EE 206: Homework #10 Solution
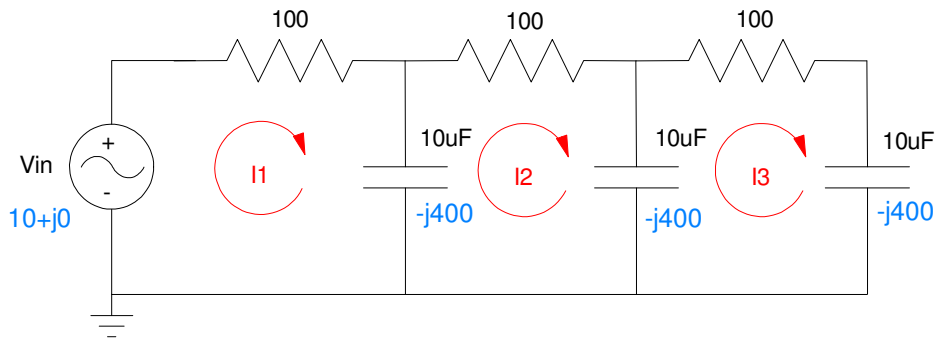
Current Loops, Op-Amps, Superposition with Phasors
Due Monday, April 20th

Problem 1) Assume

$$V_{in} = 10\cos(250t)$$

1a) Write the current-loop equations for the following circuit.

1b) Solve for I1, I2, and I3

Convert to phasor notation

$$s = j\omega = j250$$

$$V_{in} = 10 + j0$$

$$C \rightarrow \tfrac{1}{j\omega C} = -j400$$

Write the current loop equations

$$-10 + 100I_1 - j400(I_1 - I_2) = 0$$

$$-j400(I_2 - I_1) + 100I_2 - j400(I_2 - I_3) = 0$$

$$-j400(I_3 - I_2) + 100I_3 - j400I_3 = 0$$

Group terms

$$(100 - j400)I_1 + (j400)I_2 = 10$$

$$(j400)I_1 + (100 - j800)I_2 + (j400)I_3 = 0$$

$$(j400)I_2 + (100 - j800)I_3 = 0$$

Place in matrix form

$$
\begin{bmatrix}
(100 - j400) & j400 & 0 \\
j400 & (100 - j800) & j400 \\
0 & j400 & (100 - j800)
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
=
\begin{bmatrix}
5 \\
0 \\
0
\end{bmatrix}
$$

Solve using Matlab

```
A =[100-j*400, j*400, 0 ; j*400, 100-j*800, j*400 ; 0, j*400, 100-j*800]

    100. - 400.i    400.i              0
    400.i           100. - 800.i    400.i
    0               400.i           100. - 800.i

B = [10;0;0]

    10.
    0.
    0.

I = inv(A)*B

I1    0.0343 + 0.0327i
I2    0.0261 + 0.0163i
I3    0.0139 + 0.0064i
```

meaning
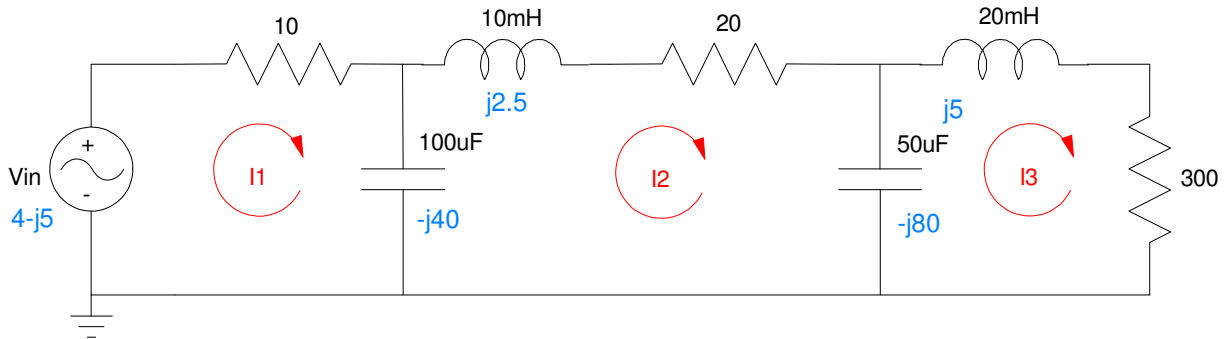
$$i_1(t) = 0.0343 \cos(250t) - 0.0327 \sin(250t)$$

$$i_2(t) = 0.0261 \cos(250t) - 0.0163 \sin(250t)$$

$$i_3(t) = 0.0139 \cos(250t) - 0.0064 \sin(250t)$$

Problem 2) Assume

$$V_{in} = 4\cos(250t) + 5\sin(250t)$$

2a) Write the voltage node equations for the following circuit

Convert to phasor notation (shown in blue)

$$V_{in} = 4 - j5$$
$$s = j\omega = j250$$
$$L \rightarrow j\omega L$$
$$C \rightarrow \frac{1}{j\omega C}$$

