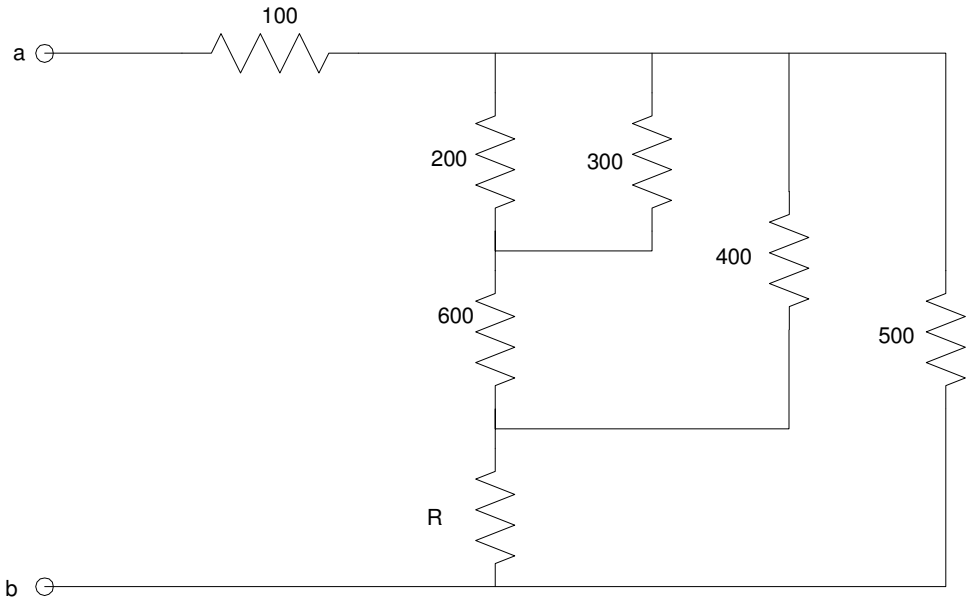
EE 206: Homework #3 Solution

Voltage and Current Division. Voltage Nodes. Due Mon, Feb 3rd

Resistors in Series and Parallel

1) Assume R = 100. Determine the total resistance, Rab



Work from the inside out

200 || 300 = 120

120 + 600 = 720

720 || 400 = 257.14

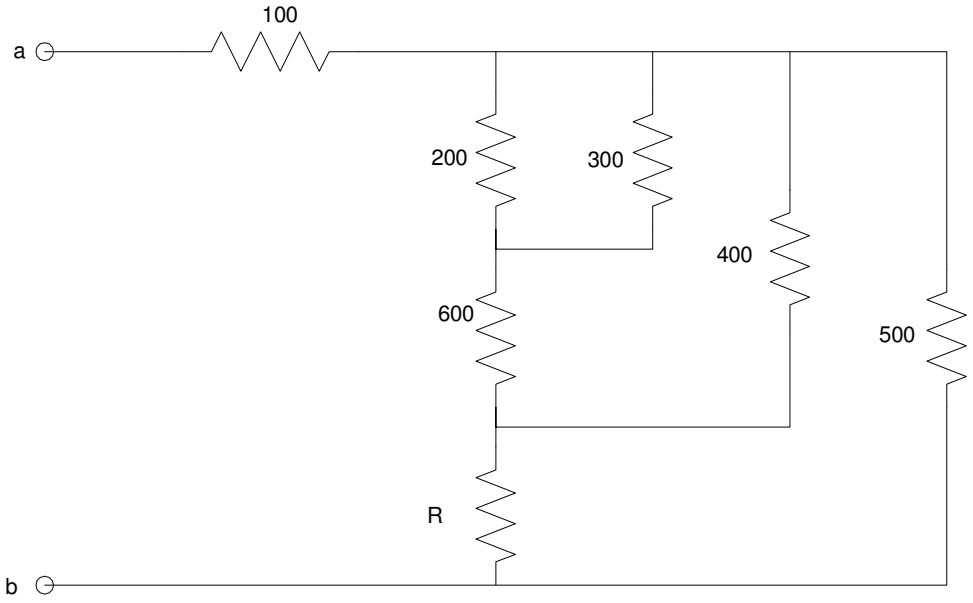
257.14 + 100 = 357.14

357.13 || 500 = 208.33

208.33 + 100 = 308.33

answer: Rab = 308.33 Ohms

2) Assume the total resistance is Rab = 400 Ohms. Determine R.



