

EE 206: Homework #4 Solution

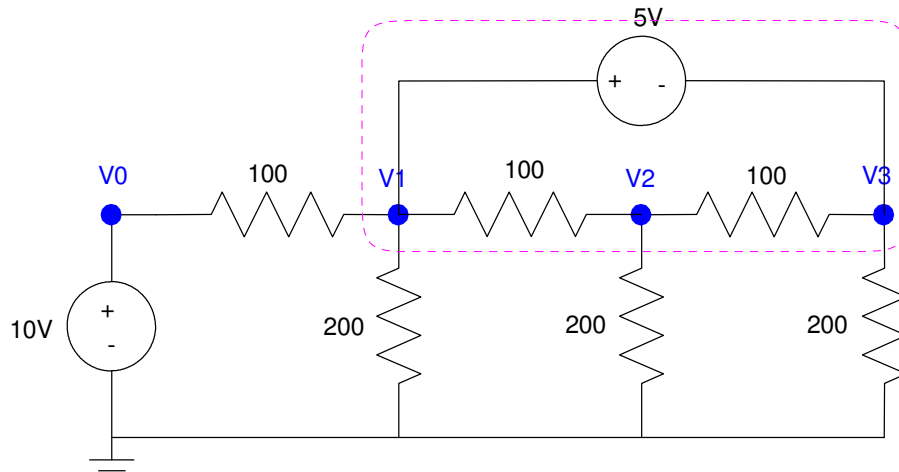
Super Nodes, Current Loops. Due Mon, Feb 10th

Note: In this assignment just write N equations for N unknowns.

Super Nodes

1) Write the voltage node equations for the following circuit (N equations for N unknowns)

- Solve for V_1 , V_2 , and V_3



Start with the easy equations

$$V_0 = 10$$

$$V_1 - V_3 = 5$$

Add two more equations

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{200}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_3}{200}\right) = 0$$

2) Solve: Group terms and place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{100}\right) & \left(\frac{-1}{100}\right) \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{200}\right) & \left(\frac{1}{200}\right) & \left(\frac{1}{200}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

Solve:

$$A = [1, 0, 0, 0 ; 0, 1, 0, -1 ; 0, -1/100, 1/100+1/200+1/100, -1/100 ; -1/100, 1/100+1/200, 1/200, 1/200]$$

$$\begin{bmatrix} 1. & 0. & 0. & 0. \\ 0. & 1. & 0. & - 1. \\ 0. & - 0.01 & 0.025 & - 0.01 \\ - 0.01 & 0.015 & 0.005 & 0.005 \end{bmatrix}$$

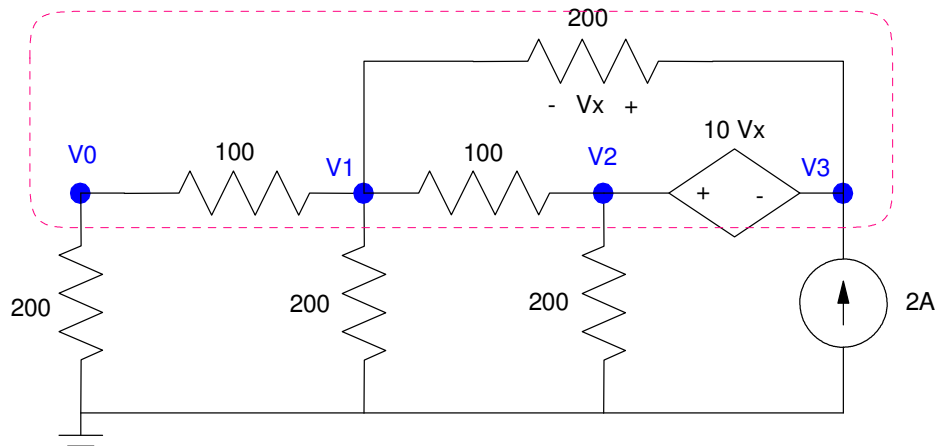
$$B = [10; 5; 0; 0]$$

$$\begin{bmatrix} 10. \\ 5. \\ 0. \\ 0. \end{bmatrix}$$

$$V = \text{inv}(A) * B$$

$$\begin{bmatrix} \mathbf{V0} & \mathbf{10.} \\ \mathbf{V1} & \mathbf{5.625} \\ \mathbf{V2} & \mathbf{2.5} \\ \mathbf{V3} & \mathbf{0.625} \end{bmatrix}$$

3) Write the voltage node equations for the following circuit (N equations for N unknowns)



