## EE 206: Homework \#5 Solution

Thevenin Equivalents, Maximum Power Transfer. Due Wed, Feb 24th

## Thevenin Equivalents

1) Find the Thevenin equivalent for the following circuit by transforming between Thevenin and Norton equivalents:


Switch back and forth between Thevenin and Norton equivalents

$$
I_{n} R_{n}=V_{t h} \quad R_{t h}=R_{n}
$$



At this point we know the answer:

$$
\begin{aligned}
& R_{t h}=200 \| 29.09=25.39 \Omega \\
& V_{t h}=\left(\frac{200}{200+29.09}\right) \cdot 9.09 \mathrm{~V}=7.9365 \mathrm{~V}
\end{aligned}
$$


2) Find the Thevenin equivalent for the following circuit by transforming between Thevenin and Norton equivalents:


3) Find the Thevenin equivalent for the following circuit:


Find the open-circuit voltage

$$
\begin{aligned}
& \left(\frac{X-5}{400}\right)+\left(\frac{X}{500}\right)-100\left(\frac{5-X}{400}\right)+\left(\frac{X}{2000}\right)=0 \\
& X=4.951 V
\end{aligned}
$$

Turn off the power supplies ( 5 V goes to 0 V ). Measure the resistance at the output.
This isn't obvious, so add a 1 V test voltage and compute the current

$$
\begin{aligned}
& I=\left(\frac{1 V}{400 \Omega}\right)+\left(\frac{1 V}{500 \Omega}\right)+100\left(\frac{1 V}{400 \Omega}\right)+\left(\frac{1 V}{2000 \Omega}\right)=255 \mathrm{~mA} \\
& R_{t h}=\frac{1 V}{255 \mathrm{~mA}}=3.9216 \Omega
\end{aligned}
$$


4) Find the Thevenin equivalent for the following circuit:


Compute the open-circuit voltage (write the voltage node equation at X )

$$
\begin{aligned}
& \left(\frac{X-5}{400}\right)+\left(\frac{X}{500}\right)-100\left(\frac{0-X}{500}\right)+\left(\frac{X}{2000}\right)=0 \\
& X=0.06097 V
\end{aligned}
$$

Turn off the power supply ( 5 V becomes 0 V ). Compute the resistance looking in.
This isn't obvious, so add a test voltage and compute the current

$$
\begin{aligned}
& I=\left(\frac{1}{400}\right)+\left(\frac{1}{500}\right)-100\left(\frac{0-1}{500}\right)+\left(\frac{1}{2000}\right) \\
& I=205 \mathrm{~mA} \\
& R=\frac{1 \mathrm{~V}}{205 \mathrm{~mA}}=4.878 \Omega
\end{aligned}
$$



## Maximum Power Transfer

5) Determine RL so that the maximum power is delivered to the load (RL)


Take the Thevenin equivalent of this circuit (problem \#1)


Maximum power transfer is when RL = Rth
answer: $\quad$ RL $=$ 25.39 Ohms

## PartSim

6) Simulate the circuit of problem 5 . Determine the voltage and current at the load You can also use the Thevenin equivalent from problem \#5:

|  | V | I | Power |
| :---: | :---: | :---: | :---: |
| $\mathrm{R}=0$ | 0 V | 313 mA | 0 mW |
| $\mathrm{R}=10$ | 2.243 V | 224 mA | 503 mW |
| $\mathrm{R}=25.39$ <br> max power | 3.968 V | 156 mA | 620 mW |
| $\mathrm{R}=50$ | 5.264 V | 105 mA | 554 mW |
| $\mathrm{R}=100$ | 6.329 V | 63 mA | 401 mW |
| $\mathrm{R}=$ infinite | 7.936 V | 0 mA | 0 mW |

Note:

$$
\begin{aligned}
& V=\left(\frac{R_{L}}{R_{L}+25.39 \Omega}\right) 7.9365 V \\
& I=\left(\frac{7.9365 V}{R_{L}+25.39 \Omega}\right) \\
& P=V I
\end{aligned}
$$

7) Plot V vs. I on a graph and draw a line between these points. How does this line relate to the Thevenin equivalent for circuit \#5?

- The y-axis interecept is I(Norton)
- The x axis intercept is V (Thevenin)
- The slope is $-1 / \mathrm{R}$ (25.39 Volts / Amp)