Write the current loop equations

$$-(4 - j5) + 10I_1 - j40(I_1 - I_2) = 0$$
$$-j40(I_2 - I_1) + (20 + j2.5)I_2 - j80(I_2 - I_3) = 0$$
$$-j80(I_3 - I_2) + j5I_3 + 300I_3 = 0$$

Group terms

$$(10 - j40)I_1 + (j40)I_2 = 4 - j5$$
$$(j40)I_1 + (20 - j117.5)I_2 + (j80)I_3 = 0$$
$$(j80)I_2 + (300 - j75)I_3 = 0$$

Place in matrix form

$$
\begin{bmatrix}
(10-j40) & j40 & 0 \\
j40 & (10-j117.5) & j80 \\
0 & j80 & (300-j75)
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
=
\begin{bmatrix}
4-j5 \\
0 \\
0
\end{bmatrix}
$$

Solve in Matlab

```
A = [10-j*40, j*40, 0 ; j*40, 10-j*117.5, j*80 ; 0, j*80, 300 - j*75]

    10. -  40.i      40.i              0
    40.i             10. - 117.5i      80.i
    0                80.i              300. - 75.i

B = [4-j*5 ; 0 ; 0]

    4. -  5.i
    0
    0

I = inv(A)*B

I1    0.2092021 + 0.0436046i
I2    0.0733009 - 0.0040949i
I3    0.0035715 - 0.0186540i
```

or in terms of mA

```
I = inv(A)*B * 1000

I1    209.20207 + 43.604619i    mA
I2    73.300916 - 4.094863i     mA
I3    3.5715428 - 18.654025i    mA
```
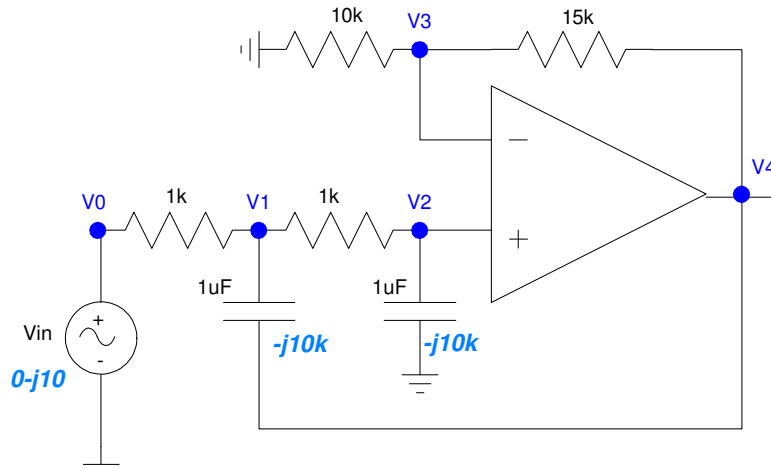
meaning
```
I1 =  209.20207 cos(250t) - 43.60461 sin(250t)    mA
I2 =   73.30091 cos(250t) + 4.094863 sin(50t)     mA
I3 =    3.57154 cos(250t) + 18.65402 sin(250t)    mA
```

Problem 3) Assume

$$V_{in} = 10 \sin(100t)$$

3a) Write the voltage node equations

$$V_2 = V_3$$

$$V_0 = 0 - j10$$

$$\left(\frac{V_1 - V_0}{1k}\right) + \left(\frac{V_1 - V_4}{-j10k}\right) + \left(\frac{V_1 - V_2}{1k}\right) = 0$$

$$\left(\frac{V_2 - V_1}{1k}\right) + \left(\frac{V_2}{-j10k}\right) = 0$$

$$\left(\frac{V_3}{10k}\right) + \left(\frac{V_3 - V_4}{15k}\right) = 0$$

3b) Solve for V1 .. V4. Group terms. Scale by 1000 for equations 3,4,5

$$V_2 - V_3 = 0$$

$$V_0 = 0 - j10$$

$$\left(\frac{-1}{1k}\right)V_0 + \left(\frac{1}{1k} + \frac{1}{-j10k} + \frac{1}{1k}\right)V_1 - \left(\frac{1}{1k}\right)V_2 - \left(\frac{1}{-j10k}\right)V_4 = 0$$

$$-\left(\frac{1}{1k}\right)V_1 + \left(\frac{1}{1k} + \frac{1}{-j10k}\right)V_2 = 0$$

$$\left(\frac{1}{10k} + \frac{1}{15k}\right)V_3 - \left(\frac{1}{15k}\right)V_4 = 0$$

Place in matrix form. Multiply the last three equations by 1000 to clear the 'k' terms

$$
\begin{bmatrix}
0 & 0 & 1 & -1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
-1 & 2+j0.1 & -1 & 0 & -j0.1 \\
0 & -1 & 1+j0.1 & 0 & 0 \\
0 & 0 & 0 & 0.1667 & -0.0667
\end{bmatrix}
\begin{bmatrix}
V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4
\end{bmatrix}
=
\begin{bmatrix}
0 \\ -j10 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