Rab = 400 = 100 + Rc

Rc = 300

300 = 500 || Rd

$$\frac{1}{300} = \frac{1}{500} + \frac{1}{R_d}$$

Rd = 750

Simplifying the inside

200 || 300 = 120

120 + 600 = 720

720 || 400 = 257.14

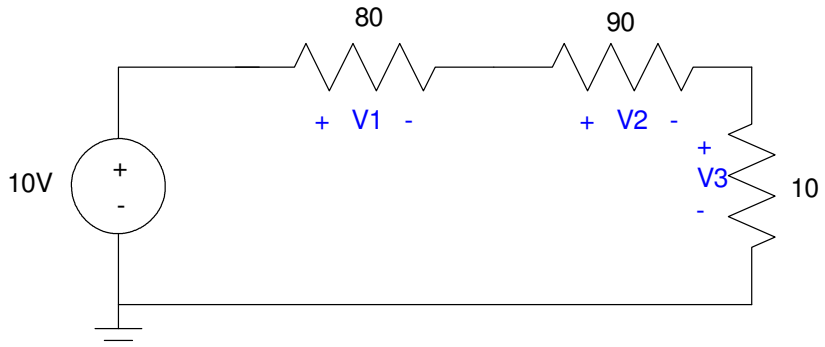
Then...

257.14 + R = 750

R = 492.86 Ohms

answer: R = 492.86 Ohms

Voltage Division:



2a) Use votlage division to determine the voltages V1, V2, and V3

Recall

$$V_x = \left(\frac{R_x}{\text{total R}}\right) V_{in}$$

Then

$$V_1 = \left(\frac{80}{80+90+10}\right) 10V = 4.444V$$

$$V_2 = \left(\frac{90}{80+90+10}\right) 10V = 5.000V$$

$$V_3 = \left(\frac{10}{80+90+10}\right) 10V = 0.556V$$

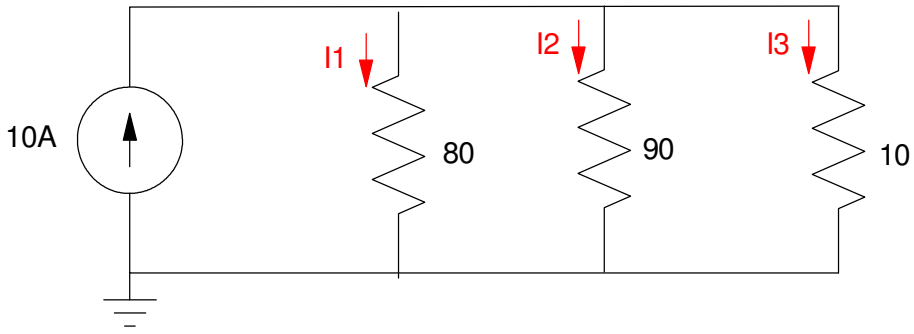
Note: V1 + V2 + V3 = 10.000V

2b) Determine the power dissipated in the 10 Ohm resistor (in Watts)

$$P = VI = \frac{V^2}{R} = I^2R$$

$$P = \frac{V^2}{R} = \frac{(0.556V)^2}{10\Omega} = 30.9mW$$

Problem 2)



2a) Use current division to determine the current I1, I2, and I3

$$I_1 = \left(\frac{\frac{1}{80}}{\frac{1}{80}+\frac{1}{90}+\frac{1}{10}} \right) 10A = 1.0112A$$

$$I_2 = \left(\frac{\frac{1}{90}}{\frac{1}{80}+\frac{1}{90}+\frac{1}{10}} \right) 10A = 0.8989A$$

$$I_3 = \left(\frac{\frac{1}{10}}{\frac{1}{80}+\frac{1}{90}+\frac{1}{10}} \right) 10A = 8.0899A$$

Note: I1 + I2 + I3 = 10A

2b) Determine the power dissipated in the 10 Ohm resistor (in Watts)

