# Superposition (take 3)

Previously, we looked at how to analyze an AC to DC converter and a Buck converter. To solve these circuits, we changed the problem so that the input contained

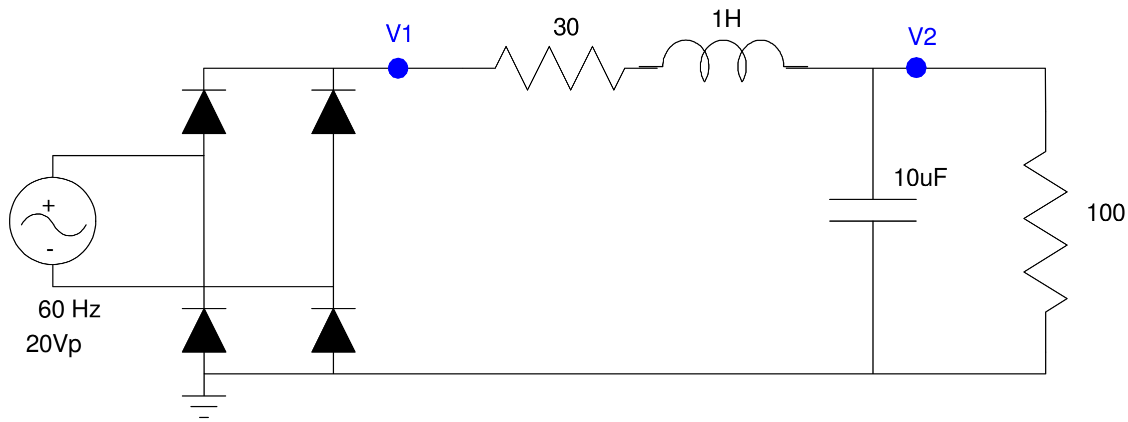
- A DC signal, and
- An AC signal

The resulting output that we calculated was close but slightly off from the actual output voltage.

A more accurate way to analyze these circuits is to express the input in terms of its Fourier Series. Then, superposition can be used to determine the output at each frequency.

## Example 1: AC to DC Converter

The following circuit is an AC to DC converter that we'll cover in ECE 320 Electronics I. Determine the voltage at V2:




AC to DC Converter covered in ECE 320 Electronics I

From PartSim, the signal at V1 is

$$v_1(t) = |20\sin(377t)| - 1.4$$

Previously, we approximated this as

- A DC term which matched the DC term of v1, and
- An AC term which was the same frequency as v1(t) (120Hz) and same peak-to-peak voltage

$$v_1(t) \approx 11.33 + 10\cos(754t)$$

A more accurate approximation would be the Fourier Series approximation.

**Step 1: Find the Fourier Series Approximation for x(t):**

```
T = pi;
t = [0:0.0001:1]' * T;
Wo = 2*pi/T;

x = 20*sin(t) - 1.4;

a0 = mean(x)

a1 = 2*mean(x .* cos(Wo*t))
a2 = 2*mean(x .* cos(2*Wo*t))
a3 = 2*mean(x .* cos(3*Wo*t))
a4 = 2*mean(x .* cos(4*Wo*t))
a5 = 2*mean(x .* cos(5*Wo*t))

b1 = 2*mean(x .* sin(Wo*t))
b2 = 2*mean(x .* sin(2*Wo*t))
b3 = 2*mean(x .* sin(3*Wo*t))
b4 = 2*mean(x .* sin(4*Wo*t))
b5 = 2*mean(x .* sin(5*Wo*t))
```

resulting in

| n | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| w (rad/sec) | 0 | 754 | 1,508 | 2,262 | 3,016 | 3,770 |
| $a_n$ | 11.331 | -8.488 | -1.698 | -0.728 | -0.404 | -0.257 |
| $b_n$ | 0 | 0 | 0 | 0 | 0 | 0 |

meaning

$$v_1(t) = 11.331 - 8.488\cos(754t) - 1.698\cos(1508t) - 0.728\cos(2262t)$$
$$-0.404\cos(3016t) - 0.257\cos(3770t)$$

