EE 206: Homework #9 Solution

Passive Circuit Elements, Series and Parallel with Phasors, Voltage Nodes
Due Monday, April 6th

1) Determine the impedance of a resistor, inductor, and capacitor at 10, 1000, and 10k rad/sec

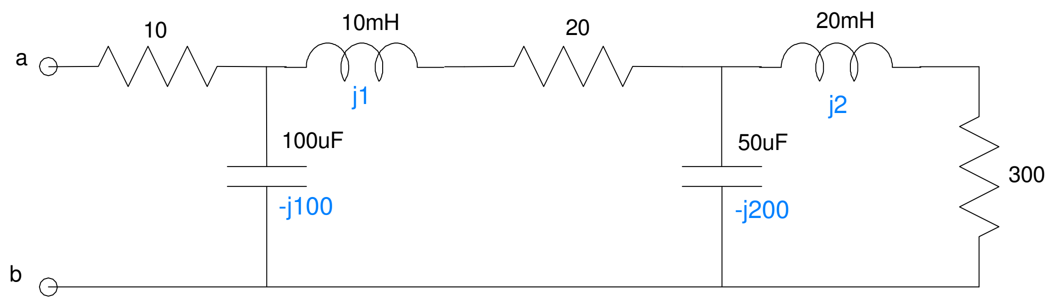
	R = 100 Ohms	L = 10mH	C = 10uF
20 rad/sec	100 Ohms	j0.20 Ohms	-j5000 Ohms
200 rad/sec	100 Ohms	j2.00 Ohms	-j500 Ohms
2000 rad/sec	100 Ohmns	j20.00 Ohms	-j50 Ohms

$$R \to R$$

$$L \to j\omega L$$

$$C \to \frac{1}{j\omega C}$$

2) Find the impedance Zab for the following circuit at 100 rad/sec (15.9Hz)



Going right to left

$300 + j2 = 300 + j2$

$(300 + j2) \parallel (-j200) = 92.876 - j138.702$

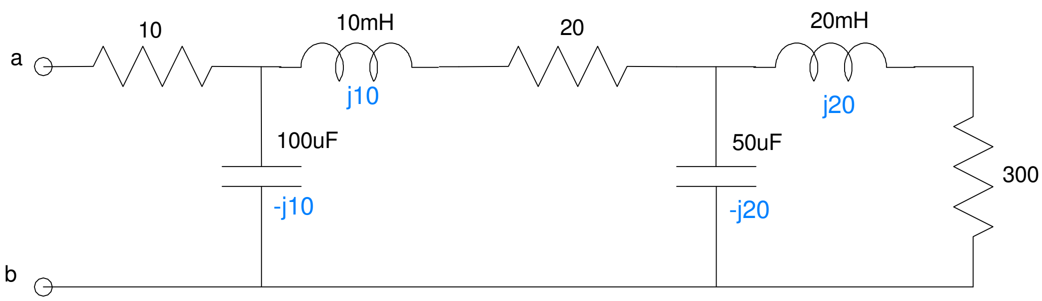
$(92.876 - j138.702) + (20 + j1) = 112.876 - j137.702$

$(112.876 - j137.702) \parallel (-j100) = 16.301 - j65.671$

$(16.301 - j65.671) + 10 = 26.301 - j65.571$

answer: **26.301 - j65.571**

3) Find the impedance Zab for the following circuit at 1000 rad/sec (159Hz)