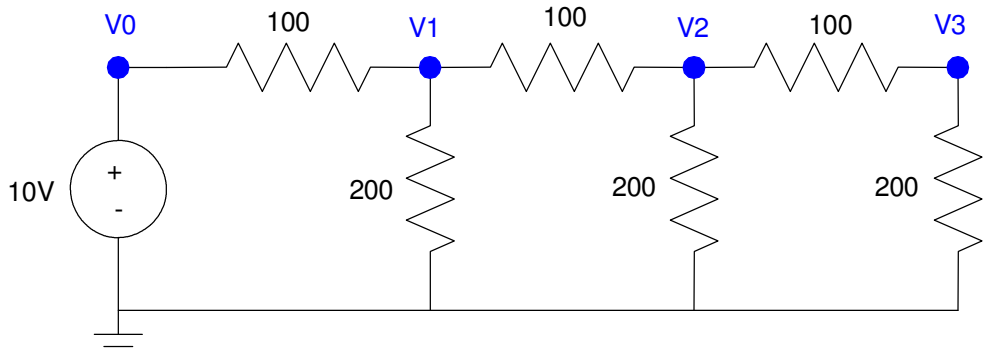
$$P = VI = \frac{V^2}{R} = I^2R$$

$$P = I^2R$$

$$P = (8.8099A)^2 \cdot 10\Omega = 654.46W$$

Voltage Nodes:

3) Write the votlage node equations for the following circuit. Solve for V1..V3 using Matlab (or similar program)



$$V_0 = 10V$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{200}\right) + \left(\frac{V_1 - V_2}{100}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{200}\right) = 0$$

Group terms

$$\left(\frac{1}{100} + \frac{1}{200} + \frac{1}{100}\right) V_1 - \left(\frac{1}{100}\right) V_2 = \left(\frac{V_0}{100}\right) = \left(\frac{10}{100}\right)$$

$$-\left(\frac{1}{100}\right) V_1 + \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{100}\right) V_2 - \left(\frac{1}{100}\right) V_3 = 0$$

$$-\left(\frac{1}{100}\right) V_2 + \left(\frac{1}{100} + \frac{1}{200}\right) V_3 = 0$$

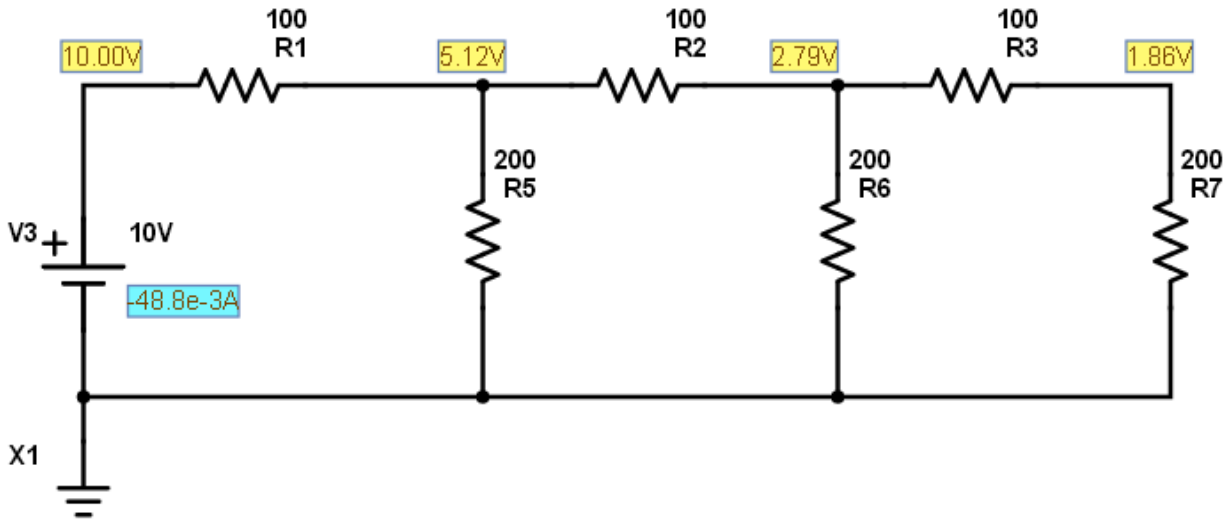
Place in matrix form

$$\begin{bmatrix} \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{100}\right) & \left(\frac{-1}{100}\right) & 0 \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{100}\right) & \left(\frac{-1}{100}\right) \\ 0 & \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{200}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{10}{100}\right) \\ 0 \\ 0 \end{bmatrix}$$

