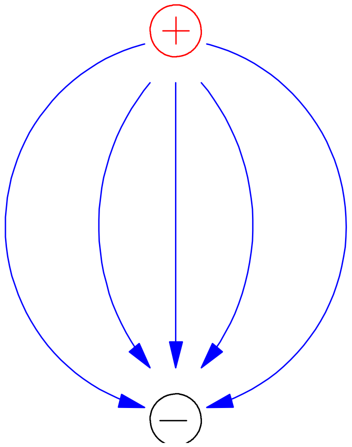
Mutual Inductance

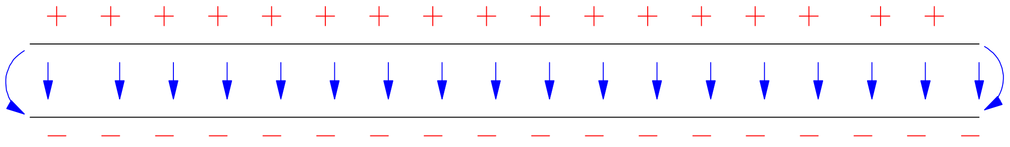
Capacitors

Since electrical charges *do* exist, electric field lines have a starting point and an ending point. For example, if you have a + and a - change, the field lines would look something like the following



The field lines flow from a + charge to a - change

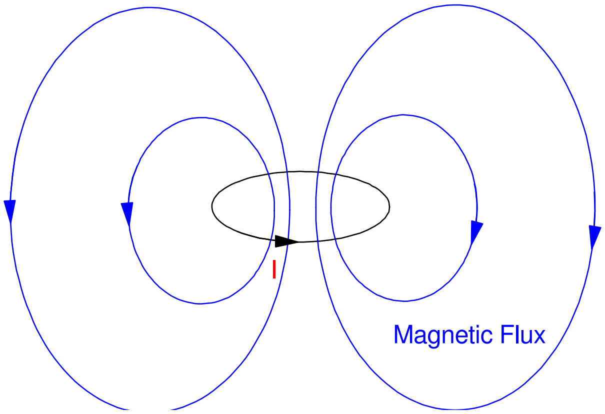
In a parallel plate capacitor, some of the field lines will extend beyond the plates of the capacitor. If these field lines overlap with another capacitor's field lines, there will be coupling. Since the spacing is so small, this coupling is effectively zero.



In a parallel plate capacitor, almost all of the field lines are confined between the plates of the capacitor

Inductors

Unlike electric fields, magnetic fields do not have a start or an end (magnetic monopoles do not exist). Instead, they form a closed path. For example, a coil with current flowing produces magnetic flux similar to the following:

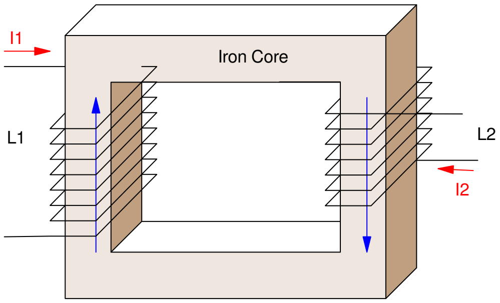


Current traveling around a loop produces a magnetic field. The field lines do not have a beginning or end.

Since magnetic fields do not have a beginning or end, they cannot be easily contained. If the field lines of two inductors overlap, the two inductors affect each other. This is called *mutual inductance*. Sometimes this is good, sometimes this is bad.

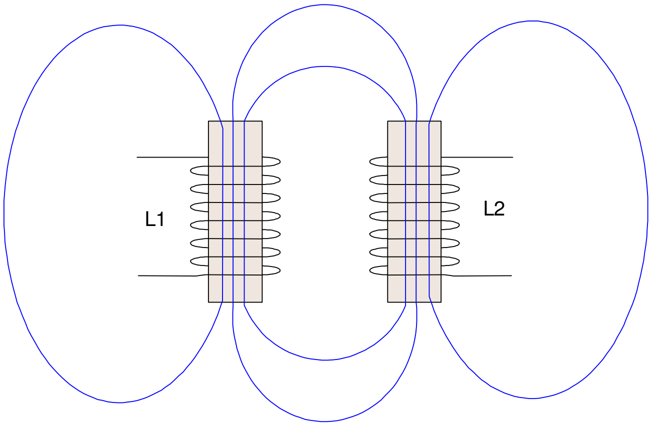
Mutual Inductance

Mutual inductance is modeled as coupling between two inductors. Let 'k' be the percentage of the field lines shared by two inductors. When k=1, all of the magnetic flux is shared. An example of this would be a transformer where an iron core keeps most of the magnetic flux lines within the iron itself



