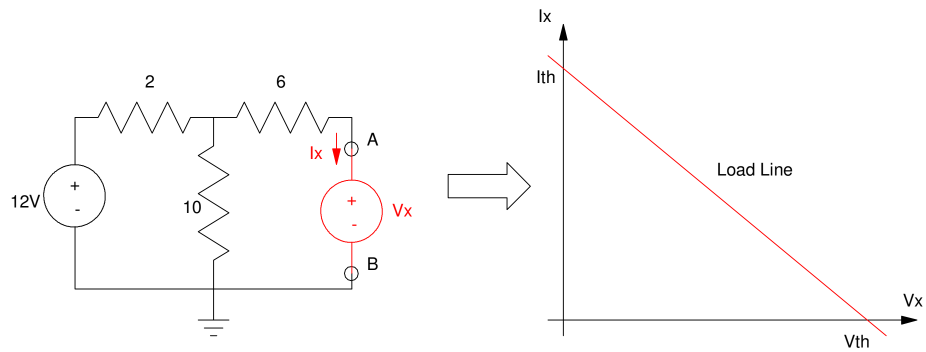
Thevenin Equivalent and Load Lines

So far we've looked at two methods to solve a circuit: current loops and voltage nodes. You can solve any circuit using these techniques.

A third option, Thevenin Equivanents, is a little more tricky - but it can make some cirucuits a lot easier to analyze if you can get the hang of it.

The idea behind Thevenin equivalents is that the voltage-current relationship for a linear circuit follows a straight line. Likewise, any circut which has the the same voltage-current relationship will behave exactly the same.

For example, consider the following circuit:

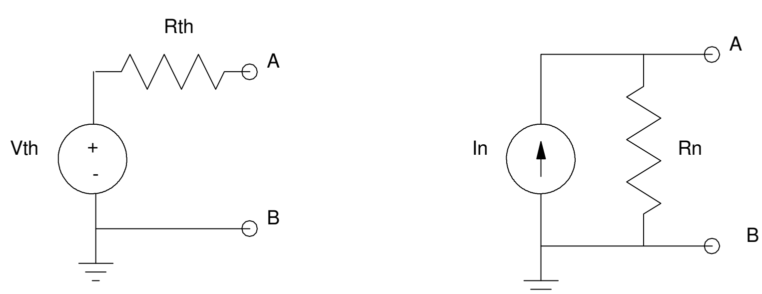


Since it's a linear circuit, the V/I relationship at the load (Vx, Ix) will follow a straight line as shown on the right. As far as the load is concerned, any circuit with the same load line is indistingushable.

Since all circuits with the same load line are indistinguishable, let's use the simplest one possible:

- A voltage source and resistance in series (termed a Thevenin equivelent), or
- A current source and resistance in parallel (termed a Norton equivelent).

The trick is to find the values ot Vth, Rth, or Ith



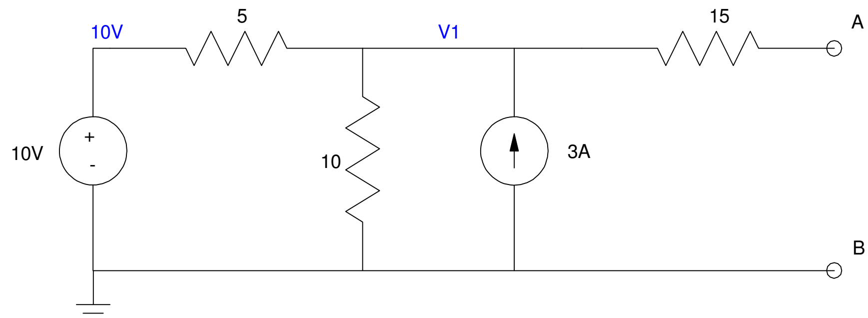
Thevenin Equivalent Norton Equivalent

Note that these circuits are equivalent to the circuit you're analyzing. This means that if you do the same thing to both circuits, you should get the same result. This leads to the following procedure to find the Thevenin and Norton equivalents:

- Vth: Measure the open-circuit voltage of your circuit.
- Rth = Rn: Turn off all sources (V = 0, I = 0). Measure the resulting resistance.
- In: Measure the short-circcuit current

It's probably easiest to illustrate this through examples.