Going right to left

300 + j20 = 300 + j20

(300 + j20) || (-j20) = 1.333 - j20.000

(1.333 - j20.000) + (20 + j10) = 21.333 - j10.000

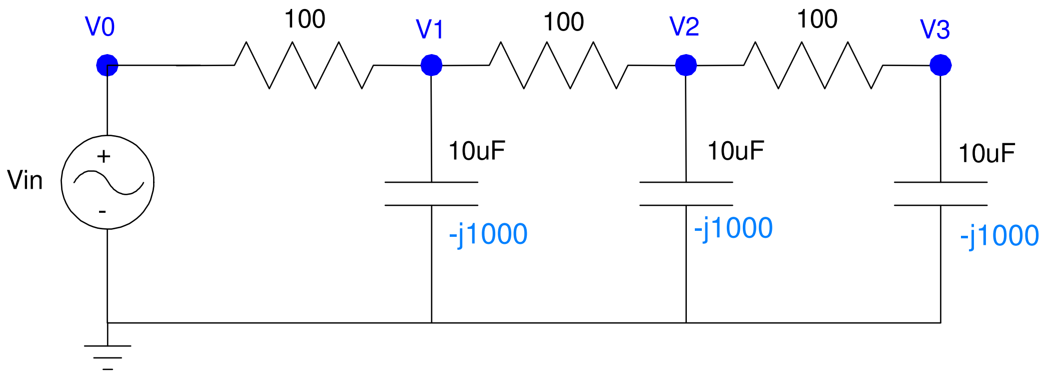
(21.333 - j10.000) || (-j10) = 2.495 - j7.661

(2.495 - j7.661) + 10 = 12.495 - j7.661

answer: **12.495 - j7.661**

Problem 4: Assume $V_{in} = 10\cos(100t)$

- Write the voltage node equations for the following circuit.
- Solve for V1, V2, and V3



Convert to phasor impedance

$$s = j\omega = j100$$

$$V_{in} = 10 + j0$$

$$C \to \frac{1}{j\omega C} = -j1000\Omega$$

Write the voltage node equations

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{-j1000}\right) + \left(\frac{V_1 - V_2}{100}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{-j1000}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{-j1000}\right) = 0$$

Group terms

$$\left(\frac{1}{100} + \frac{1}{-j1000} + \frac{1}{100}\right)V_1 - \left(\frac{1}{100}\right)V_2 = \left(\frac{1}{100}\right)V_0$$

$$-\left(\frac{1}{100}\right)V_1 + \left(\frac{1}{100} + \frac{1}{-j1000} + \frac{1}{100}\right)V_2 - \left(\frac{1}{100}\right)V_3 = 0$$

$$-\left(\frac{1}{100}\right)V_2 + \left(\frac{1}{100} + \frac{1}{-j1000}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} \left(\frac{1}{100}+\frac{1}{-j1000}+\frac{1}{100}\right) & \left(\frac{-1}{100}\right) & 0 \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100}+\frac{1}{-j1000}+\frac{1}{100}\right) & \left(\frac{-1}{100}\right) \\ 0 & \left(\frac{-1}{100}\right) & \left(\frac{1}{100}+\frac{1}{-j1000}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{100}\right) \\ 0 \\ 0 \end{bmatrix} V_0$$

Solve using Matlab

```
A = [1/100 + 1/(-j*1000) + 1/100, -1/100, 0];
A = [A ; -1/100, 1/100 - 1/(j*1000) + 1/100, -1/100];
A = [A ; 0, -1/100, 1/100 - 1/(j*1000)]

    0.02 + 0.001i  - 0.01             0
  - 0.01             0.02 + 0.001i  - 0.01
    0              - 0.01             0.01 + 0.001i

B = [1/100 ; 0 ; 0]

    0.01
    0.
    0.

V0 = 10 + j*0;

V = inv(A)*B*V0
```

V1   8.8813059 - 2.4420023i
V2   8.006812 - 3.9958741i
V3   7.5319055 - 4.7490647i

Convert back to time

$$v_1(t) = 8.88\cos(100t) + 2.44\sin(100t)$$

$$v_2(t) = 8.00\cos(100t) + 3.99\sin(100t)$$

$$v_3(t) = 7.53\cos(100t) + 4.74\sin(100t)$$

If you prefer polar form...