5 unknowns (V_1, V_2, V_2, V_4, V_x) means write 5 equations. Start with the easy equations

$$V_x = V_3 - V_1$$

$$10V_x = V_2 - V_3$$

Write 3 more equations

$$\left(\frac{V_0}{200}\right) + \left(\frac{V_0 - V_1}{100}\right) = 0$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{200}\right) + \left(\frac{V_1 - V_2}{100}\right) + \left(\frac{V_1 - V_3}{200}\right) = 0$$

$$\left(\frac{V_0}{200}\right) + \left(\frac{V_1}{200}\right) + \left(\frac{V_2}{200}\right) - 2 = 0$$

Problem 4) Solve

Group terms

$$V_x - V_3 + V_1 = 0$$

$$10V_x - V_2 + V_3 = 0$$

$$\left(\frac{1}{200} + \frac{1}{100}\right)V_0 - \left(\frac{1}{100}\right)V_1 = 0$$

$$-\left(\frac{1}{100}\right)V_0 + \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{100} + \frac{1}{200}\right)V_1 - \left(\frac{1}{100}\right)V_2 - \left(\frac{1}{200}\right)V_3 = 0$$

$$\left(\frac{1}{200}\right)V_0 + \left(\frac{1}{200}\right)V_1 + \left(\frac{1}{200}\right)V_2 = 2$$

Place in matrix form

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 & 10 \\ \left(\frac{1}{200} + \frac{1}{100}\right) & \left(\frac{-1}{100}\right) & 0 & 0 & 0 \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{100} + \frac{1}{200}\right) & \left(\frac{-1}{100}\right) & \left(\frac{-1}{200}\right) & 0 \\ \left(\frac{1}{200}\right) & \left(\frac{1}{200}\right) & \left(\frac{1}{200}\right) & 0 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

Solve

```
A = [0,1,0,-1,1 ; 0,0,-1,1,10];
A = [A ; 1/200+1/100,-1/100,0,0,0];
A = [A ; -1/100,1/100+1/200+1/100+1/200,-1/100,-1/200,0];
A = [A ; 1/200,1/200,1/200,0,0]
```

```
0.      1.      0.      - 1.      1.
0.      0.      - 1.      1.      10.
0.015  - 0.01   0.      0.      0.
- 0.01   0.03  - 0.01  - 0.005  0.
0.005   0.005  0.005   0.      0.
```

```
B = [0;0;0;0;2]
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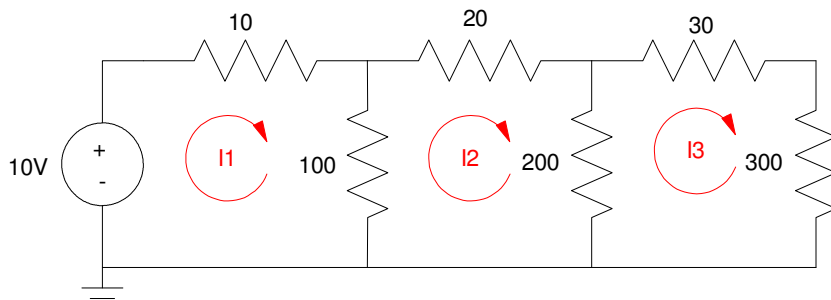
```
0.
0.
0.
0.
2.
```