Example when the coupling coefficient (k) is one. Almost all of the flux lines from inductor L1 go through inductor L2

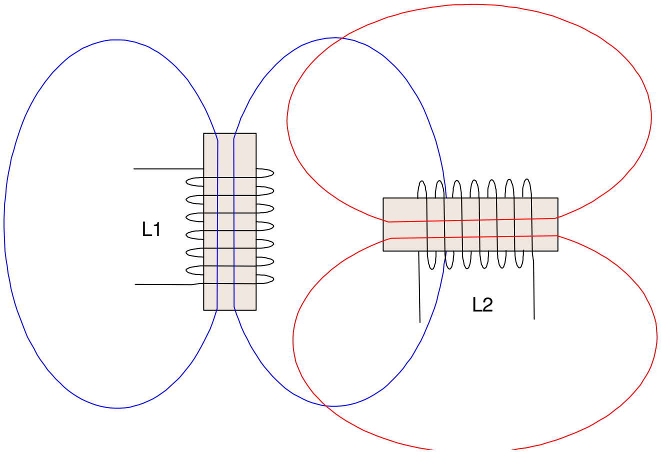
If two transformers are close to each other, some of the magnetic flux lines will be shared. In this case, 0<k<1.



Example where 0 < k < 1. Some of the magnetic flux lines are shared between two inductors.

As the inductors are moved further and further away, the coupling drops. Magnetic field strength drops as the cube of the distance, so coupling drops fairly quickly.

If two inductors are far apart or are perpendicular to each other, than the coupling is almost zero (k = 0).



Example where k = 0. The inductors are perpendicular to each other, so they share no flux lines.

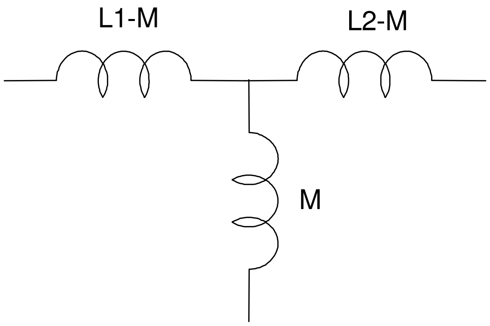
This makes circuit board design with inductors somewhat annoying: the placement of the inductors can have a significant influence on how the circuit behaves.

Electric Model for Mutual Inductance

If 'k' is the percentage of field lines that are common between two inductors, the mutual inductance, M, is

$$M = k\sqrt{L_1 L_2}$$

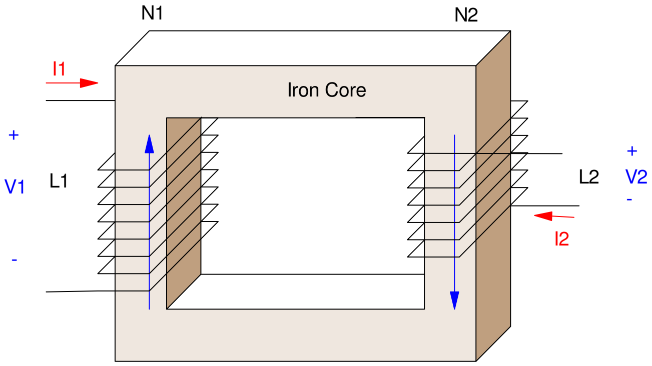
The T-circuit model for two couples inductors is then



T-Circuit model for a circuit with two inductors and mutual inductance

Usage of Mutual Inductance: Transformers

Transformers are essentially two inductors which share a common iron core. The permeability of iron is much higher than air, resulting in most of the magnetic flux from one inductor going through the other inductor.



Transformer with N1 turns on inductor L1 and N2 turns on inductor L2

Since the magnetic field is the same for both inductors

$$\phi_1 = \phi_2$$

$$N_1 I_1 = N_2 I_2$$

$$I_2 = \left(\frac{N_1}{N_2}\right) I_1$$

Inductors are a way to increase or decrease current flow. Power has to balance

$$P_1 = P_2$$

$$V_1 I_1 = V_2 I_2$$

This results in

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1$$

Transformers are a convenient way to change voltages and currents. This is why the power grid operates at 60Hz: transformers only work at AC (you need a changing flux to induce voltage and current).

