Passive Circuit Elements

Passive circuit elements are circuit elements which don't need a power supply or battery. The ones we consider here are

- Inductors,
- Capacitors, and
- Resistors.

The phasor impedance for these are:

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Resistors:

For a resistor,

$$\boxed{V = IR}$$

The power dissipated by a resistor as heat is

$$P = I^2R = \frac{V^2}{R} = VI \quad \text{Watts}$$

where we're using DC voltages and currents of rms units for AC soruces.

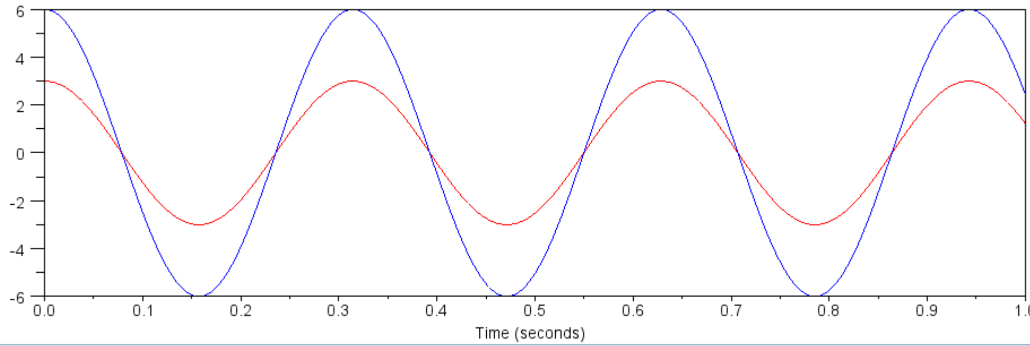
Note that votlage and current are in phase for resistors. For example, if you have a 2 Ohm resistor and

$$i(t) = 3\cos(20t)$$

then

$$v(t) = i(t) \cdot R$$

$$v(t) = 6\cos(20t)$$



Current (red) and Voltage (blue) for a 2 Ohm resistor. Note that current and voltage are in-phase for a resistor.

Inductors:

For an inductor, the energy stored is

$$E = \frac{1}{2}LI^2 \qquad \text{Joules}$$

Differentiating to get Watts gives

$$P = \frac{dE}{dt} = LI \cdot \frac{dI}{dt} \qquad \text{Watts}$$

This is also

$$P = VI = LI \cdot \frac{dI}{dt}$$

which gives the basic equation for an inductor:

$$\boxed{V = L \cdot \frac{dI}{dt}}$$

Note that this applies to any current going through an inductor: the voltage will be the derivative of the current, scaled by L. For example, if the current through a 100mH inductor is a sine wave

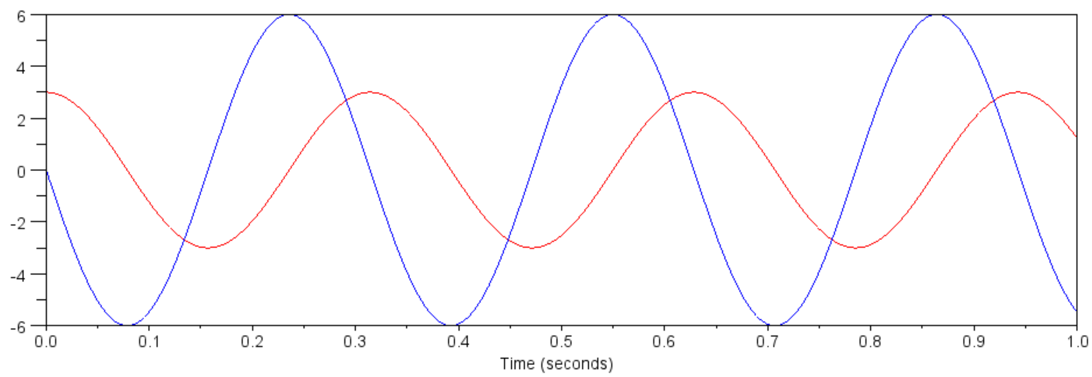
$$i(t) = 3\cos(20t)$$

the voltage awill be

$$v(t) = L\frac{di}{dt}$$

$$v(t) = -6\sin(20t)$$

Note that the voltage leads the current by 90 degrees (the peak votlage is 90 degrees before the peak current).



