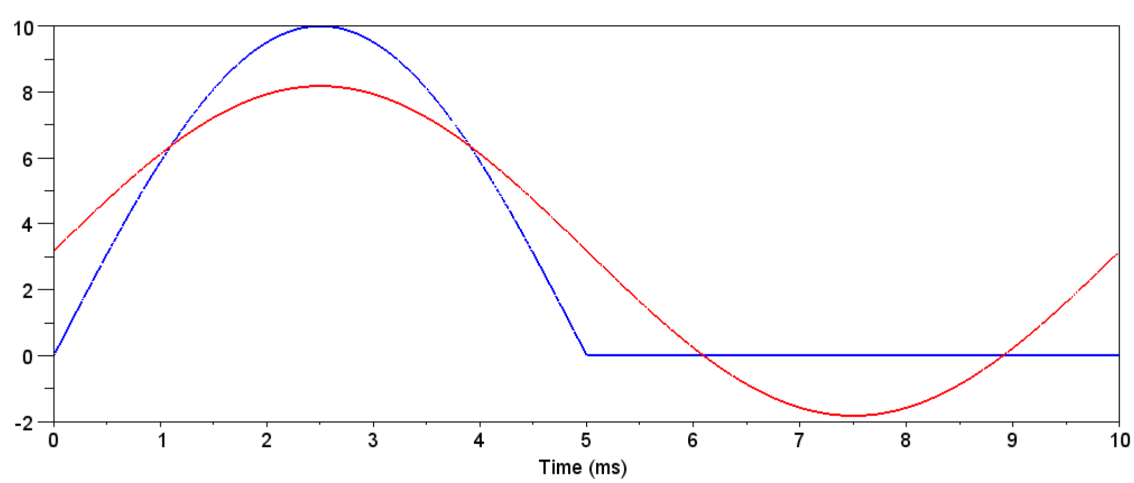
where

- a = average(Vin)
- b = 1/2 of the peak-to-peak votlage of Vin

The DC voltage is (using Matlab)

```
w = 628;
T = 2*pi/w

 T  = 0.01

t = [0:0.0001:1]' * T;
Vin = max(0, 10*sin(200*t));
a = mean(Vin)

 a  = 3.1827805

b = (max(Vin) - min(Vin)) / 2

 b  =    5.
```

$$V_{in} \approx 3.18 + 5\sin(628t)$$



Vin (blue) and an approximation for Vin (red)
The red line has i) The same DC voltage, ii) The same frequency, and iii) The same peak-to-peak ripple as Vin

2) Determine the first 3 terms of the Fourier series approximation for Vin

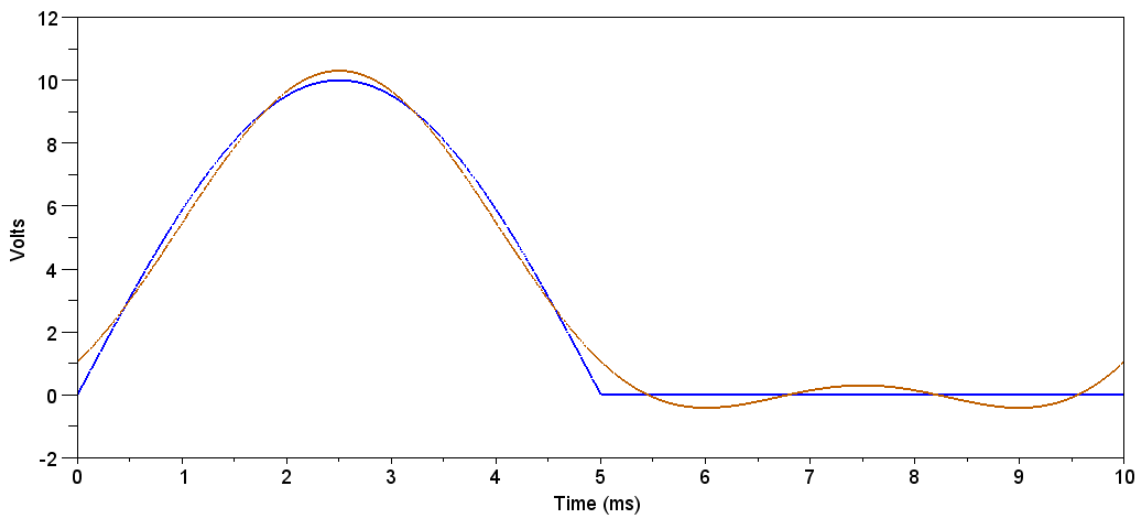
$$V_{in} \approx a_0 + a_1\cos(628t) + b_1\sin(628t) + a_2\cos(1256t) + b_2\sin(1256t)$$