Suppose you wanted to transmit power from western North Dakota to Minneapolis (1000km). Assume copper wire, 1cm in diameter. The resistance of this wire would be

$$R = \frac{\rho L}{A}$$

$$R = \left(\frac{\left(1.68 \cdot 10^{-8}\ \Omega m\right)(1000km)}{\pi \cdot (0.005m)^2}\right)$$

$$R = 213\Omega$$

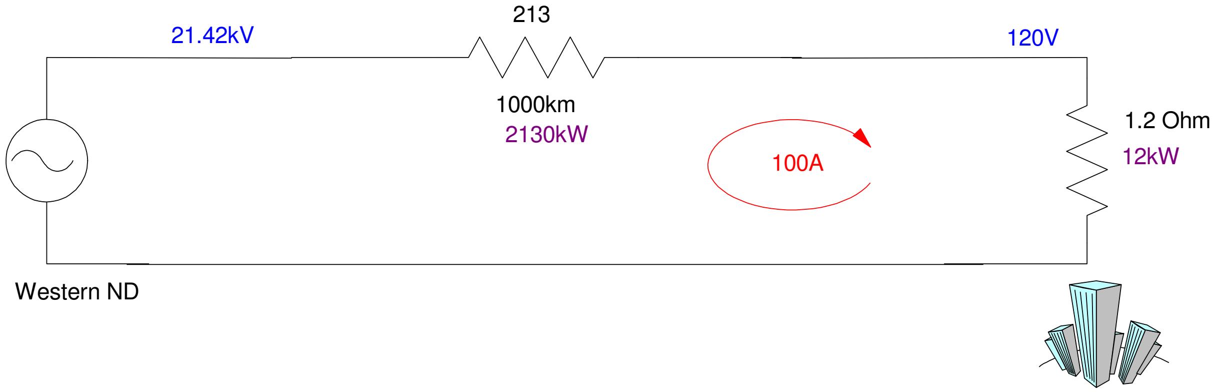
If you want to transmit 12kW at 120V (I = 100A), the line losses would be

$$P_{line} = I^2 R$$

$$P_{line} = (100A)^2 \cdot 213\Omega$$

$$P_{line} = 2130kW$$

99.94% of the energy is dissipated in the transmission lines while only 0.056% of the energy gets to the customer.



If you transmit power 1000km using DC, only 0.056% of the energy gets to the customer.

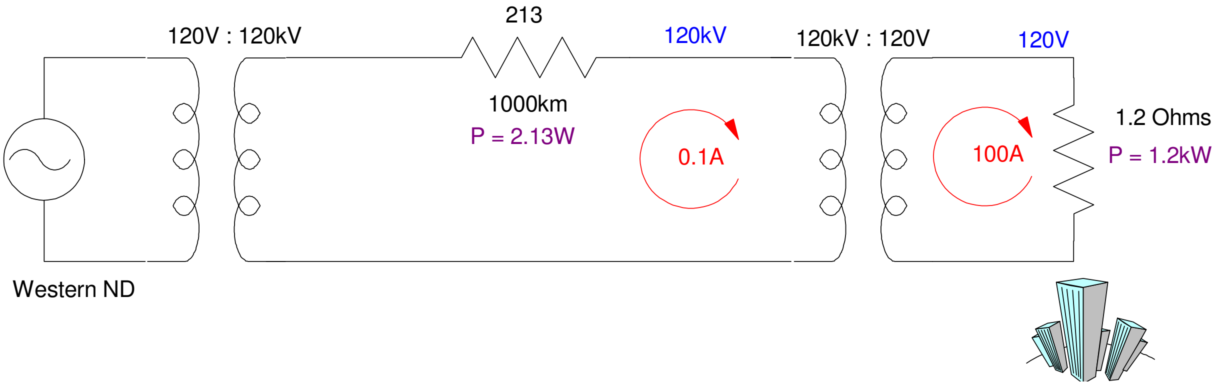
If you use a transformer to bump the voltage up to 120kV instead, the current drops by 1000x (keeping the power constant)

$$I = 0.1A$$

$$P_{line} = (0.1A)^2 \cdot 213\Omega$$

$$P_{line} = 2.13W$$

In this case, 99.8% of the energy gets to the customer while only 0.2% is dissipated by the transmission lines.