Example 1: Determine the Thevenin and Norton equivalent for the following circuit:



Exampl e 1: To find V(thevenin), compute the open-circuit voltage

Vth: Measure the open-circuit voltage.

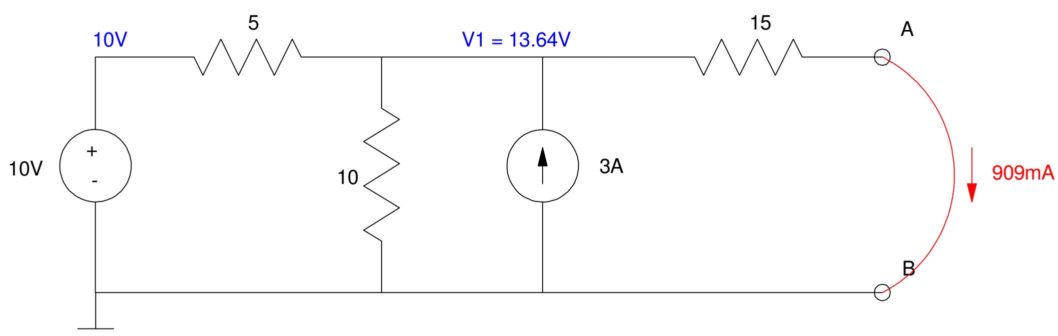
This isn't obvious so let's write the voltage node equation

$$\left(\frac{V_1-10}{5}\right)+\left(\frac{V_1}{10}\right)-3=0$$

$$V_{th}=V_1=16.67V$$

This is Vth

In: Short AB and measure the current. Again, this isn't obvious so write the node equation at V1



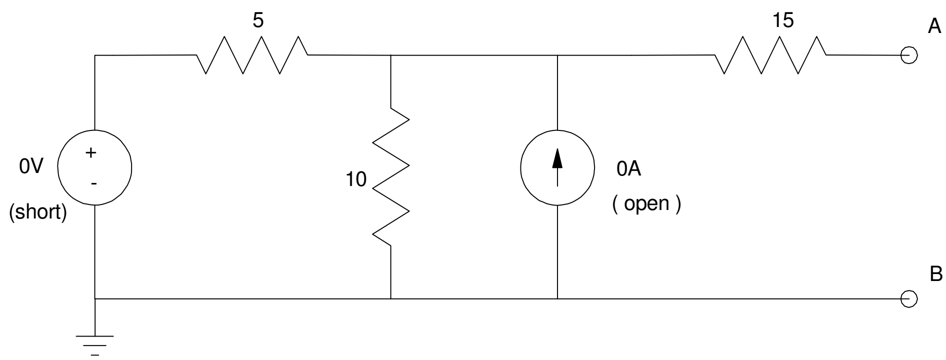
To find I(Norton), short the output and compute the short-circuit current

$$\left(\frac{V_1-10}{5}\right)+\left(\frac{V_1}{10}\right)-3+\left(\frac{V_1}{15}\right)=0$$

$$V_1=13.64V$$

$$I_{short}=I_N=\frac{13.64V}{15\Omega}=909.1mA$$

Rth: Turn off the sources (V = 0, I = 0). Measure the resistance between A and B



To measure the Thevenin resistance, turn off all sources and compute the resistance between A and B

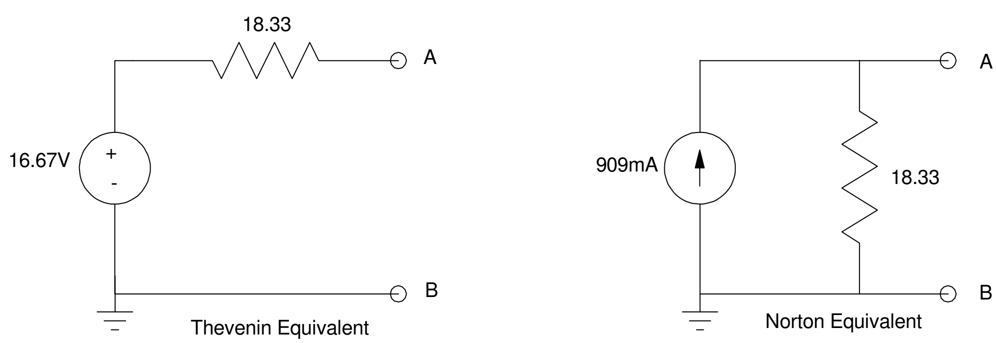
The resistance is

$$R_{AB} = 15 + 5||10$$

$$R_{AB} = 18.333\Omega$$

Note that you only need to compute two of these: the third redundant.