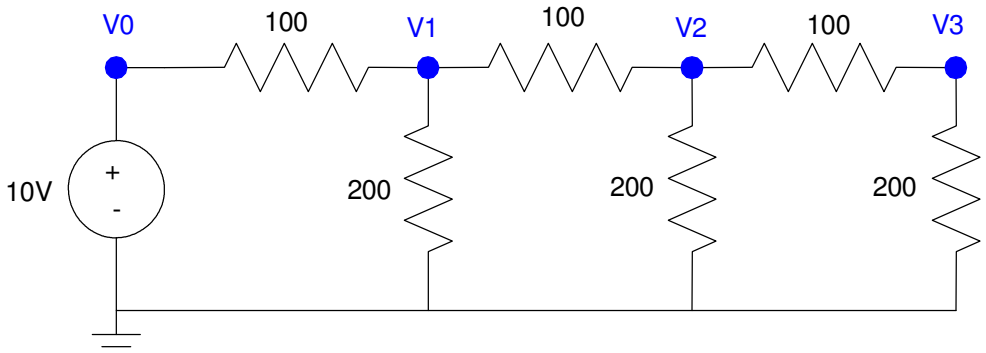
Solve using Matlab

```
A = [1/100+1/200+1/100,-1/100,0 ; -1/100,1/100+1/200+1/100,-1/100 ;
0,-1/100,1/100+1/200]


    0.025   - 0.01      0.
  - 0.01      0.025   - 0.01
    0.      - 0.01      0.015

B = [10/100 ; 0 ; 0]

    0.1
    0.
    0.

V = inv(A)*B

V1     5.1162791
V2     2.7906977
V3     1.8604651
```

4) Check your answers in PartSim (or similar program)

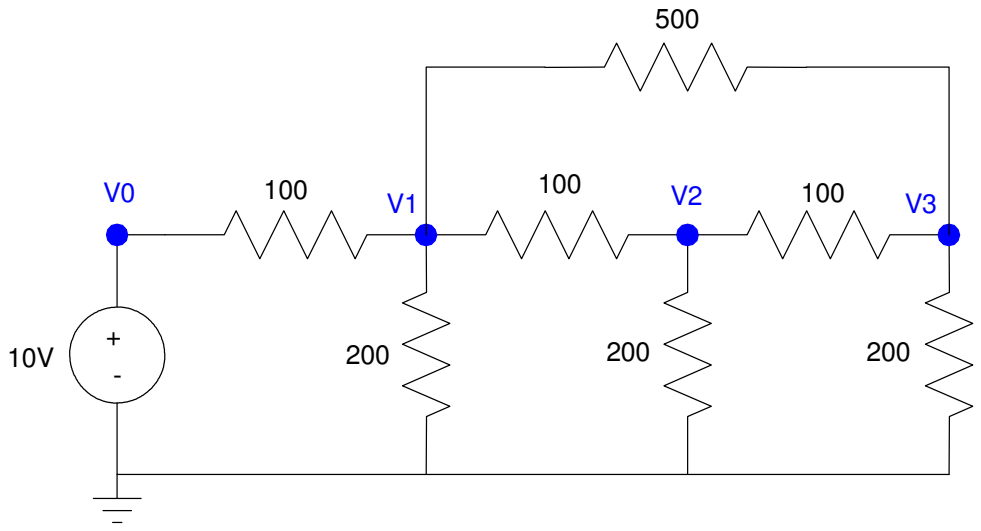


	Calculated prob 3	Simulated prob 4
V1	**5.1162791**	**5.12**
V2	**2.7906977**	**2.79**
V3	**1.8604651**	**1.86**



Circuit for Problem 3 - 4

5) Write the votlage node equations for the following circuit. Solve for V1 .. V3 using Matlab (or similar program)



Write the voltage node equations

$$V_0 = 10V$$

$$\left(\frac{V_1-V_0}{100}\right)+\left(\frac{V_1}{200}\right)+\left(\frac{V_1-V_2}{100}\right)+\left(\frac{V_1-V_3}{500}\right)=0$$

$$\left(\frac{V_2-V_1}{100}\right)+\left(\frac{V_2}{200}\right)+\left(\frac{V_2-V_3}{100}\right)=0$$

$$\left(\frac{V_3-V_2}{100}\right)+\left(\frac{V_3}{200}\right)+\left(\frac{V_3-V_1}{500}\right)=0$$

Group terms

$$\left(\frac{1}{100}+\frac{1}{200}+\frac{1}{100}+\frac{1}{500}\right)V_1-\left(\frac{1}{100}\right)V_2-\left(\frac{1}{500}\right)V_3=\left(\frac{V_0}{100}\right)=\left(\frac{10}{100}\right)$$

$$-\left(\frac{1}{100}\right)V_1+\left(\frac{1}{100}+\frac{1}{200}+\frac{1}{100}\right)V_2-\left(\frac{1}{100}\right)V_3=0$$

$$-\left(\frac{1}{500}\right)V_1-\left(\frac{1}{100}\right)V_2+\left(\frac{1}{100}+\frac{1}{200}+\frac{1}{500}\right)V_3=0$$

