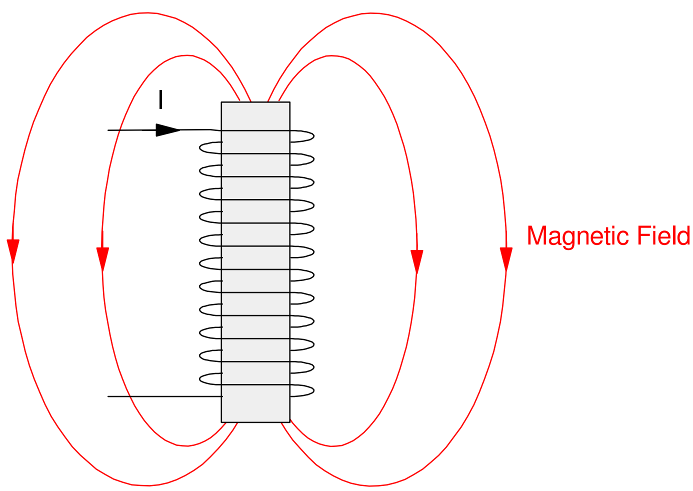
Inductors

Capacitors deal with voltage and store energy in an electric field. Inductors, on the other hand, deal with current and store energy in a magnetic field.



Inductors: Coils of wire produce a magnetic field, which in turns produces an inductor

In general, inductors are avoided if at all possible:

- Unlike electric fields and capacitors, magnetic fields do not have starting point and an ending point. That means that the magnetic fields extend outside the inductor and affect other components in a circuit.
- The core of an inductor almost has to be iron. Iron is about the only material which is strongly magnetic. This dependence upon iron makes inductors heavy, lossy, and only capable of supporting a limited amount of current until the iron saturates.
- The large amount of windings necessary to make an inductor creates a large resistive element with inductors, also reducing the efficiency.

Sometimes, you just can't avoid using inductors:

- Speakers rely upon electromagnets to work. An electromagnet is inherently an inductor.
- Motors rely upon electromagnetics to work. Again, any time you have an electromagnet you inherently have an inductor.
- Transmission lines are wires that carry electricity large distances. Current through a wire creates a magnetic field. This magnetic field stores energy, and again, created inductance.

Example 1: Determine the inductance and resistance of an inductor like the one shown above. Assume

- 100 windings of 36 gage wire copper wire (1.38 Ohms / meter)[1]
- Cross sectional area = $25mm^2$
- Iron core with relative permeability of 800

[1] https://www.engineeringtoolbox.com/copper-wire-d_1429.html

Solution: From Electronics Tutorials[2]

$$L = \mu N^2 A / l$$

$$L = \left(4\pi \cdot 10^{-7} \tfrac{H}{m}\right)(800)(100)^2(25 \cdot 10^{-6} m^2)/(0.025 m)$$

$$L = 0.0101 H$$

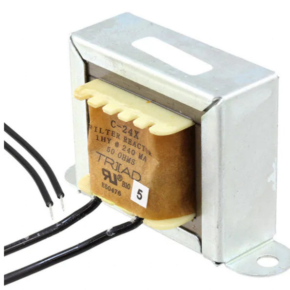
The resistance is

$$R = (100 \text{ windings})\left(0.02 \tfrac{\text{m}}{winding}\right)(1.38 \tfrac{\Omega}{m})$$

$$R = 2.77\Omega$$

This shows up in inductors you can get from Digikey:

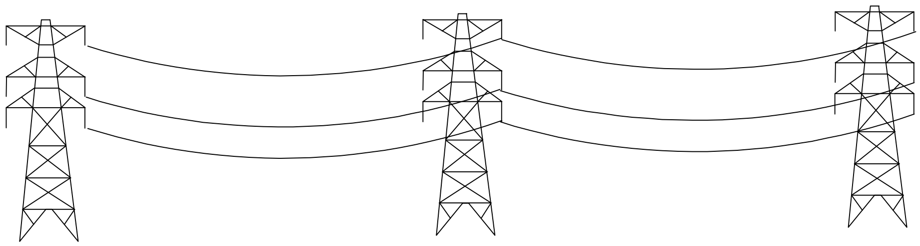
- If you want large inductance, you have high resistance
- If you want low resistance, you get low inductance
- If you want a small inductor, you get both: low inductance and high resistance