Not that this is a little different from what we previously approximated the waveform at V1 as:

$$v_1(t) \approx 11.33 + 10\cos(754t)$$

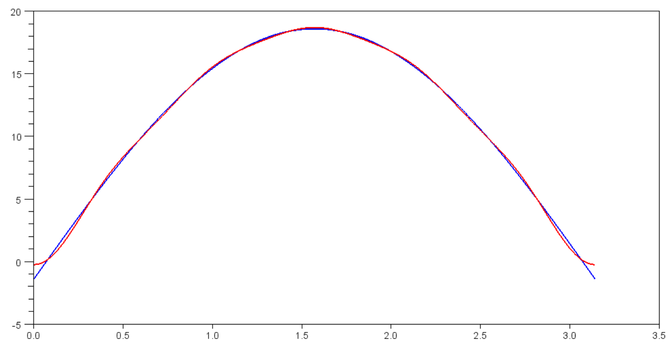
The Fourier series approximation is

- More accurate, but
- Much harder to determine.

Proceeding with the Fourier Series approximation, the actual waveform at V1 and it's approximation taken out to the 5th harmonic are:

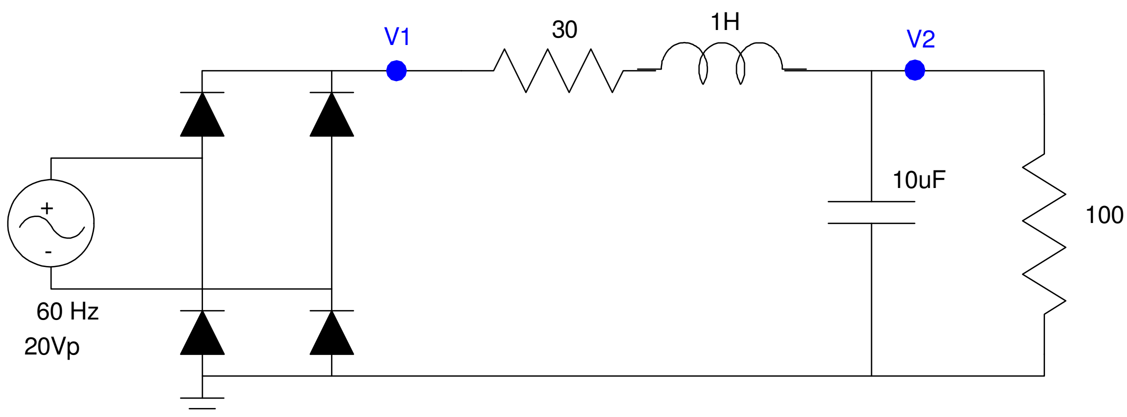
```
y = a0 + a1*cos(Wo*t) + a2*cos(2*Wo*t) + a3*cos(3*Wo*t) + a4*cos(4*Wo*t) + a5*cos(5*Wo*t);

y = y + b1*sin(Wo*t) + b2*sin(2*Wo*t) + b3*sin(3*Wo*t) + b4*sin(4*Wo*t) + b5*sin(5*Wo*t);
```

```
plot(t,x,'b',t,y,'r');
```




Signal at V1 (blue) and Fourier Series Approximation (red)

**Step 2: Use Superposition at Each Frequency**




$$v_1(t) = 11.331 - 8.488\cos(754t) - 1.698\cos(1508t) - 0.728\cos(2262t)$$
$$-0.404\cos(3016t) - 0.257\cos(3770t)$$

DC:

$$V1 = 11.331$$

$$V_2 = \left(\frac{100}{100+30}\right) \cdot 11.331$$

$$V_2 = 8.716$$

754 rad/sec (120Hz: fundamental)

$$v_1(t) = -8.488\cos(754t)$$

$$V_1 = -8.488 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j132.6\Omega$$

$$L \rightarrow j\omega L = j754\Omega$$

$$-j132.6 || 100 = 63.76 - j48.07$$

$$V_2 = \left(\frac{(63.76-j48.07)}{(63.76-j48.07)+(30+j754)}\right) \cdot (8.488 + j0)$$

$$V_2 = -0.468 - j0.829$$

$$v_2(t) = -0.468\cos(754t) + 0.829\sin(754t)$$

At 1508 rad/sec (240Hz: 2nd harmonic)

$$v_1 = -1.698\cos(1508t)$$

$$V_1 = -1.698 + j0$$

$$\omega = 1508$$

$$C \rightarrow \frac{1}{j\omega C} = -j66.31\Omega$$

$$L \rightarrow j\omega L = j1508\Omega$$

$$100 \; || \; -j66.31 = 30.54 - j46.05$$

$$V_2 = \left(\frac{30.54-j46.05}{(30.54-j46.05)+(30+j1508)}\right) \cdot (-1.698 + j0)$$