Using Matlab:

```
a0 = mean(Vin)

 a0  =  3.1827805

a1 = 2*mean(Vin .* cos(628*t))

 a1  =  0

b1 = 2*mean(Vin .* sin(628*t))

 b1  = 4.9995

a2 = 2*mean(Vin .* cos(1256*t))

 a2  = - 2.1218539

b2 = 2*mean(Vin .* sin(1256*t))
 b2  =  0
```

This means

$$V_{in} \approx 3.18 + 5\sin(628t) - 2.12\cos(1256t)$$



Vin (blue) and it's Fourier Series apprximation (red)

Note: Using the DC level and the peak-to-peak value (problem #1) was

- Exact for the DC term, and
- Close for the 1st harmonic (in amplitude) but not phase, and
- *Much* easier to compute than the Fourier series approximation,

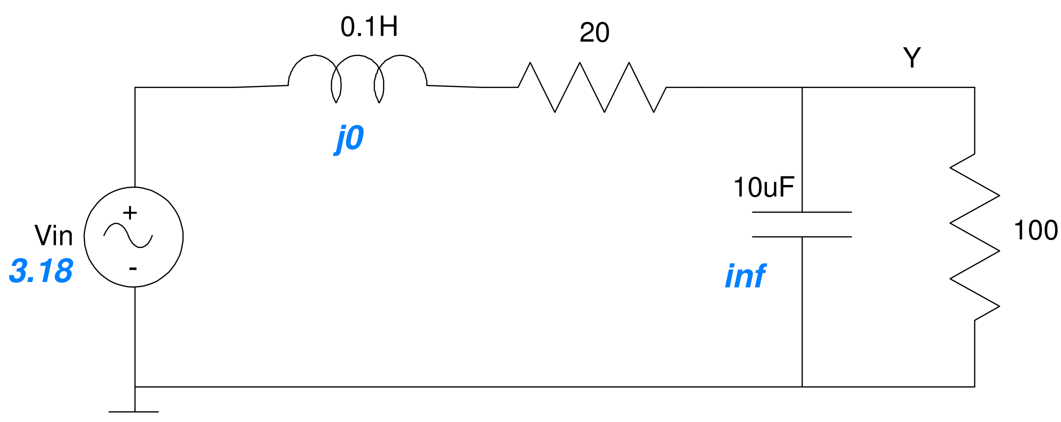
3) For your result of problem #2, determine y(t)

DC: Vin = 3.18

$$\omega = 0$$

$$L \rightarrow j\omega L = 0$$

$$C \rightarrow \frac{1}{j\omega C} = \infty$$



By voltage division

$$Y = \left(\frac{100}{100+20}\right) \cdot 3.18$$

$$Y = 2.65$$

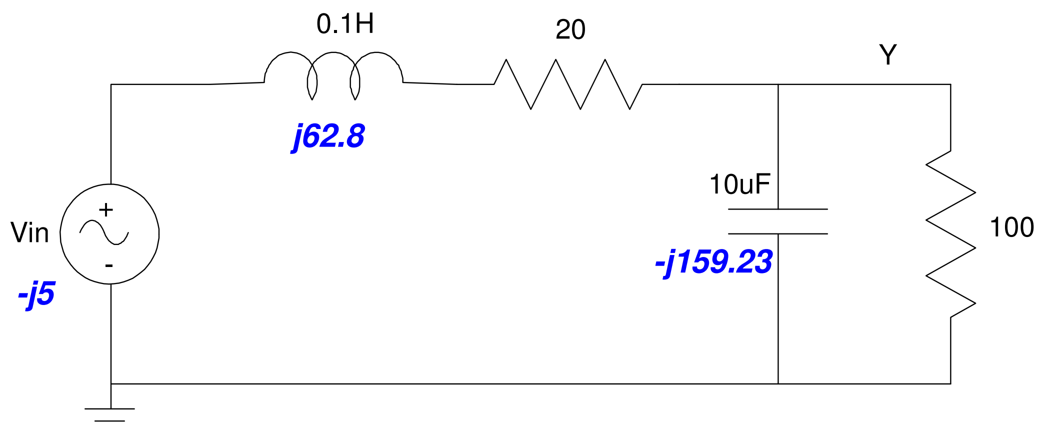
ii) $V_{in} = 5\sin(628t)$

$$\omega = 628$$

$$V_{in} \to 0 - j5$$

$$L \to j\omega L = j62.8$$

$$C \to \frac{1}{j\omega C} = -j159.23$$



Adding R and C in parallel:

$$100 || -j159.23 = \left(\frac{1}{100} + \frac{1}{-j159.23}\right)^{-1} = 71.71 - j45.04$$