Place in matrix form

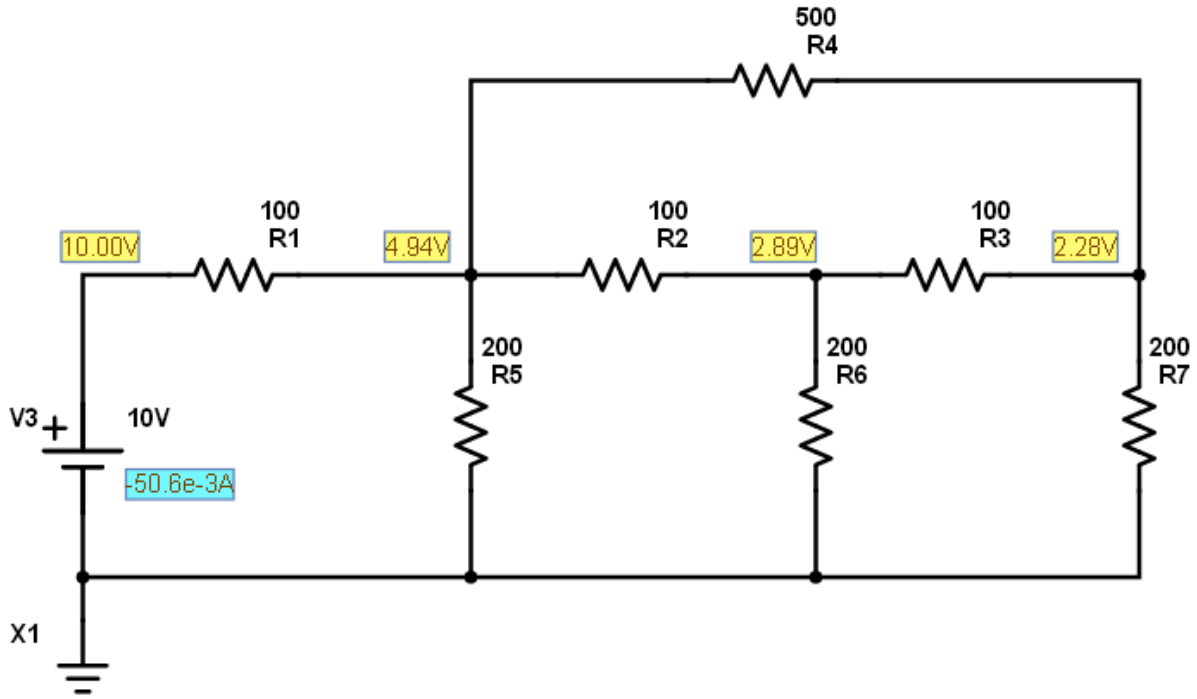
$$\begin{bmatrix} \left(\frac{1}{100}+\frac{1}{200}+\frac{1}{100}+\frac{1}{500}\right) & \left(\frac{-1}{100}\right) & \left(\frac{-1}{500}\right) \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100}+\frac{1}{200}+\frac{1}{100}\right) & \left(\frac{-1}{100}\right) \\ \left(\frac{-1}{500}\right) & \left(\frac{-1}{100}\right) & \left(\frac{1}{100}+\frac{1}{200}+\frac{1}{500}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{10}{100}\right) \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
A = [1/100+1/200+1/100+1/500,-1/100,-1/500];
A = [A ; -1/100, 1/100+1/200+1/100, -1/100];
A = [A ; -1/500, -1/100, 1/100+1/200+1/500]

    0.027  - 0.01   - 0.002
  - 0.01     0.025  - 0.01
  - 0.002  - 0.01     0.017

B = [10/100 ; 0 ; 0]

    0.1
    0.
    0.

V = inv(A)*B
```

V1 4.9429658
V2 2.8897338
V3 2.2813688

6) Check your answers in PartSim (or similar program)



Circuit for Problem 5 - 6

	Calculated prob 5	Simulated prob 6
V1	**4.9429658**	**4.94**
V2	**2.8897338**	**2.89**
V3	**2.2813688**	**2.28**