CX24	5250-RC
	
1 H	0.0001H
50 Ohms	0.216 Ohms
240mA (max)	2 A
$10.23 ea	$2.11

[2] https://www.electronics-tutorials.ws/inductor/inductor.html

Example 2: Determine the inductance of a copper transmission line:

- Length = 1km
- radius = 1cm
- frequency = 60Hz



From Wikipedia[3]

$$L = \frac{\mu_0}{2\pi} l (A - B + C)$$

$$A = \ln\left(\frac{l}{r} + \sqrt{\left(\frac{l}{r}\right)^2 + 1}\right) = 12.20$$

$$B = \frac{1}{\frac{r}{l} + \sqrt{1 + \left(\frac{r}{l}\right)^2}} \approx 1$$

$$C = \frac{1}{4 + r\sqrt{\frac{2}{\rho}\omega\mu}} = 0.1569$$

giving

$$L = 2.30 \ \frac{mH}{km}$$

The inductance isn't a lot, but when a transmission line travels thousands of kilometers, it adds up.

[3] https://en.wikipedia.org/wiki/Inductor#Inductance_formulas

VI Characteristics for an Inductor

The basic equations for an inductor are

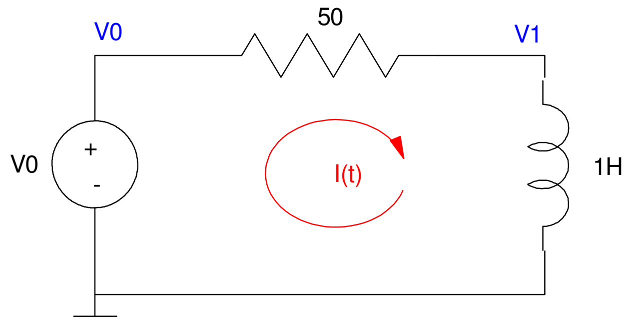
$$V = L\frac{dI}{dt}$$

$$E = \frac{1}{2}LI^2$$

Like capacitors, each inductor adds a 1st-order differential equation to a circuit. In this case, the differential equation is in terms of current.

Example 3: Determine the current and voltage for the following circuit. Assume

$$V_0(t) = 10\,u(t)$$



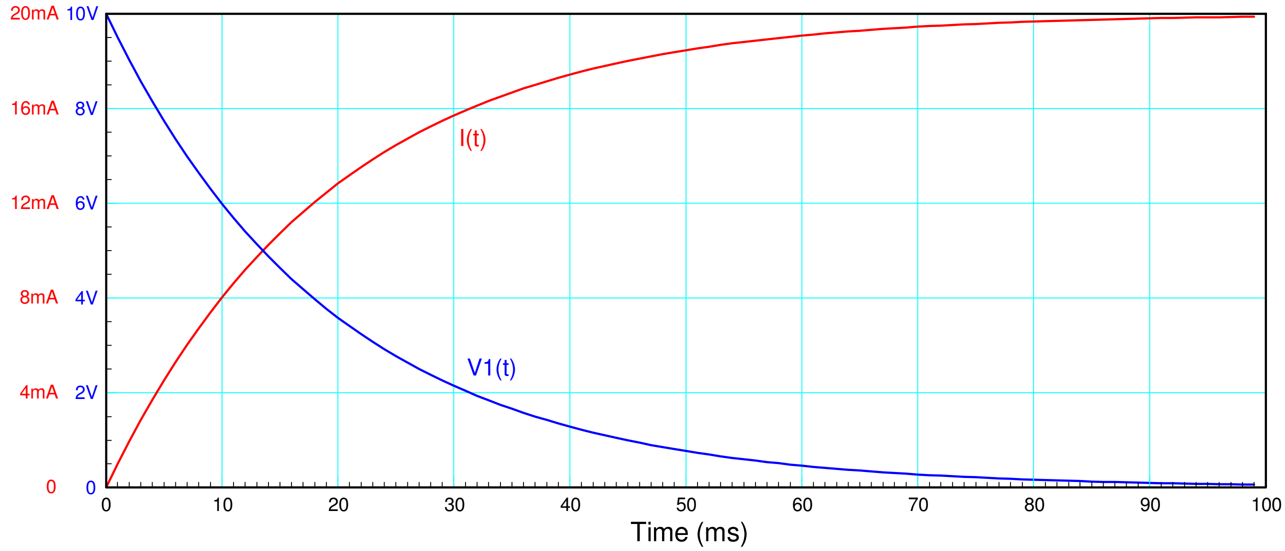
Solution: Write the differential equation

$$V_1 = L\frac{dI}{dt} = V_0 - 50I$$

$$\frac{dI}{dt} = V_0 - 50I$$

Solve using numerical integration like we did with capacitors:

```
V0 = 10;
I = 0
dt = 0.001;
t = 0;
Y = [];

while(t < 0.1)
   dI   = V0 - 50*I;

   I = I + dI * dt;
   t = t + dt;
   V1 = V0 - 50*I;

   Y = [Y ; [I, V1] ];
   end

t = [0:length(Y)-1]' * dt;
plot(t,Y)
```



