Series and Parallel with Phasors

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

You can also simplify circuits with inductors and capacitors using phasor notation. All you need is a fixed frequency (so that the phasor impedance is fixed). The only catch is that you'll be using complex numbers.

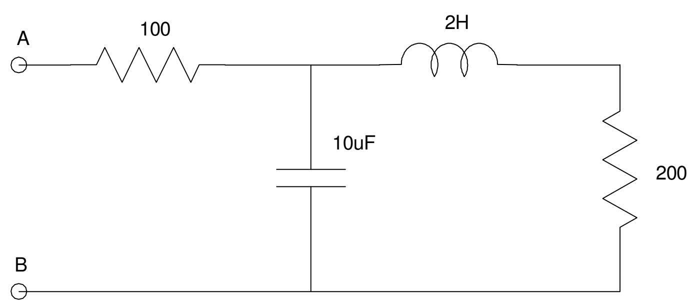
Impedances in series add as

$$Z_{total} = Z_1 + Z_2 + Z_3$$

Impedances in parallel add as

$$\frac{1}{Z_{total}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

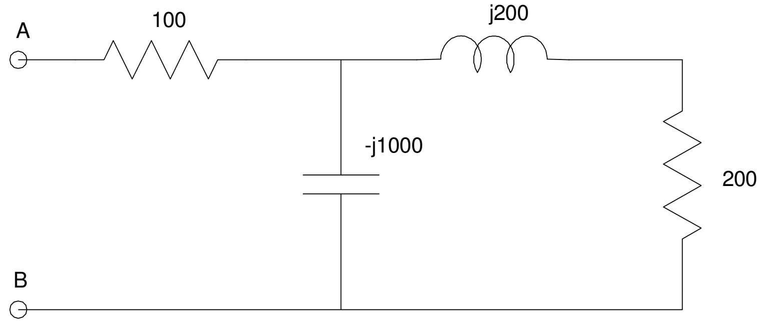
Example: Fine the impedance between terminals A and B assuming the frequency is 100 rad/sec.



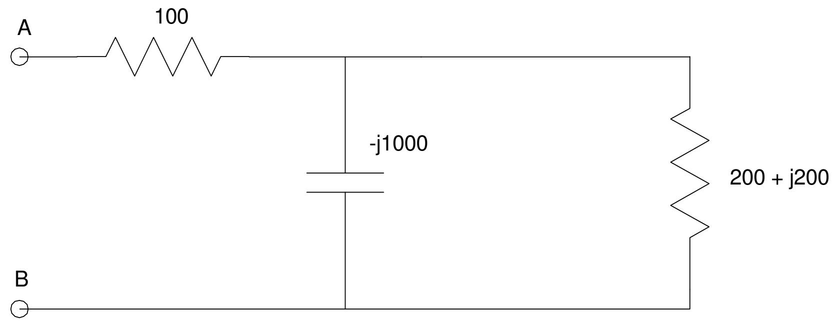
Step 1: Convert to phasor impedance. Note that $\omega = 100$ rad/sec

$$L \rightarrow j\omega L = j200$$

$$C \rightarrow \frac{1}{j\omega C} = -j1000$$

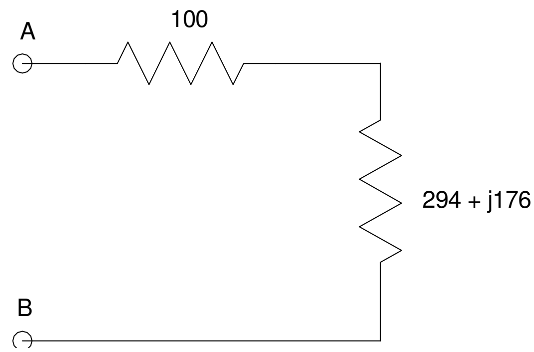


Resistors in series add. Add 200 to j200 together:



Resistors in parallel add as the sum of the inverses, inverted

$$\left(\frac{1}{-j1000}+\frac{1}{200+j200}\right)^{-1}=294+j176$$



Resistors in series add. The total impedance is thus

$$Z_{AB}=394+j176$$

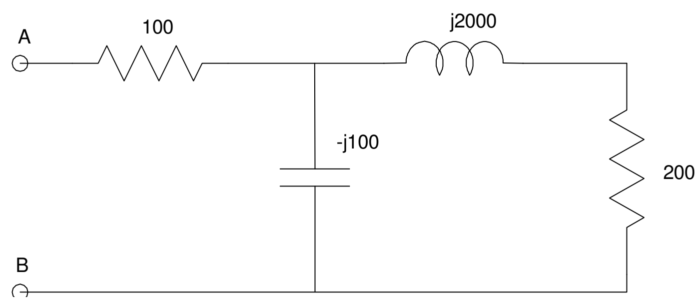
Note that you get a different answer if you change the frequency.

Example 2: Repeat but find the impedance at 1000 rad/sec.

Step 1: Convert to phasor notation

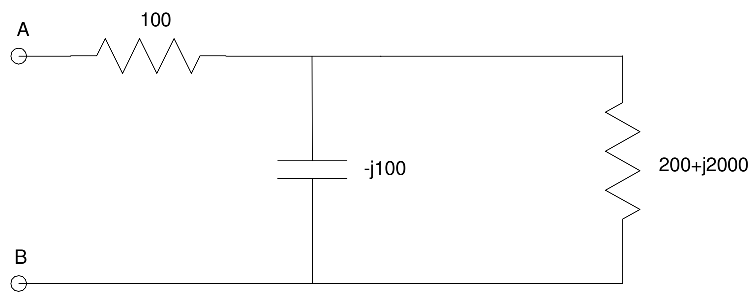
$$L \rightarrow j\omega L = j2000$$

$$C \rightarrow \frac{1}{j\omega C} = -j100$$



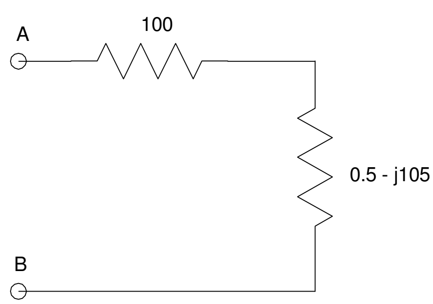
Add the resistor and inductor in series

$$Z = 200 + j2000$$



Add the capacitor in parallel with the 200+j200 impedance

$$Z = \left(\frac{1}{-j100} + \frac{1}{200+j200}\right)^{-1} = 0.5 - j105.2$$



The net impedance is

$$Z_{AB} = 100.5 - j105 \quad \text{Ohms}$$