Solve

```
A = [0,0,1,-1,0 ; 1,0,0,0,0 ; -1,2+j*0.1,-1,0,-j*0.1 ; 0,-1,1+j*0.1,0,0 ;
0,0,0,0.1667,-0.0667]

     0        0           1.          - 1.          0
     1.       0           0             0           0
  -  1.       2. - 0.1i  - 1.           0           0.1i
     0      - 1.          1. + 0.1i     0           0
     0        0           0             0.1667    - 0.0667

B = [0;-j*10;0;0;0]

     0
  - 10.i
     0
     0
     0

V = inv(A)*B

          0 -10.0000i
     0.4979 -10.1262i
    -0.5096 -10.0752i
    -0.5096 -10.0752i
    -1.2737 -25.1805i
```

meaning

```
Vo =   10 sin(100t)
V1 =    0.4879 cos(100t)   + 10.1262 sin(100t)
V2 = -0.5096 cos(100t) + 10.0752 sin(100t)
V3 = -0.5096 cos(100t) + 10.0752 sin(100t)
V4 = -1.2737 cos(100t) + 25.1805 sin(100t)
```

Problem 4) Assume

$$V_{in} = 12\cos(200t)$$

4a) Write the voltage node equations

Everything stays the same except for V0 and the capacitors (complex terms)

$$V_2 - V_3 = 0$$

$$V_0 = 12 + j0$$

$$\left(\frac{-1}{1k}\right)V_0 + \left(\frac{1}{1k} + \frac{1}{-j5k} + \frac{1}{1k}\right)V_1 - \left(\frac{1}{1k}\right)V_2 - \left(\frac{1}{-j5k}\right)V_4 = 0$$

$$-\left(\frac{1}{1k}\right)V_1 + \left(\frac{1}{1k} - \frac{1}{-j5k}\right)V_2 = 0$$

$$\left(\frac{1}{10k} + \frac{1}{15k}\right)V_3 - \left(\frac{1}{15k}\right)V_4 = 0$$

In matrix form

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 2+j0.2 & -1 & 0 & -j0.2 \\ 0 & -1 & 1+j0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.1667 & -0.0667 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 12+j0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving in Matlab

```
A = [0,0,1,-1,0 ; 1,0,0,0,0 ; -1,2+j*0.2,-1,0,-j*0.2 ; 0,-1,1+j*0.2,0,0 ;
0,0,0,0.1667,-0.0667]

    0      0              1.          - 1.          0
    1.     0              0           0             0
  - 1.    2. - 0.2i    - 1.           0             0.2i
    0    - 1.             1. + 0.2i   0             0
    0      0              0           0.1667  - 0.0667

B = [0;12+j*0;0;0;0]

    0
    12.
    0
    0
    0

V = inv(A)*B

   12.0000
   12.6234 + 1.1831i
   12.3654 - 1.2900i
   12.3654 - 1.2900i
   30.9043 - 3.2240i
```
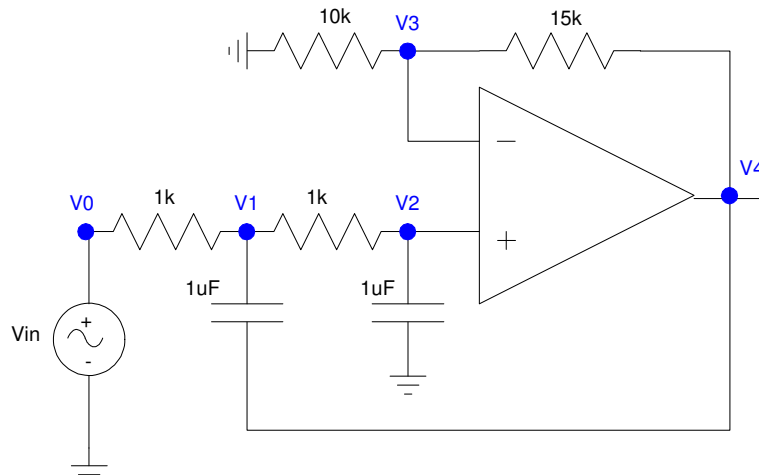
meaning
```
V0 =   12.0000 cos(200t)
V1 =   12.6234 cos(200t) - 1.1831 sin(200t)
V2 =   12.3654 cos(200t) + 1.2900 sin(200t)
V3 =   12.3654 cos(200t) + 1.2900 sin(200t)
V4 =   30.9043 cos(200t) + 3.2240 sin(200t)
```

Problem 5) Assume

$$V_{in} = 10\sin(100t) + 12\cos(200t)$$

Determine V1 .. V4

Use superposition; add solution #4 to solution #5

```
problem 5 =          problem 3              +         problem 4

  Vo =   10 sin(100t) + 12.0000 cos(200t)

  V1 =   0.4879 cos(100t)   + 10.1262 sin(100t)
       + 12.6234 cos(200t)  -  1.1831 sin(200t)

  V2 = -0.5096 cos(100t)  + 10.0752 sin(100t)
       + 12.3654 cos(200t)  +  1.2900 sin(200t)

  V3 = -0.5096 cos(100t)  + 10.0752 sin(100t)
       + 12.3654 cos(200t)  +  1.2900 sin(200t)

  V4 = -1.2737 cos(100t)  + 25.1805 sin(100t)
       + 30.9043 cos(200t)  +  3.2240 sin(200t)
```