EE 206: Homework #8

Sinusoidal Source, Complex Numbers, Complex Impedance. Due Monday, March 30th

Please make the subject "EE 206 HW#8" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Sine Waves

1) Convert to Vp, Vpp, Vrms

Vp (peak)	Vpp (peak-to-peak)	Vrms
15Vp	**30 Vpp**	**10.607 Vrms**
7.5 Vp	15Vpp	**5.303 Vrms**
21.213 Vp	**42.426 Vpp**	15Vrms

Note:

$$V_{pp} = 2V_p$$

$$V_{rms} = \frac{1}{\sqrt{2}} V_p$$

2) Solve the following differential equation for y(t)

$$\frac{dy}{dt} + 3y = x$$

$$x(t) = 2\cos(5t)$$

hint: assume y(t) is in the form of

$$y(t) = a\cos(5t) + b\sin(5t)$$

substitute and match coefficients for sine and cosine.

Solution:

$$\frac{dy}{dt} = -5a\sin(5t) + 5b\cos(5t)$$

Substitute:

$$\frac{dy}{dt} + 3y = x$$

$$(-5a\sin(5t) + 5b\cos(5t)) + 3(a\cos(5t) + b\sin(5t)) = 2\cos(5x)$$

Group terms

$$(-5a + 3b)\ \sin(5t) + (5b + 3a)\ \cos(5t) = 2\cos(5t)$$

Both sine and cosine have to match

$$-5a + 3b = 0 \qquad \textit{sine}$$

$$5b + 3a = 2 \qquad \textit{cosine}$$

Solving 2 equtions for 2 unknowns gives

$$\begin{bmatrix} -5 & 3 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

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a =     0.1764706
b =     0.2941176
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meaning

$$\boxed{y(t) = 0.1764\cos(5t) + 0.2941\sin(5t)}$$

note: If you don't use complex numbers, you get 2 equations for 2 unknowns (one for sine, one for cosine). Complex numbers reduces this to 1 equation for 1 unknown,

Complex Numbers:

3) Find Y as a complex number

3a) $Y = \left(\frac{8+j3}{2+j7}\right) + \left(\frac{4-j3}{8+j5}\right)$

Y = 0.889 - j1.438

3b) $Y = \left(\frac{100(s+3)}{s(s+5)(s+10)}\right)_{s=j3}$

Y = -0.108 - j2.321

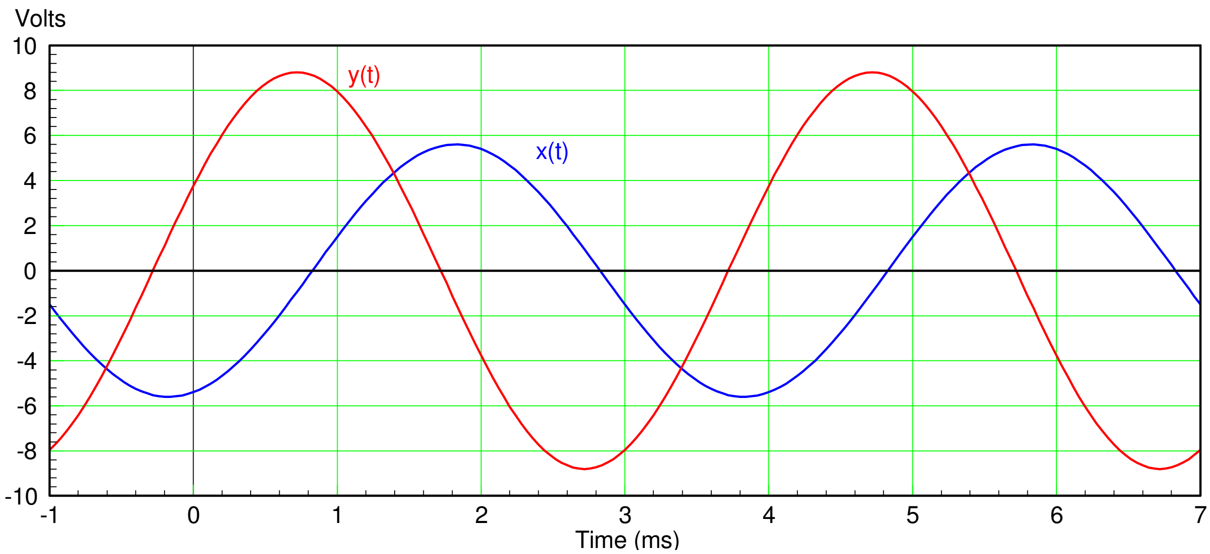
3c) $Y = \left(\frac{5s^2+10s+20}{s^3+6s^2+11s+6}\right)_{s=j4}$