Current and Voltage for a 1-Stage RL Circuit

Note that

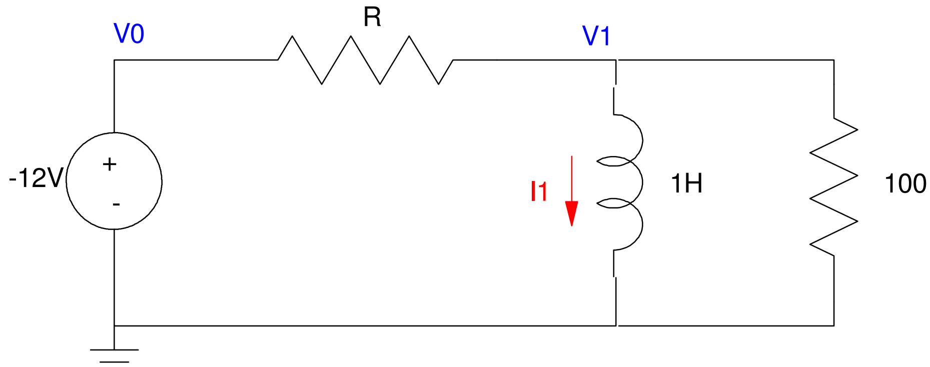
- The current increases to it's steady-state value (20mA) the same way the voltage across a capacitor increased to its steady-state value, and
- At steady-state, the voltage across the inductor is zero

With inductors, you can get more voltage out than you put in. The voltage across an inductor is

$$V = L\frac{dI}{dt}$$

If the current suddenly goes to zero, the voltage goes to infinity. This is how alternators and spark plugs work in your car.

Example 4: Let R vary from 5 Ohms to 1000 Ohms (switch closed or switch open). Determine V1(t)



Solution: Write the differential equation for this circuit

$$\left(\frac{V_1 - V_0}{R}\right) + \left(\frac{V_1}{100}\right) + I_1 = 0$$

$$V_1 = L\frac{dI_1}{dt}$$

giving

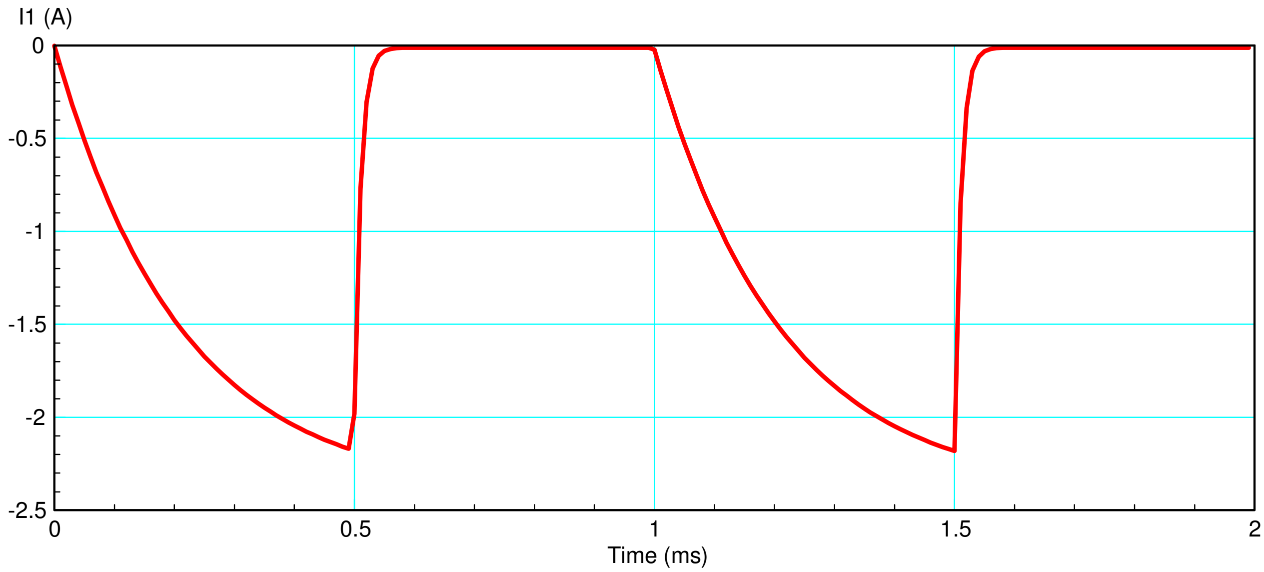
$$\left(\frac{1}{R} + \frac{1}{100}\right)V_1 = \left(\frac{V_0}{R}\right) - I_1$$