```
V = inv(A)*B
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```
V1    76.987448
V2    115.48117
V3    207.53138
V4    123.84937
Vx    8.3682008
```

Current Loops

5) Write the current loop equations for the following circuit (N equations for N unknowns).



$$-10 + 10I_1 + 100(I_1 - I_2) = 0$$

$$100(I_2 - I_1) + 20I_2 + 200(I_2 - I_3) = 0$$

$$200(I_3 - I_2) + 30I_3 + 300I_3 = 0$$

6) Solve. Group terms

$$110I_1 - 100I_2 = 10$$

$$-100I_1 + 320I_2 - 200I_3 = 0$$

$$-200I_2 + 530I_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 110 & -100 & 0 \\ -100 & 320 & -200 \\ 0 & -200 & 530 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

Solve

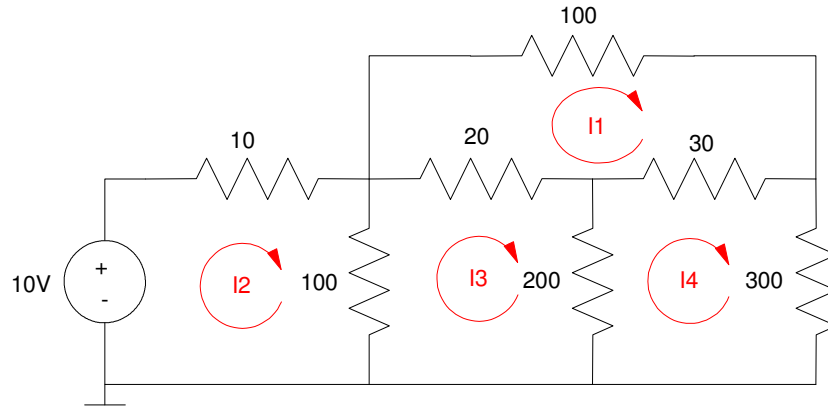
$$A = [110, -100, 0 ; -100, 320, -200 ; 0, -200, 530]$$
$$\begin{matrix} 110. & -100. & 0. \\ -100. & 320. & -200. \\ 0. & -200. & 530. \end{matrix}$$

$$B = [10; 0; 0]$$

$$I = \text{inv}(A) * B$$

$$\begin{matrix} I1 & 0.1447075 \\ I2 & 0.0591782 \\ I3 & 0.0223314 \end{matrix}$$

7) Write the current loop equations for the following circuit



$$100I_1 + 30(I_1 - I_4) + 20(I_1 - I_3) = 0$$

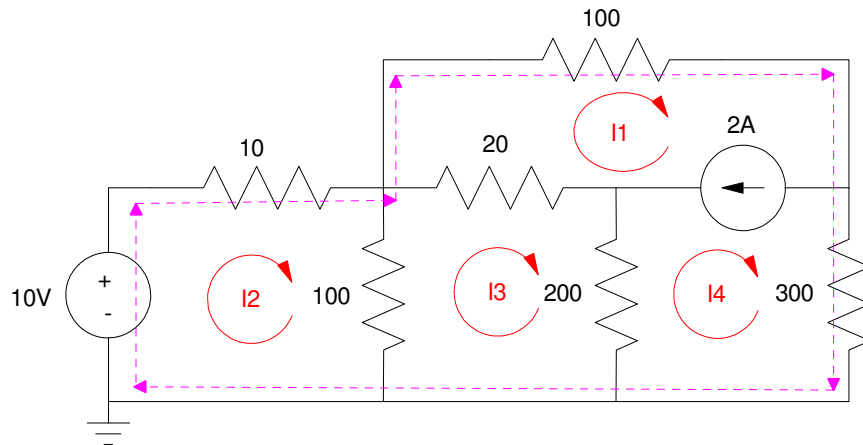
$$-10 + 10I_2 + 100(I_2 - I_3) = 0$$

$$100(I_3 - I_2) + 20(I_3 - I_1) + 200(I_3 - I_4) = 0$$

$$200(I_4 - I_3) + 30(I_4 - I_1) + 300I_4 = 0$$

Super Loops

8) Write the current loop equations for the following circuit



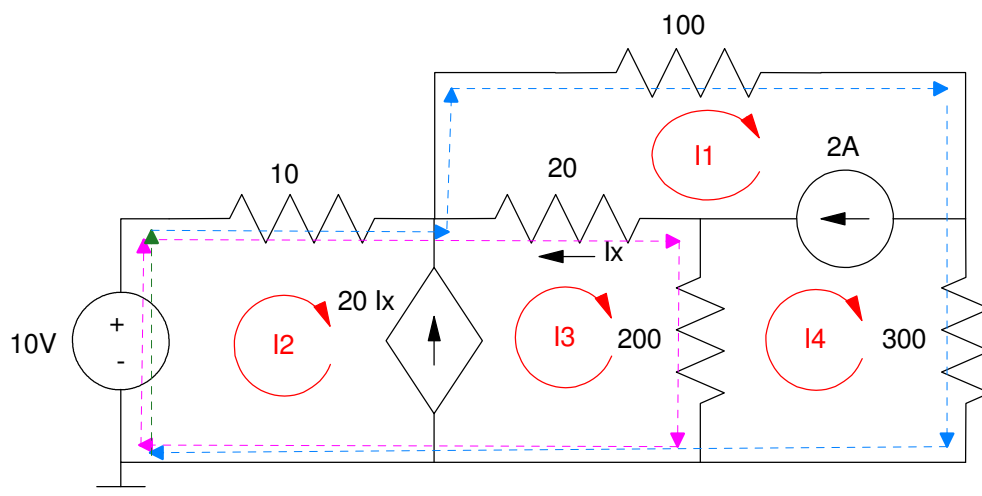
$$I_1 - I_4 = 2$$

$$-10 + 10I_2 + 100(I_2 - I_3) = 0$$

$$100(I_3 - I_2) + 20(I_3 - I_1) + 200(I_3 - I_4) = 0$$

$$-10 + 10I_2 + 100I_1 + 300I_4 = 0$$

9) Write the current loop equations for the following circuit



There are 5 unknowns (I_1, I_2, I_3, I_4, I_x). Write 5 equations for 5 unknowns.

Start with the easy ones:

$$I_x = I_1 - I_3$$

$$I_1 - I_4 = 2$$

$$20I_x = I_3 - I_2$$

Add two more loops

$$-10 + 10I_2 + 20(I_3 - I_1) + 200(I_3 - I_4) = 0 \quad \textit{pink}$$

$$-10 + 10I_2 + 100I_1 + 300I_4 = 0 \quad \textit{blue}$$