Transformers also change impedances. Suppose an impedance is connected to side 2

$$V_2 = R_2 I_2$$

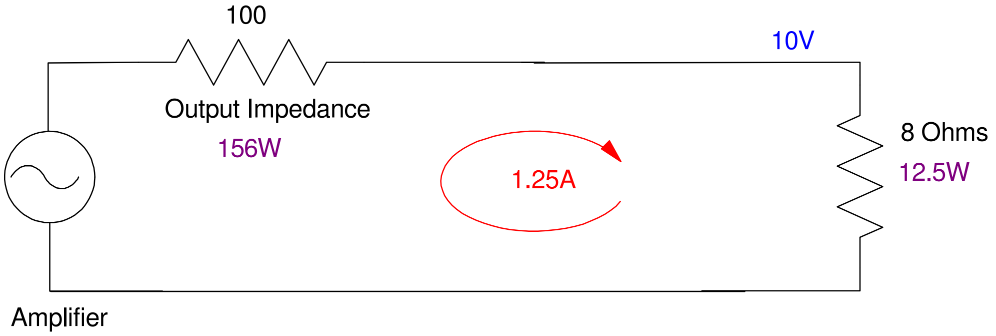
Substituting, on side 1 you see

$$\left(\frac{N_2}{N_1}\right) V_1 = R_2 \left(\frac{N_1}{N_2}\right) I_1$$

$$V_1 = \left(\left(\frac{N_1}{N_2}\right)^2 R_2 \right) I_1$$

The impedance seen through a transformer is changed by the square of the turns ratio.

This is used in some older stereos. Suppose the Thevenin resistance of your amplifier is 100 Ohms. If you drive an 8-Ohm speaker, only 7% of the energy gets to the speaker



If an amplifier has an output impedance of 100 Ohms, only 7% of the energy gets to the speaker.

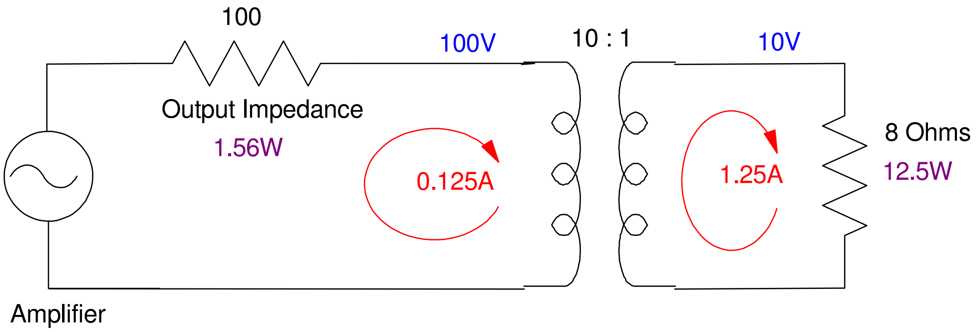
If you insert a 10:1 transformer before the speaker, the impedance as seen by the amplifier is

$$R = \left(\frac{10}{1}\right)^2 \cdot 8\Omega = 800\Omega.$$

Now, 89% of the energy gets to the speaker

$$eff = \eta = \frac{P_{load}}{P_{total}}$$

$$\eta = \left(\frac{I \cdot 800}{I \cdot (100+800)}\right) = \left(\frac{800}{900}\right) = 89\%$$



By inserting a transformer with a 10:1 turn ratio, the efficiency is increased to 89%.
However, the voltage before the transformer needs to be 10x higher than the voltage at the speaker.

Note however that the voltage inside the amplifier is 10x higher than at the speaker. If you're driving the speaker at 100W