$$\left(\frac{1}{R} + \frac{1}{100}\right)\left(\frac{dI_1}{dt}\right) = \left(\frac{V_0}{R}\right) - I_1$$

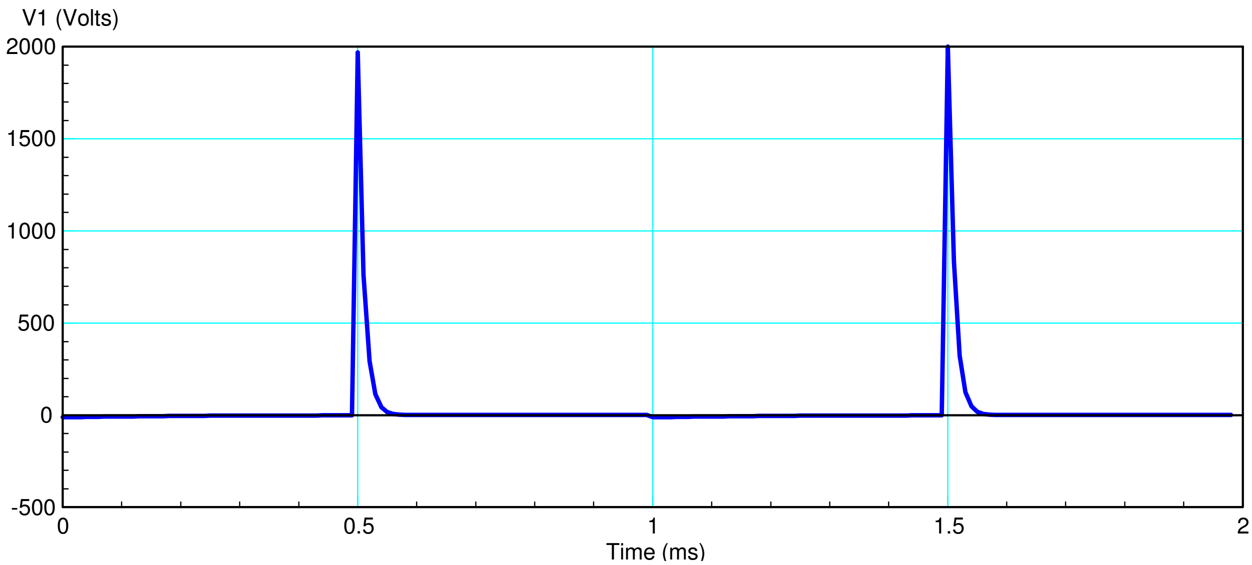
$$\frac{dI_1}{dt} = \left(\frac{1}{R} + \frac{1}{100}\right)^{-1}\left(\left(\frac{V_0}{R}\right) - I_1\right)$$

Solve numerically:

```
V0 = -12;
I = 0
dt = 0.001;
t = 0;
Y = [];

while(t < 2)
   if(sin(2*pi*t) > 0)
       R = 5;
   else
       R = 1000;
   end

   dI = (V0/R - I) / ( 1/R + 1/100);

   I = I + dI * dt;
   t = t + dt;
   V1 = V0 - I*R;

   Y = [Y ; [I, V1] ];
end
```



I1 vs. Time. Current slowly builds up between (0, 0.5) seconds when R = 5 Ohms
Current quickly decays between (0.5 and 1.0) seconds when R = 1000 Ohms.



V1 vs. Time. Note that a 12V battery can produce a 2000V spike.
When the switch opens (R = 1000), the voltage jumps to 2000V.

This is how spark plugs work:

- A 12V battery provides current to the alternator, storing energy in the magnetic field.
- When the battery is disconnected, the magnetic field collapses, creating a large voltage.

The path to ground is the spark plug.