

Voltage Nodes

Kirchoff's Current Law (KCL)

To solve a circuit, we primarily use three tools:

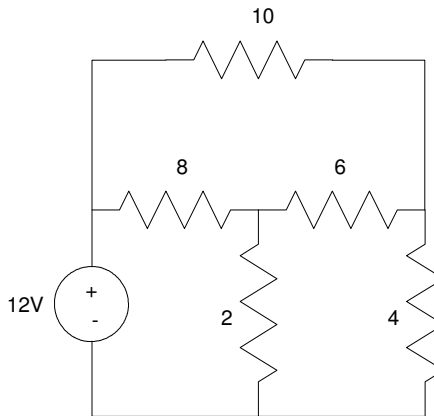
- Kirchoff's voltage law (voltage nodes)
- Kirchoff's current law (current mesh)
- Thevenin equivalents

The goal of the first two is to obtain N equations so solve for N unknowns.

Voltage Nodes

Voltage Nodes uses the notion that the sum of the current to (or from) a node must sum to zero.

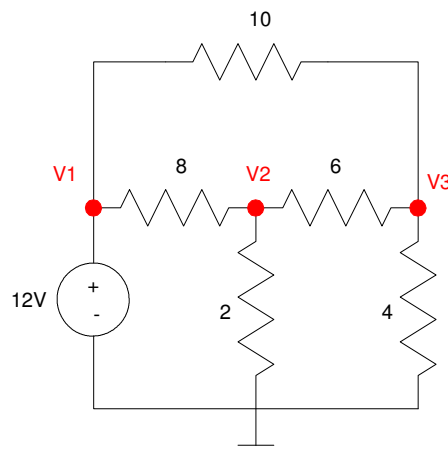
For example, find the voltages and currents in the following circuit:



Example Circuit: Find the voltages at each node.

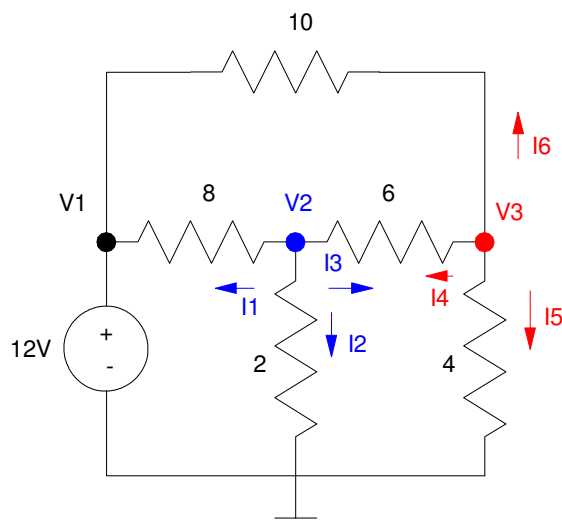
Step 1. Define your ground reference. This is normally the bottom of the diagram.

Step 2. Label the remaining voltage nodes



Step 1: Ground is defined as the node at the bottom (somewhat arbitrary)
Step 2: Each other voltage node is assigned a name: V1, V2, V3

Step 3. Write N equations to solve for N unknowns. If you know the voltage already, such as $V_1 = 12V$, use that as an equation. For the remaining nodes, the current from the node must sum to zero



Step 3: Write N equations for N unknown voltages.
The total current from each node must add to zero.

At node V_1 :

$$(1) \quad V_1 = 12$$

At node V_2 : (blue arrows)

$$I_1 + I_2 + I_3 = 0$$

$$(2) \quad \left(\frac{V_2 - V_1}{8} \right) + \left(\frac{V_2 - 0}{2} \right) + \left(\frac{V_2 - V_3}{6} \right) = 0$$

At node V_3 : (red arrows)

$$I_4 + I_5 + I_6 = 0$$

$$(3) \quad \left(\frac{V_3 - V_2}{6} \right) + \left(\frac{V_3 - 0}{4} \right) + \left(\frac{V_3 - V_1}{10} \right) = 0$$

This gives you three equations to solve for three unknowns. We'll do this more in Chapter 4. For now, just come up with N equations for N unknowns.