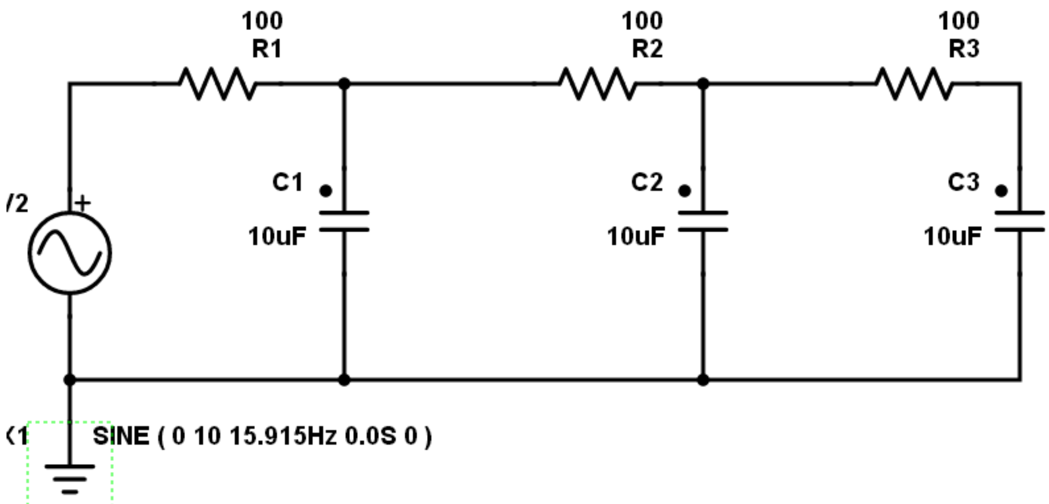
$$v_1(t) = 9.21\cos(100t - 15.4^0)$$

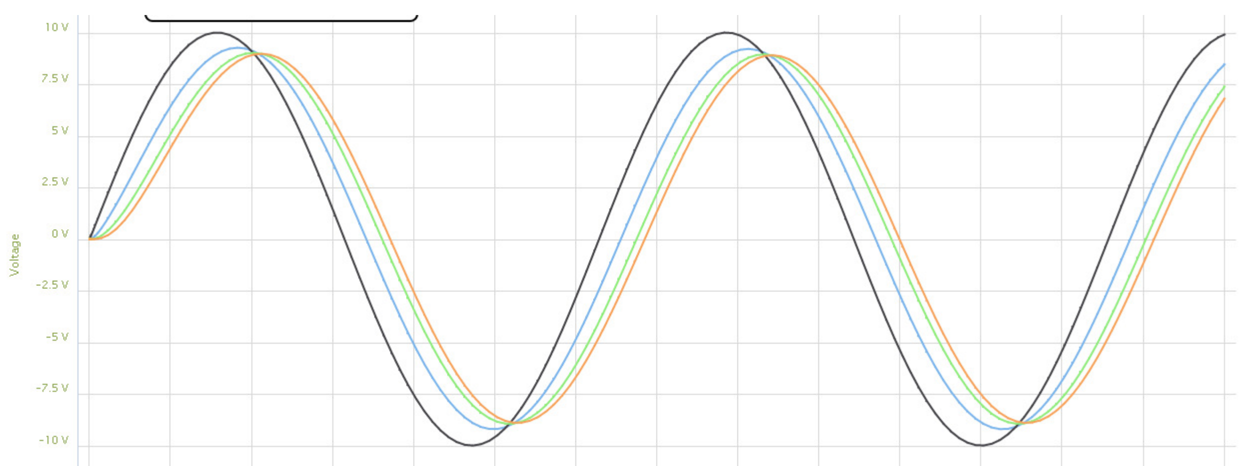
$$v_2(t) = 8.95\cos(100t - 26.5^0)$$

$$v_1(t) = 8.90\cos(100t - 32.2^0)$$

Problem 5) Simulate the circuit of problem #4 in PartSim (or similar program) and compare the simulation results to your results from problem #4.

