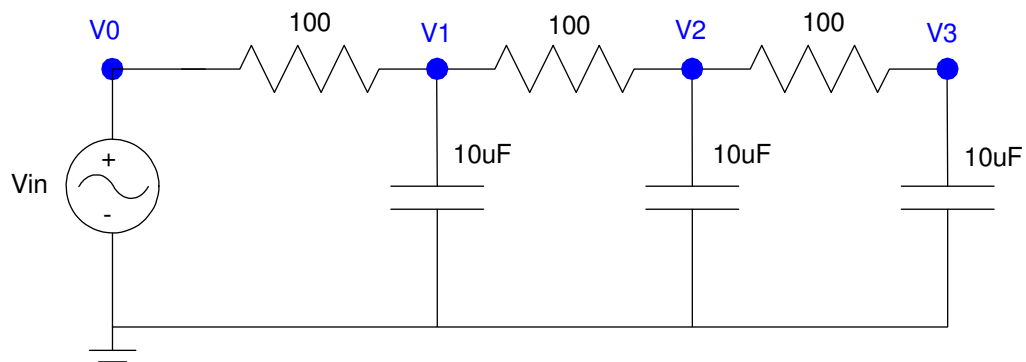# EE 206: Homework #11

Superposition with Phasors. Due Monday, April 27th

Please make the subject "EE 206 HW#11" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Assume Vin is a 10Vpp, 477Hz (3000 rad/sec) square wave

$$V_{in} = \begin{cases} +10 & \cos(3000t) > 0 \\ 0 & \cos(3000t) < 0 \end{cases}$$

Find V3(t) using superposition

Problem 1) Approximate Vin(t) as

$$V_{in} \approx a_0 + a_1\cos(3000t) + b_1\sin(3000t) + a_2\cos(6000t) + b_2\sin(6000t) + a_3\cos(9000t) + b_3\sin(9000t)$$

In Matlab

```
wo = 3000;
f = wo / (2*pi);
T = 1/f;
t = [0:0.0001:1]' * T;
Vin = 10*(cos(3000*t) > 0);
a0 = mean(Vin)

a0 =     5.0005

a1 = 2*mean(Vin .* cos(3000*t))

a1 =     6.3676

b1 = 2*mean(Vin .* sin(3000*t))

b1 =     0.0020

a2 = 2*mean(Vin .* cos(6000*t))

a2 =     0.0020
```
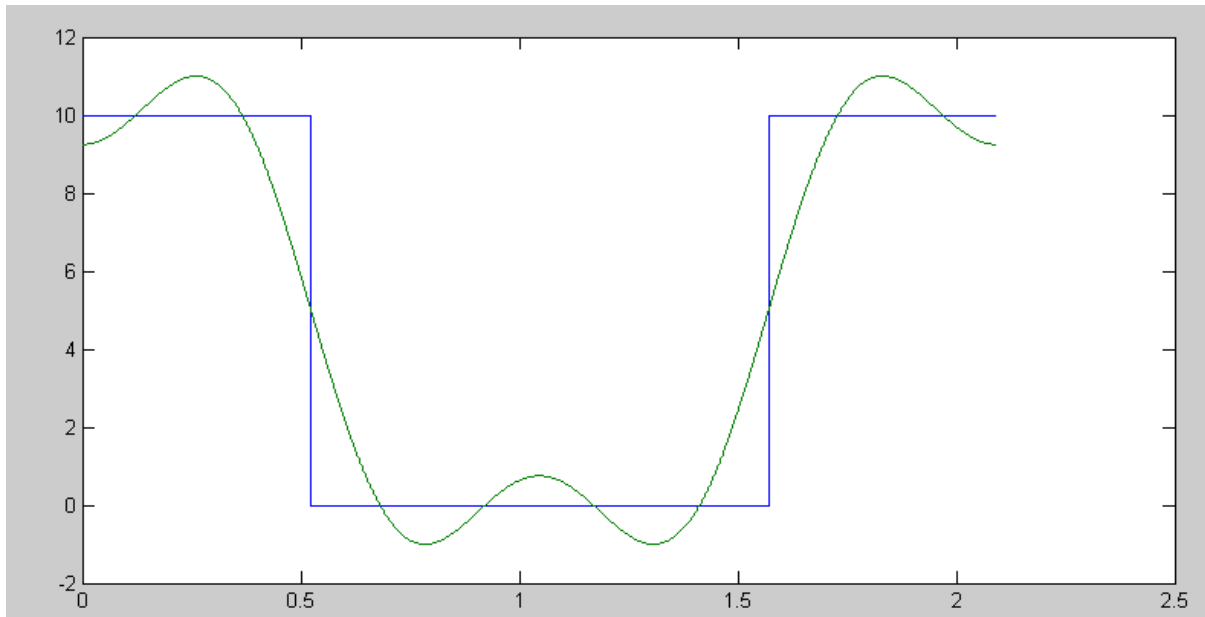
```
b2 = 2*mean(Vin .* sin(6000*t))

b2 = -1.6161e-015

a3 = 2*mean(Vin .* cos(9000*t))

a3 =   -2.1199

b3 = 2*mean(Vin .* sin(9000*t))

b3 =   -0.0020

>> plot(t,Vin,t,a0 + a1*cos(3000*t) + a3*cos(9000*t))
```