To solve, throw this into MATLAB.

Write this in matrix form:

$$V_1 = 12$$

$$-\left(\frac{1}{8}\right)V_1 + \left(\frac{1}{8} + \frac{1}{2} + \frac{1}{6}\right)V_2 - \left(\frac{1}{6}\right)V_3 = 0$$

$$-\left(\frac{1}{10}\right)V_1 - \left(\frac{1}{6}\right)V_2 + \left(\frac{1}{6} + \frac{1}{4} + \frac{1}{10}\right)V_3 = 0$$

or

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{8} & \left(\frac{1}{8} + \frac{1}{2} + \frac{1}{6}\right) & \left(-\frac{1}{6}\right) \\ \left(-\frac{1}{10}\right) & \left(-\frac{1}{6}\right) & \left(\frac{1}{6} + \frac{1}{4} + \frac{1}{10}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve

note: type in 'diary z:Jake.txt' in MATLAB and everything that goes to the screen will go the file 'Jake.txt' on your z-drive. This saves you a lot of writing. When you're done with your homework set, edit this text file, remove the mistakes (making it look like the homework was easy), write in pencil what the numbers mean (like $V_1 = 12V$, $V_2 = 2.55V$, etc.), and turn this in for your homework.

$$A = [1, 0, 0; -1/8, 1/8+1/2+1/6, -1/6; -1/10, -1/6, 1/6+1/4+1/10]$$

$$\begin{array}{ccc} 1. & 0. & 0. \\ -0.125 & 0.79166667 & -0.16666667 \\ -0.1 & -0.16666667 & 0.51666667 \end{array}$$

$$B = [12; 0; 0]$$

$$\begin{array}{c} 12. \\ 0. \\ 0. \end{array}$$

$$V = \text{inv}(A) * B$$

$$\begin{array}{c} 12. \\ 2.557377 \\ 3.147541 \end{array}$$

So

$$V_1 = 12V$$

$$V_2 = 2.5573 V$$

$$V_3 = 3.1475V$$

CircuitLab: A good way to check your answers are

- Simulate the circuit using CircuitLab (or like program), or
- Build the circuit in lab and measure the voltages.

The latter is done in the lab portion of this course.

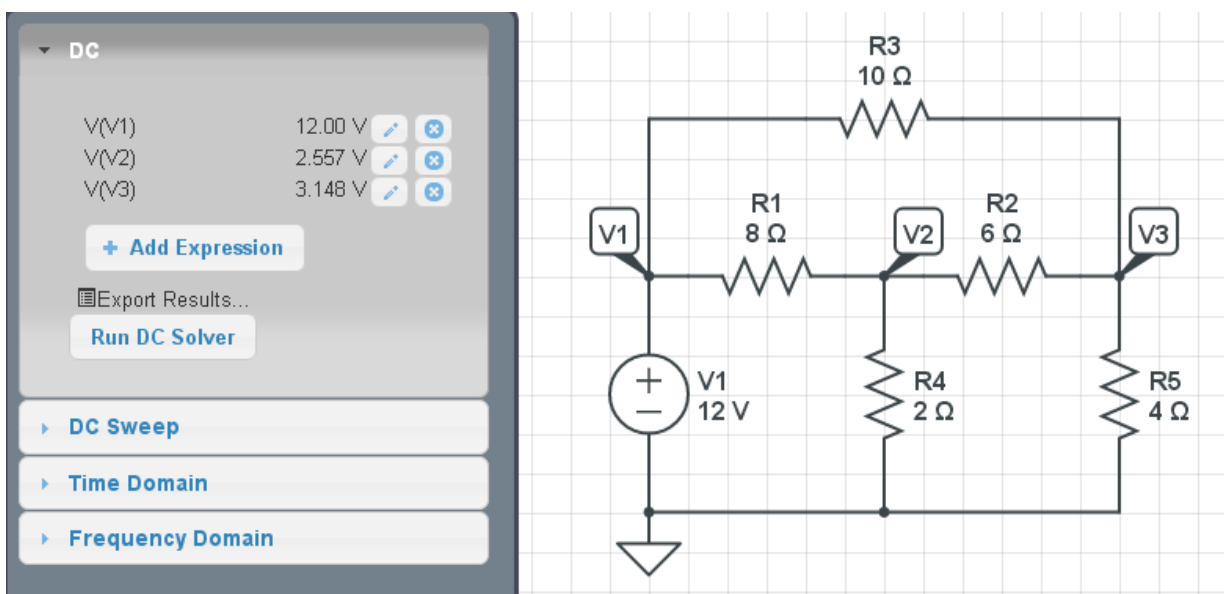
The results from CircuitLab are shown below. Note that the results match up with our Matlab calculations:

- $V_1 = 12V$
- $V_2 = 2.5573 V$
- $V_3 = 3.1475V$

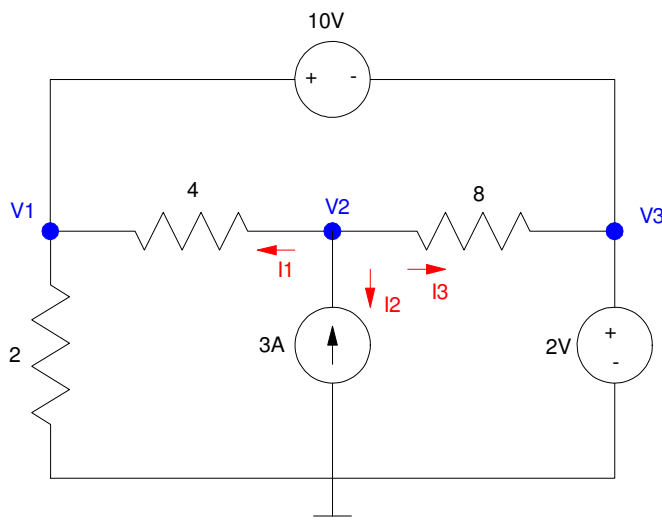
If the results don't match, most likely

- There is an error in your equations, or
- You input the matrices wrong, or
- Your circuit in CircuitLab is wrong.

I have yet to find a wrong solution in CircuitLab



Example 2: Find V_1 , V_2 , and V_3 :



Example 2: Three nodes means we need 3 equations so solve for 3 unknowns.

There are three voltage nodes: we need to write 3 equations to solve for 3 unknowns.

Start with the easy ones: the voltage sources. Each voltage source defines the voltage between the + and - terminal:

$$V_+ - V_- = X$$

For the two in this circuit:

$$V_3 - 0 = 2 \quad (1)$$

$$V_1 - V_3 = 10 \quad (2)$$

That's two equations: we need one more. At V_2 , the sum of the current from the node must add to zero:

$$I_1 + I_2 + I_3 = 0$$

$$\left(\frac{V_2 - V_1}{4}\right) + (-3) + \left(\frac{V_2 - V_3}{8}\right) = 0 \quad (3)$$

We now have 3 equations to solve for the 3 unknown voltages. Grouping terms:

$$-\left(\frac{1}{4}\right)V_1 + \left(\frac{1}{4} + \frac{1}{8}\right)V_2 - \left(\frac{1}{8}\right)V_3 = 3$$

Place in matrix form:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ -0.25 & 0.375 & -0.125 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 3 \end{bmatrix}$$

Solving in Matlab:

```
>> A = [0,0,1 ; 1,0,-1 ; -0.25,0.375,-0.125]
```

```
      0      0      1.0000
      1.0000      0      -1.0000
     -0.2500      0.3750     -0.1250
```

```
>> B = [2;10;3]
```

```
      2
     10
      3
```

```
>> V = inv(A) * B
```

```
V1  12.0000
```

```
V2  16.6667
```

```
V3  2.0000
```

This checks with Circuitlab:

Circuitlab results agree with our computed results.