$$P = 100W = \frac{V^2}{8\Omega}$$

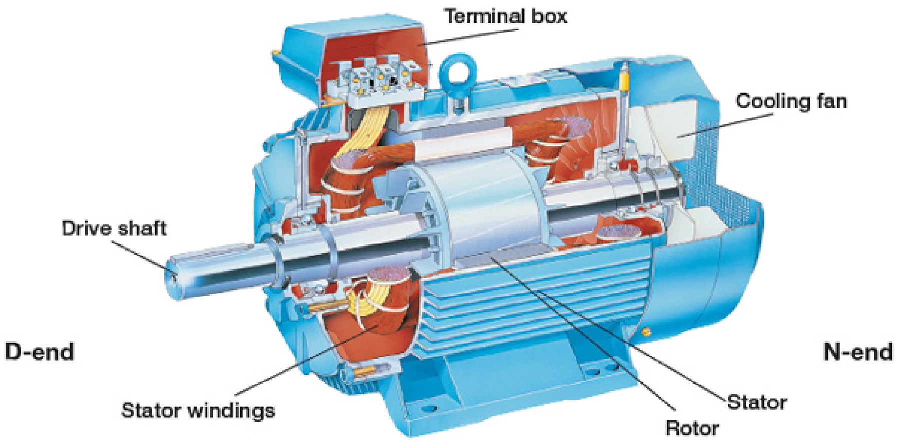
at the speaker you have 28V. At the amplifier side, the voltage will be 10x larger (280V).

Moral: If you see transformers in your stereo

- They're there to improve the efficiency of your stereo
- They also imply that the voltages inside the stereo can be very high.

Use of Mutual Inductance: AC Induction Motor

Another use of mutual inductance is the AC induction motor. Most AC motors you'll encounter are AC induction motors. These kind of look like black magic: there are no electrical connections to the rotor



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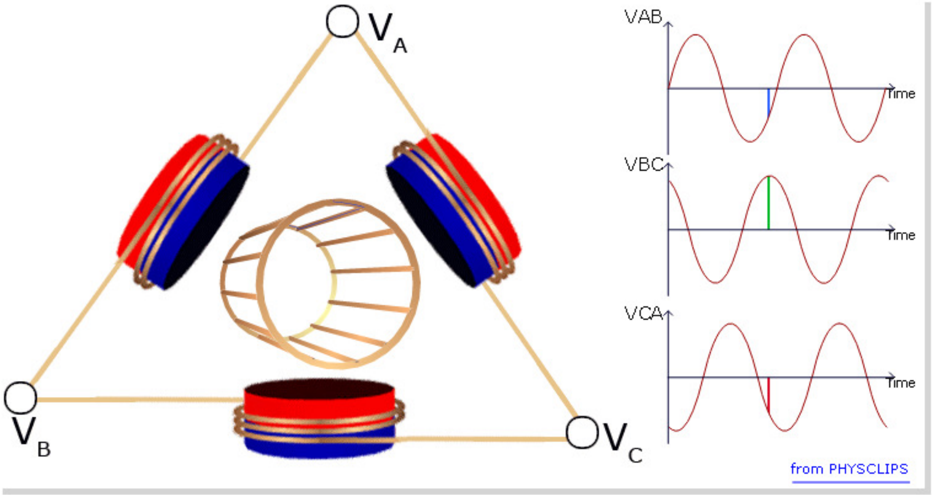
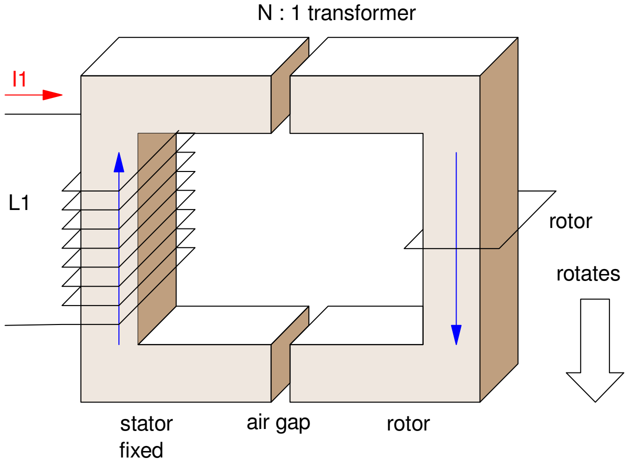
At the heart of an AC induction motor is a stator (an N-turn transformer) along with a rotor. The rotor contains aluminum bars forming a closed-path, creating a 1-turn transformer. When AC voltage powers the stator, current is induced in the rotor (the squirrel cage in the figure below). This in turns creates a magnetic field in the rotor, producing torque. This torque causes the rotor to spin, following the rotating magnetic field in set up by the stator



http://www.animations.physics.unsw.edu.au/jw/electricmotors.html

The electrical model of an AC induction motor is essentially a N:1 transformer where the secondary side rotates



The electrical model for an AC induction motor is almost identical to the electrical model for a transformer. This will be covered in more detail in ECE 331 Energy Conversion.