meaning

$$V_{in} \approx 5 + 6.3676\cos(3000t) - 2.1199\cos(9000t)$$

Problem 2) Use superposition and phasor analysis to determine V3(t)

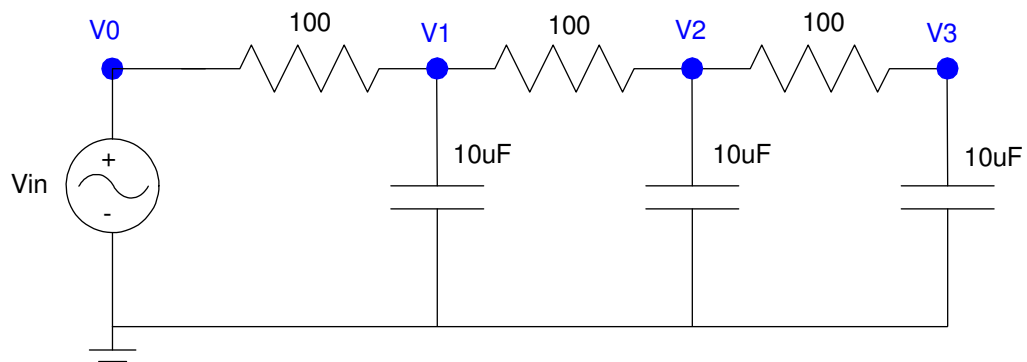$$V_{in} \approx 5 + 6.3676 \cos(3000t) - 2.1199 \cos(9000t)$$

DC Analysis

$$V_{in} = 5$$

$$\omega = 0$$

$$C \rightarrow \frac{1}{j\omega C} = \infty$$

(ignore capacitors at DC)

Since there is no path to ground

V3 = V2 = V1 = V0 = 5.0000

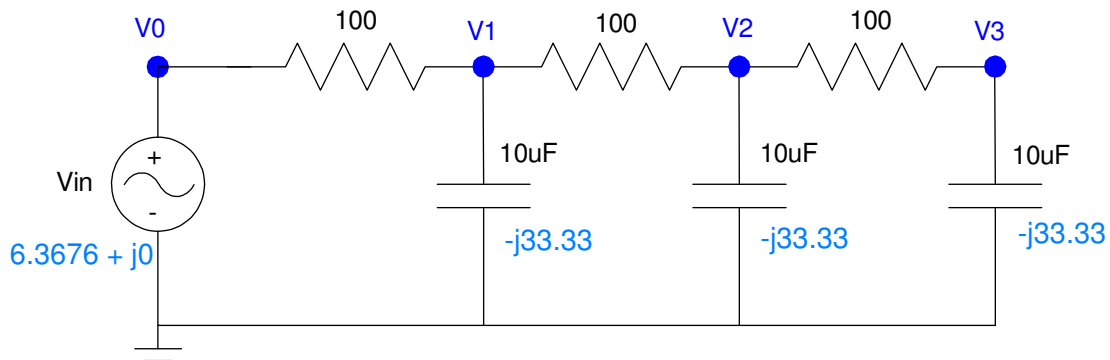$$V_{in} \approx 5 + 6.3676\cos(3000t) - 2.1199\cos(9000t)$$

At w = 3000:

$$V_{in} = 6.3676\cos(3000t)$$

In phasor form

$$V_{in} = 6.3676 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j33.333\,\Omega$$