Y = 0.541 - j0.565

Phasor Voltages

4) For the following waveforms, determine

- The frequency in rad/sec
- The phasor representation for X and Y



The period (one cycle) is 4ms

$$f = \frac{1}{T} = \frac{1}{4ms} = 250Hz$$

$$\boxed{\omega = 2\pi f = 500\pi \ \frac{rad}{\sec}}$$

X:

- The peak is 5.8V
- The peak is delayed from t=0 by 1.8ms

$$\theta_x = -\left(\frac{delay}{period}\right)360^0 = -\left(\frac{1.8ms}{4ms}\right)360^0 = -162^0$$

$$\boxed{X = 5.8\angle - 162^0}$$

Y:

- The peak is 8.8V
- The peak is delayed from t = 0 by 0.7ms

$$\theta_y = -\left(\frac{0.7ms}{4ms}\right)360^0 = -63^0$$

$$\boxed{Y = 8.8\angle - 63^0}$$

5) Express V in phasor form.

a) $V = 6\cos(10t) - 7\sin(10t)$

$$V = 6 + j7$$

real = cosine

-imag = sine

b) $V = 2\cos(20t - 30^0) + 5\cos(20t + 15^0)$

$= 2\angle -30^0 + 5\angle 15^0$

$$= 6.562 + j0.294$$

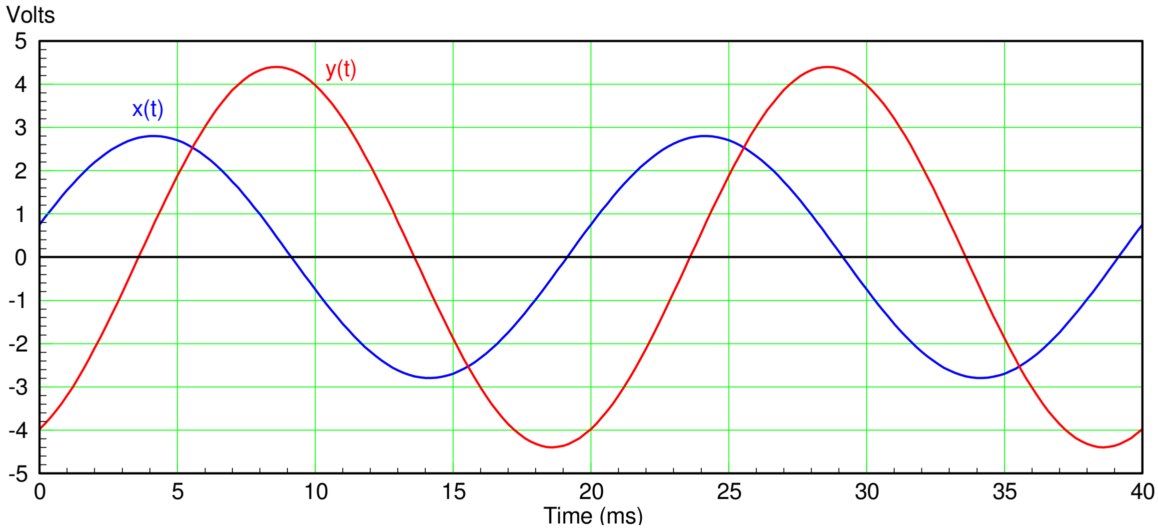
c) $V = 7\cos(5t - 20^0) + 9\sin(5t)$

$= 7\angle -20^0 - j\cdot(9\angle 0^0)$

real = cosine, -j = sine

$$= 6.578 - j11.394$$

6) Assume Y = G*X. Determine frequency and the phasor representation for G



Period = 20ms

$$f = \frac{1}{T} = \frac{1}{20ms} = 50Hz$$

$$\omega = 2\pi f = 100\pi \ \frac{rad}{\sec}$$

Y lags X (negative phase) by 5ms

$$\theta = -\left(\frac{5ms}{20ms}\right) 360^0 = -90^0$$

The gain is

$$|G| = \frac{\text{output}}{\text{input}} = \left(\frac{4.3V_p}{2.8V_p}\right) = 1.536$$

note: the units cancel so you could use rms voltage, Vpp, or Vp. Just be consistent,

So

$$G = 1.536\angle -90^0$$