Note that 100 rad/sec = 15.915Hz

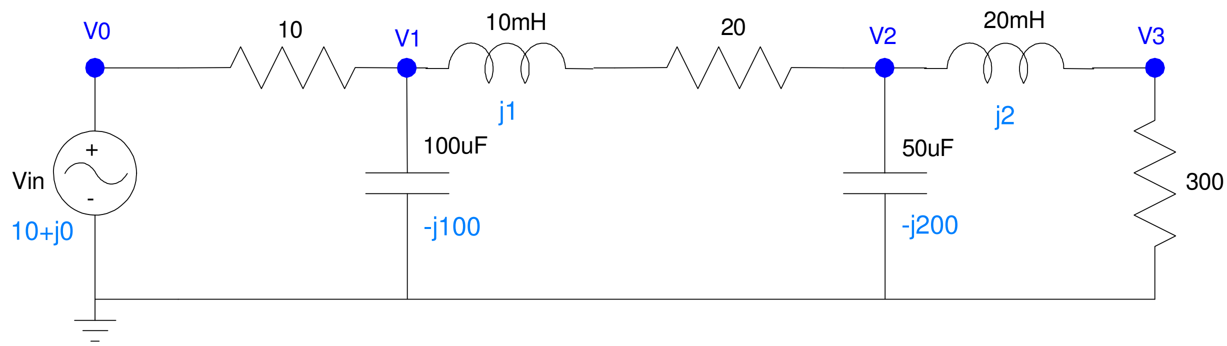




Problem 6: Assume

$$V_{in} = 10\cos(100t)$$

- Write the voltage node equations for the following circuit.
- Solve for V1, V2, and V3



$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{-j100}\right) + \left(\frac{V_1 - V_2}{20+j}\right) = 0$$

$$\left(\frac{V_2 - V_1}{20+j}\right) + \left(\frac{V_2}{-j200}\right) + \left(\frac{V_2 - V_3}{j2}\right) = 0$$

$$\left(\frac{V_3 - V_2}{j2}\right) + \left(\frac{V_3}{300}\right) = 0$$

Group terms

$$\left(\frac{1}{10} + \frac{1}{-j100} + \frac{1}{20+j}\right)V_1 - \left(\frac{1}{20+j}\right)V_2 = \left(\frac{1}{10}\right)V_0$$

$$-\left(\frac{1}{20+j}\right)V_1 + \left(\frac{1}{20+j} + \frac{1}{-j200} + \frac{1}{j2}\right)V_2 - \left(\frac{1}{j2}\right)V_3 = 0$$

$$-\left(\frac{1}{j2}\right)V_2 + \left(\frac{1}{j2} + \frac{1}{300}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} \left(\frac{1}{10} + \frac{1}{-j100} + \frac{1}{20+j}\right) & \left(\frac{-1}{20+j}\right) & 0 \\ \left(\frac{-1}{20+j}\right) & \left(\frac{1}{20+j} + \frac{1}{-j200} + \frac{1}{j2}\right) & \left(\frac{-1}{j2}\right) \\ 0 & \left(\frac{-1}{j2}\right) & \left(\frac{1}{j2} + \frac{1}{300}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{10}\right) \\ 0 \\ 0 \end{bmatrix} V_0$$

Solve in Matlab

```
A = [1/10 - 1/(j*100) + 1/(20+j), -1/(20+j), 0];
A = [A ; -1/(20+j), 1/(20+j) - 1/(j*200) + 1/(j*2), -1/(j*2)];
A = [A ; 0, -1/(j*2), 1/(j*2) + 1/300]

    0.1498753 + 0.0075062i  - 0.0498753 + 0.0024938i    0
  - 0.0498753 + 0.0024938i    0.0498753 - 0.4824938i    0.5i
    0                         0.5i                      0.0033333 - 0.5i

B = [1/10; 0; 0];

V0 = 10 + j*0;

V = inv(A)*B*V0

V1   9.4744 -1.3122i
V2   8.7223 -2.0813i
V3   8.7080 -2.1393i
```

meaning

$$v_1(t) = 9.47\cos(100t) + 1.31\sin(100t)$$

$$v_2(t) = 8.72\cos(100t) + 2.08\sin(100t)$$

$$v_3(t) = 8.71\cos(100t) + 2.13\sin(100t)$$

If you prefer polar form

$$v_1(t) = 9.56\cos(100t - 7.88^0)$$

$$v_2(t) = 8.96\cos(100t - 13.42^0)$$

$$v_3(t) = 8.96\cos(100t - 13.80^0)$$

Problem 7) Simulate the circuit of problem #6 in PartSim (or similar program) and compare the simulation results to your results from problem #6.