Write the voltage node equtations

$$V_0 = 6.3676$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{-j33.33}\right) + \left(\frac{V_1 - V_2}{100}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{-j33.33}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{-j33.33}\right) = 0$$

Group terms and place in matrix form

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
\left(\frac{1}{100} + \frac{1}{-j33.33} + \frac{1}{100}\right) & \left(\frac{-1}{100}\right) & 0 \\
\left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{-j33.33} + \frac{1}{100}\right) & \left(\frac{-1}{100}\right) \\
0 & \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{-j33.33}\right)
\end{bmatrix}
\begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix}
=
\begin{bmatrix} 6.3676 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Solve in Matlab

```
A = [1,0,0,0 ; -a,b,-a,0 ; 0,-a,b,-a ; 0,0,-a,b-a]

   1.0000                     0                    0                    0
  -0.0100         0.0200 + 0.0300i  -0.0100                             0
        0        -0.0100            0.0200 + 0.0300i  -0.0100
        0                     0    -0.0100            0.0100 + 0.0300i

B = [6.3676;0;0;0]

   6.3676
        0
        0
        0

V = inv(A)*B

   6.3676
   0.8554 - 1.4774i
  -0.2241 - 0.3882i
  -0.1389 + 0.0284i
```

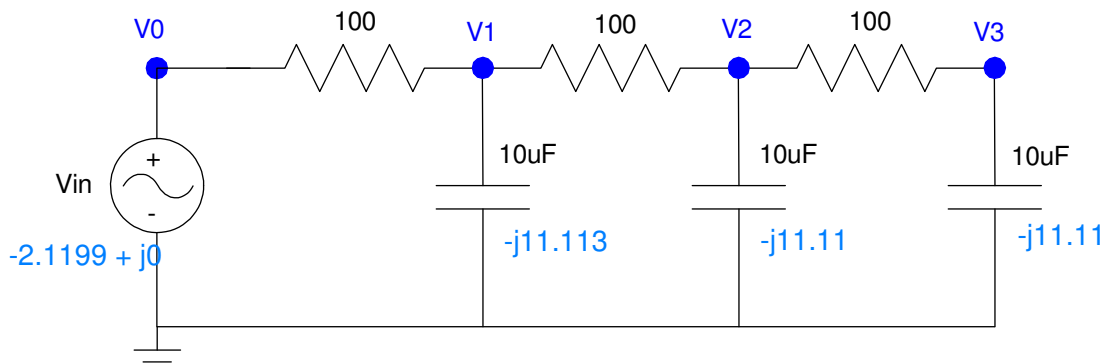$$V_{in} \approx 5 + 6.3676 \cos(3000t) - 2.1199 \cos(9000t)$$

Assume

$$V_{in} = -2.1199 \cos(9000t$$

w = 9000

In phasor form

$$V_{in} = -2.1199 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j11.111\Omega$$

Write the node equations

$$V_0 = -2.1199$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{-j11.11}\right) + \left(\frac{V_1 - V_2}{100}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{-j11.11}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{-j11.11}\right) = 0$$