$$R_{AB} = \frac{V_{th}}{I_n} = \frac{16.67V}{909mA} = 18.33\Omega$$



Resulting Thevenin and Norton equivalents of Example 1

Circuit Simplification using Thevenin and Norton Equivalent

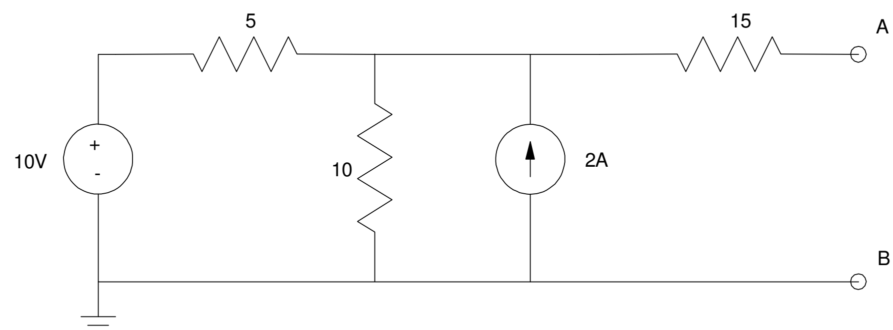
Given a circuit, you can change between Thevenin and Norton equivalents using the relationship

$$R_{Thevenin} = R_{Norton}$$

$$V_{Thevenin} = I_{Norton} \cdot R$$

$$I_{Norton} = \frac{V_{Thevenin}}{R}$$

For example, determine the Thevenin equivalent seen at terminal AB

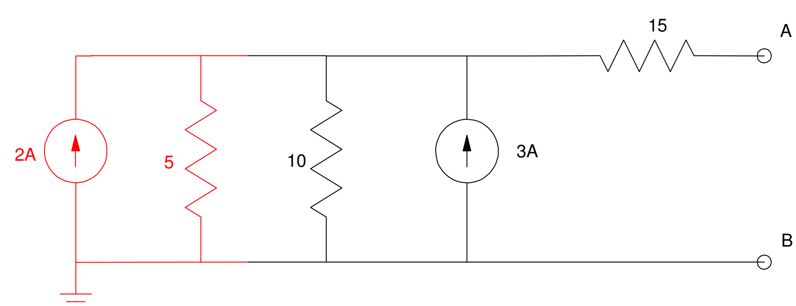


Example 2: Find the Thevenin equivalent at AB

Step 1: Convert the 10V / 5 Ohm resistor to its Norton equivalent

$$I_N = \frac{10V}{5\Omega} = 2A$$

$$R_N = R_{Th} = 5\Omega$$



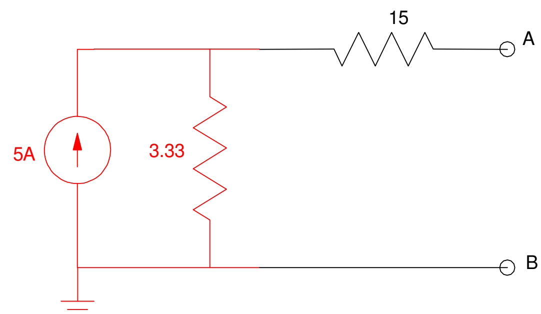
Convert the 10V : 5 Ohm source to its Norton equivalent (shown in red)

Add the resistors in parallel

$$R_{net} = 5||10 = \left(\frac{1}{5} + \frac{1}{10}\right)^{-1} = 3.333\Omega$$

Add the current soruces

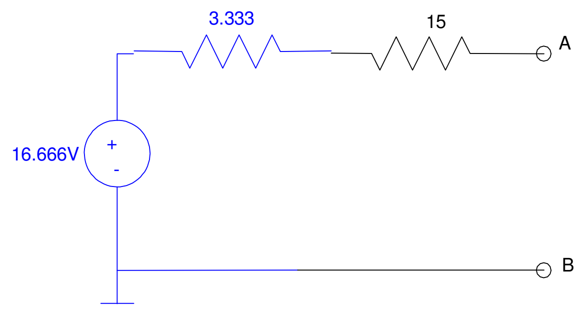
$$I_{net} = 2 + 3 = 5$$



Add the resistors and current sources in paraleel:

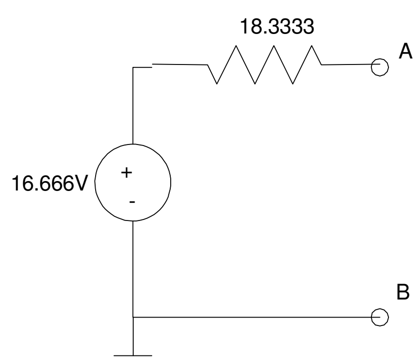
Convert back to a Thevenin equivalent

$$V_{th} = 5A \cdot 3.3333\Omega = 16.666V$$



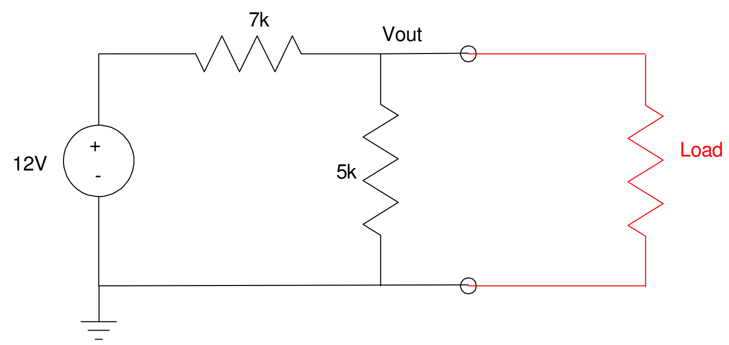
Convert to a Thevenin equivalent (shown in blue)

Add the resistors in series and you have the Thevenin equivalent looking in from terminals AB



Thevenin Equivalet of Example 2

Example 3: Thevenin equivalents can sometimes provide insight as to why something doesn't work. For example, the following circuit will convert 12VDC to 5VDC:



Voltage divider used to convert 12V to 5V

If you build this circuit and test it without a load, it works: Vout = 5V. If you then connect it to your iPod, Vout dissappears. Why?

To explain that, think Thevenin equivalents. If you convert the circuit to its Thevenin equivalent, you vet

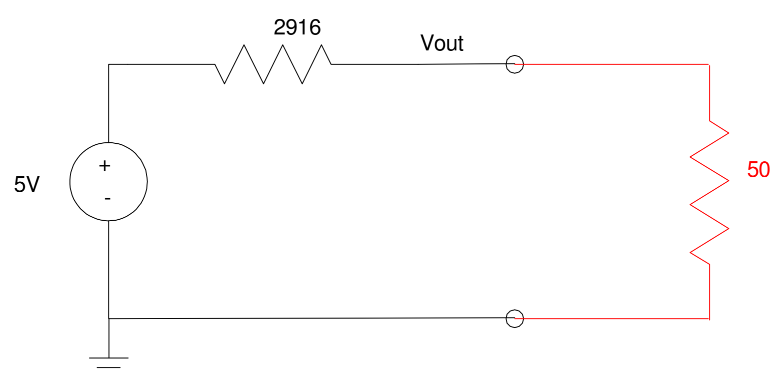
$$V_{th} = V_{open} = \left(\frac{5k}{5k+7k}\right) 12V = 5V$$

$$R_{th} = 7k || 5k = 2916\Omega$$

If your load draws 100mA @ 5V, it looks like a 50 Ohm resistor

$$R_{load} = \frac{5V}{100mA} = 50\Omega$$

The circuit them becomes



Thevenin eqivalent of the voltage divider driving a load which draws 100mA @ 5V (i.e. a 50 Ohm resistor)

By voltage division, Vout is now

$$V_{out} = \left(\frac{50}{50+2916}\right) \cdot 5V$$

$$V_{out} = 0.0843V$$

This circuit works as a 5V source as long as you don't use it. In Electronics, we'll cover other circuits which *do* work.