By voltage division

$$Y = \left(\frac{(71.71 - j45.04)}{(71.71 - j45.04) + (20 + j62.8)}\right)(0 - j5) = -3.10 - j3.31$$

meaning

$$y(t) = -3.10\cos(628t) + 3.31\sin(628t)$$

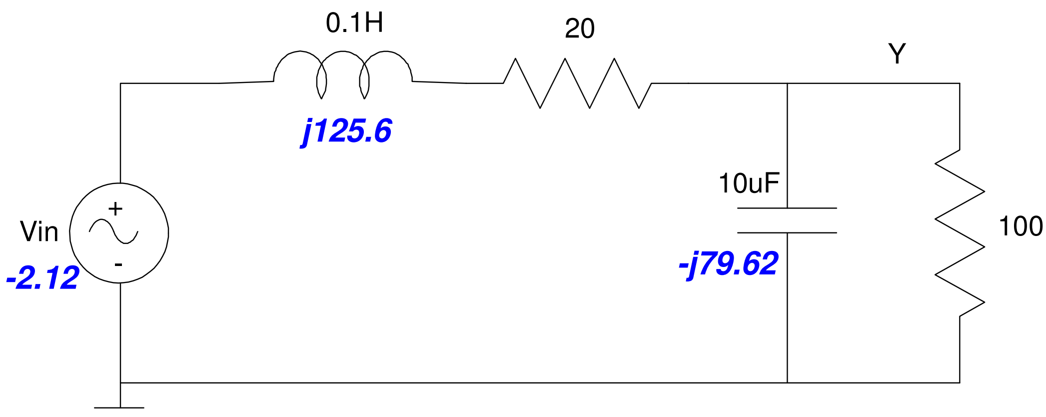
iii) $V_{in} = -2.12\cos(1256t)$

$\omega = 1256$

$V_{in} \rightarrow -2.12 + j0$

$L \rightarrow j\omega L = j125.6$

$C \rightarrow \frac{1}{j\omega C} = -j79.62$



Adding R and C in parallel

$$100 || -j79.62 = \left(\frac{1}{100} + \frac{1}{-j79.62}\right)^{-1} = 38.80 - j48.73$$

From voltage division

$$Y = \left(\frac{(38.80 - j48.73)}{(38.80 - j48.73) + (20 + j125.6)}\right)(-2.12 + j0)$$

$$Y = 0.33 + j1.32$$

meaning

$$y(t) = 0.33\cos(1256t) - 1.32\sin(1256t)$$

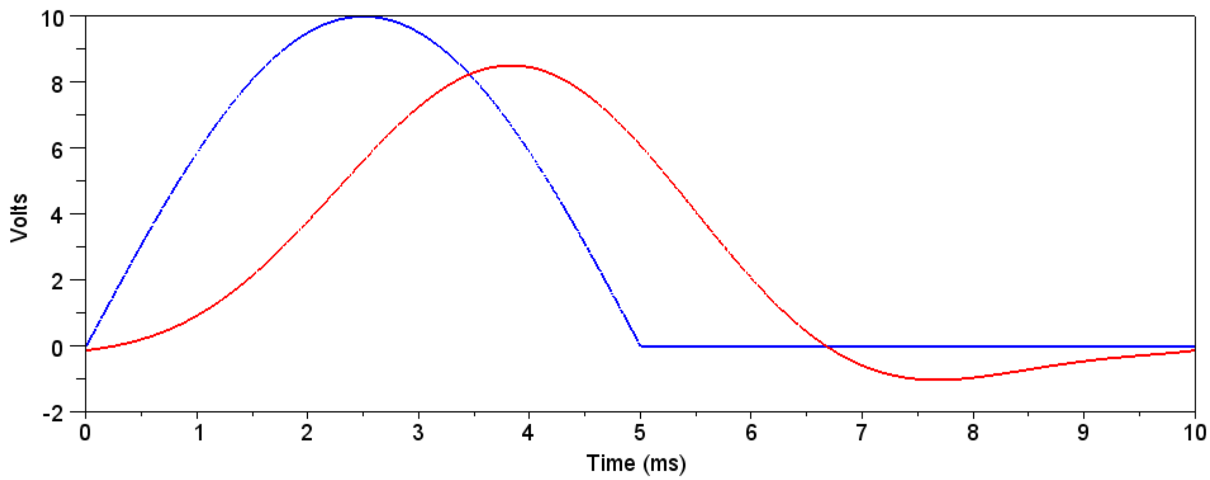
The total input is the sum of all three terms (DC, 628, 1256 rad/sec)

The total output is the sum of all three outputs

$$y(t) = 2.65 - 3.10\cos(628t) + 3.31\sin(628t) + 0.33\cos(1256t) - 1.32\sin(1256t)$$

In Matlab

```
y1 = 2.65;

y2 = -3.10*cos(628*t) + 3.31*sin(628*t);

y3 = 0.33*cos(1256*t) - 1.32*sin(1256*t);

plot(t*1000,Vin,t*1000,y1+y2+y3);
xlabel('Time (ms)');
ylabel('Volts');
```



Vin (blue) and Y (red)