Group terms and place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{1}{100} + \frac{1}{-j11.11} + \frac{1}{100}\right) & \left(\frac{-1}{100}\right) & 0 \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{-j11.11} + \frac{1}{100}\right) & \left(\frac{-1}{100}\right) \\ 0 & \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{-j11.11}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 6.3676 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in matlab

```
a = 1/100;
b = 1/100 + 1/(-j*11.11) + 1/100;
A = [1,0,0,0 ; -a,b,-a,0 ; 0,-a,b,-a ; 0,0,-a,b-a]

    1.0000                    0                    0                    0
   -0.0100        0.0200 + 0.0900i  -0.0100                    0
         0             -0.0100        0.0200 + 0.0900i  -0.0100
         0                    0             -0.0100        0.0100 + 0.0900i

B = [-2.1199;0;0;0]

   -2.1199
         0
         0
         0

V = inv(A)*B

   -2.1199
   -0.0483 + 0.2223i
    0.0222 + 0.0101i
    0.0014 - 0.0023i
```
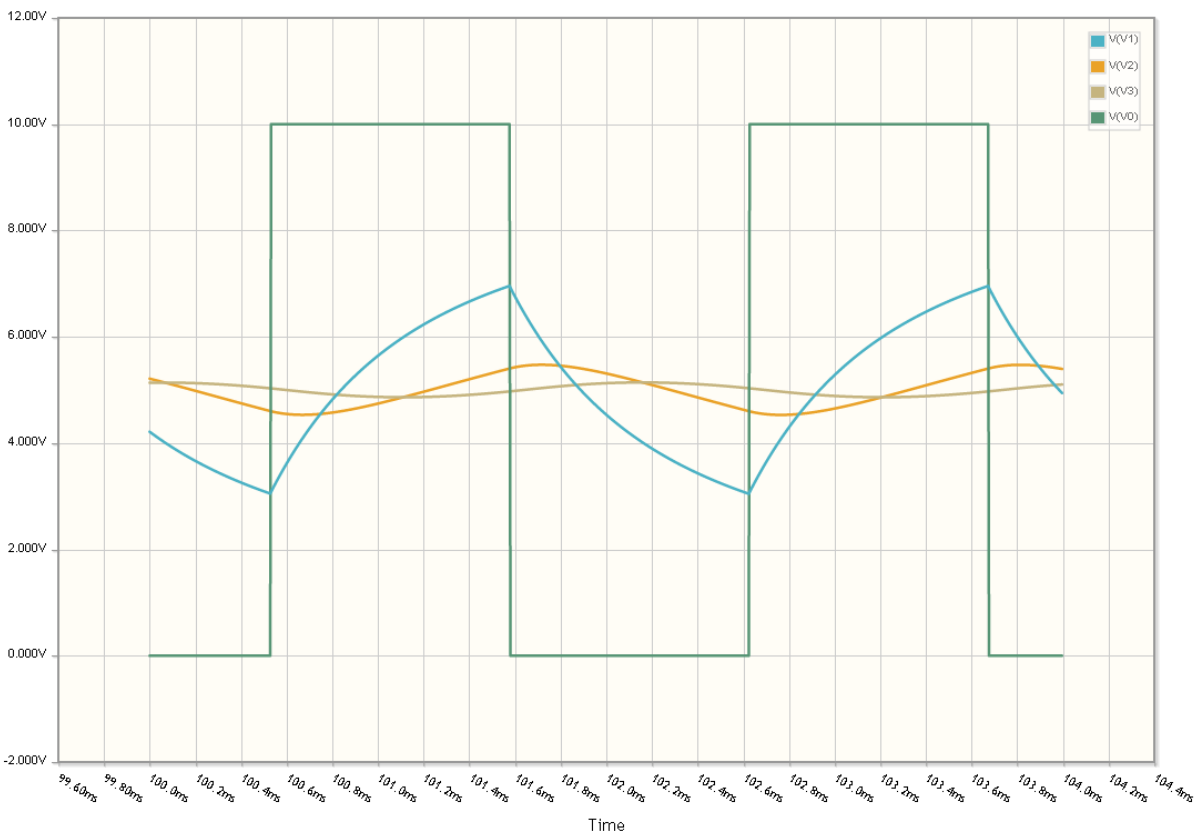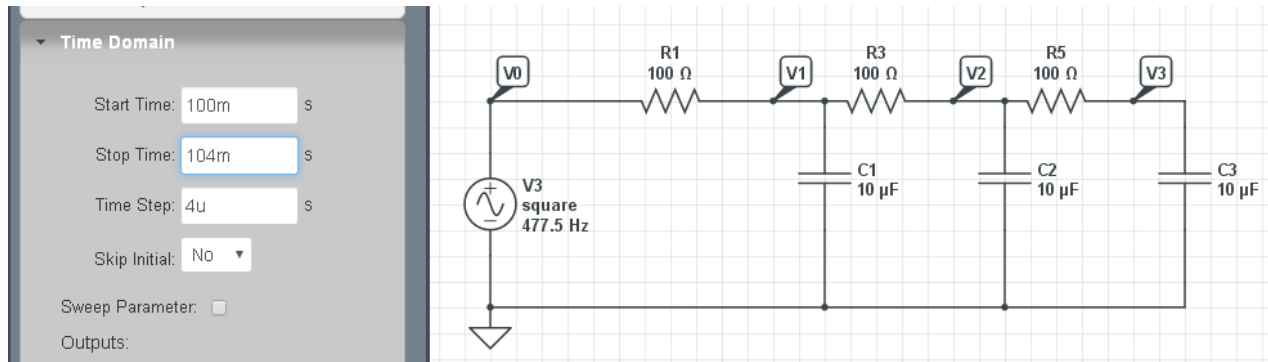
The total answer is then the sum of the three