Current (red) and Voltage (blue) for a 100mH Inductor. Note that voltage leads current by 90 degrees for inductors.

When using phasor analysis, the basic assumption is that all signals are in the form of

$$i(t) = e^{j\omega t}$$

If you have an inuctor, then the voltage is

$$v(t) = L \cdot \frac{di}{dt}$$

$$v(t) = j\omega L \cdot e^{j\omega t}$$

$$v(t) = j\omega L \cdot i(t)$$

Using the analogy of a resistor where

$$v(t) = R \cdot i(t)$$

you get the phasor impedance of an inductor:

$$\boxed{Z_L = j\omega L}$$

Note that the 'j' tells you that voltage will lead current by 90 degrees for an inductor.

Capacitors:

For a capacitor, the energy stored is

$$E = \frac{1}{2} C V^2 \qquad \text{Joules}$$

Differentiating to get Watts gives

$$P = \frac{dE}{dt} = CV \cdot \frac{dV}{dt} \qquad \text{Watts}$$

This is also

$$P = VI = CV \cdot \frac{dV}{dt}$$

which gives the basic equation for a capacitor:

$$\boxed{I = C\,\frac{dV}{dt}}$$

or

$$v(t) = \frac{1}{C}\int i(t)\cdot dt$$

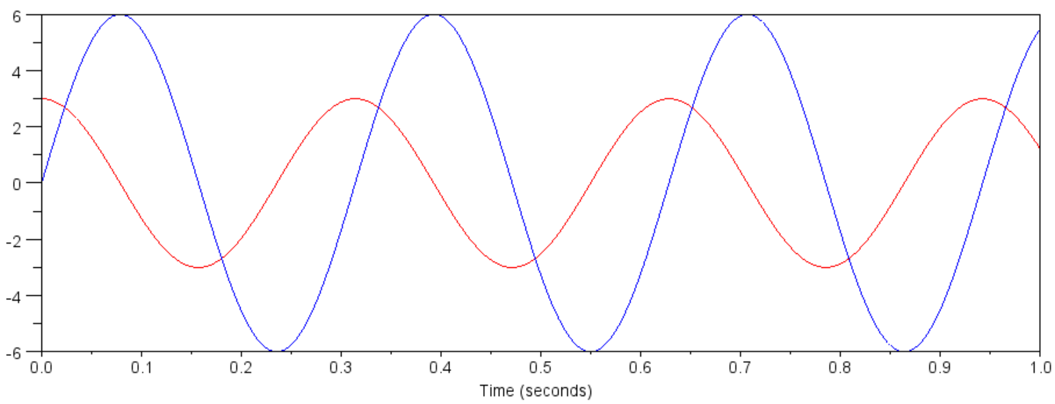
Note that this applies to any current through a capacitor: the voltage will be the integral of the current, scaled by 1/C. For example, if the current through a 25mF capacitor is a sine wave

$$i(t) = 3\cos(20t)$$

the voltage will be

$$v(t) = \frac{1}{0.025}\int 3\cos(20t)\cdot dt$$

$$v(t) = 6\sin(20t)$$



Current (red) and Voltage (blue) through a 25mF capacitor. Note that votlage lags current by 90 degrees for capacitors.

In phasors, we assume that

$$v(t) = e^{j\omega t}$$

The current is then

$$i(t) = C\cdot\frac{dv}{dt}$$

$$i(t) = j\omega C\cdot e^{j\omega t}$$

$$i(t) = j\omega C\cdot v(t)$$

or

$$v(t) = \left(\frac{1}{j\omega C}\right) i(t)$$

The phasor impedance of a capacitor is

$$Z_c = \left(\frac{1}{j\omega C}\right) = \left(\frac{-j}{\omega C}\right)$$

Note that the '-j' means that the voltage will lag the current by 90 derees for a capacitor.

The net results is, when using phasors, you can analyze circuits with inductors and capacitors just like you analyze resistor circuits, only you need to use complex numbers