  DC: 5.0000

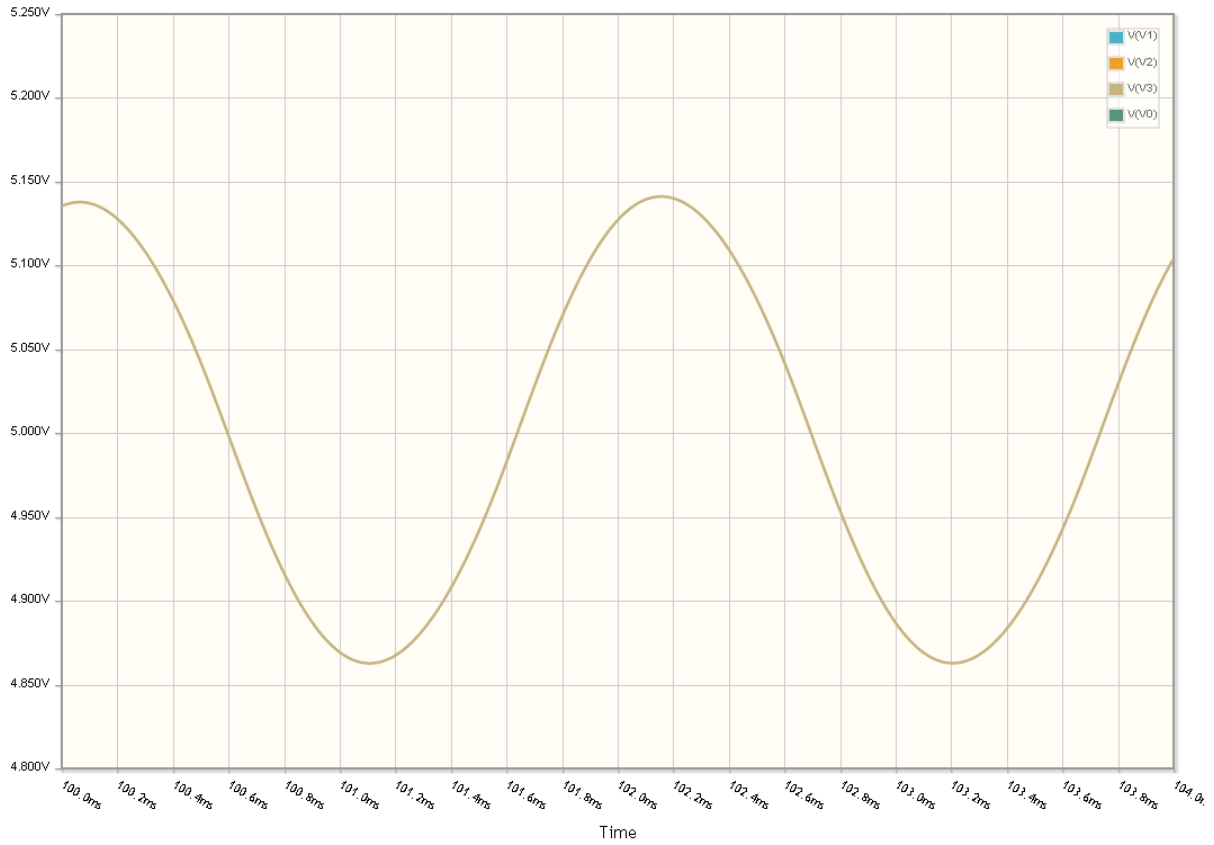 w = 3000:   -0.1389 + 0.0284i

  w = 9000   0.0014 - 0.0023i

$$V_3(t) = 5.000 - 0.1389\cos(3000t) - 0.0284\sin(3000t)$$
$$+0.0014\cos(9000t) + 0.0023\sin(9000t)$$

Problem 3) Simulate this circuit using PartSim and compare the transient response in PartSim to your computed results in problem #2

Looking at V3(t)

Max = 5.141V

Min = 4.863V

Average = 5.002V   (5.000V computed)

Vpp = 278mVpp   (283.5mV computed)

Almost a pure sine wave (meaning the 3rd harmonic is negligible)

Note: 3000 rad/sec term compued is

$$-0.1389\cos(3000t) - 0.0284\sin(3000t)$$

$$\left|-0.1389 + j0.0284\right| = 0.1417$$

The peak to peak voltage is double this

$$V_{pp} = 283.5mV_{pp}$$