$$V_2 = 0.052 + j0.038$$

$$v_2(t) = 0.052\cos(1508t) - 0.038\sin(1508t)$$

At 2262 rad/sec: (360Hz: 3rd harmonic)

$$v_1(t) = -0.728 \cos(2262t)$$

$$\omega = 2262$$

$$V_1 = -0.728 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j44.21\Omega$$

$$L \rightarrow j\omega L = j2262\Omega$$

$$100 \,||\, -j44.21 = 16.35 - j36.98$$

$$V_2 = \left(\frac{(16.35-j36.98)}{(16.35-j36.98)+(30+j2262)}\right) \cdot (-0.728 + j0)$$

$$V_2 = 0.012 + j0.006$$

$$v_2(t) = 0.012 \cos(2262t) - 0.006 \sin(2262t)$$

At 3016 rad/sec (480Hz: 4th harmonic)

$$v_1(t) = -0.404 \cos(3016t)$$

$$V_1 = -0.404 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j33.15\Omega$$

$$L \rightarrow j\omega L = j3016\Omega$$

$$100 \,||\, -j33.15\Omega = 9.90 - j29.87$$

$$V_2 = \left(\frac{(9.90-j29.87)}{(9.90-j29.87)+(30+j3016)}\right) \cdot (-0.404 + j0)$$

$$V_2 = 0.004 + j0.001$$

$$v_2(t) = 0.004 \cos(3016t) - 0.001 \sin(3016t)$$

At 3770 rad/sec (5th harmonic)

$$v_1(t) = -0.257\cos(3770t)$$

$$V_1 = -0.257 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j26.53\Omega$$

$$L \rightarrow j\omega L = j3770\Omega$$

$$100 \,||\, -j26.53\Omega = 6.58 - j24.79$$

$$V_2 = \left(\frac{(6.58-j24.79)}{(6.58-j24.79)+(30+j3770)}\right)\cdot(-0.257 + j0)$$

$$V_2 = 0.0017 + j0.0005$$

$$v_2(t) = 0.0017\cos(3370t) - 0.0005\sin(3370t)$$

The total answer will be the sum of all the terms

$$v_2(t) = 8.716$$

$$-0.468\cos(754t) + 0.829\sin(754t)$$

$$+0.052\cos(1508t) - 0.038\sin(1508t)$$

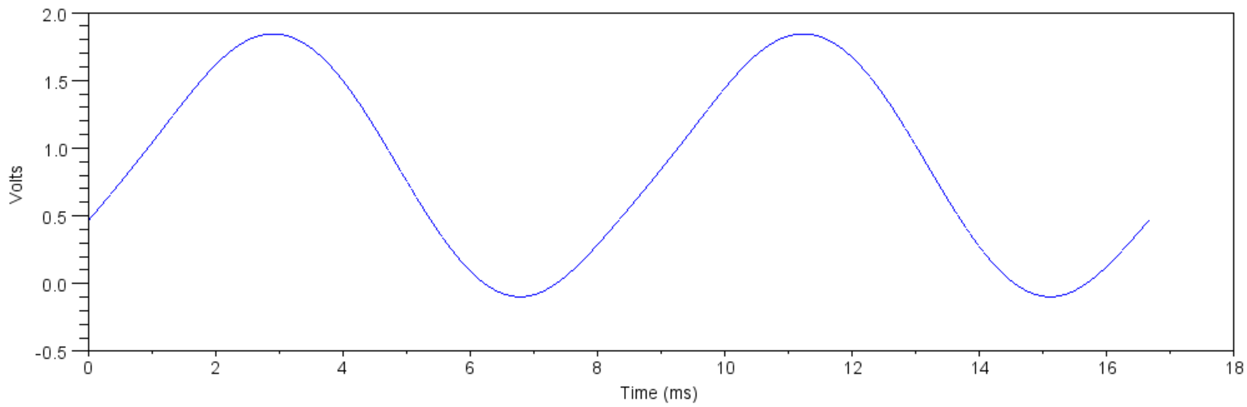
$$+0.012\cos(2262t) - 0.006\sin(2262t)$$

$$+0.004\cos(3016t) - 0.001\sin(3016t)$$

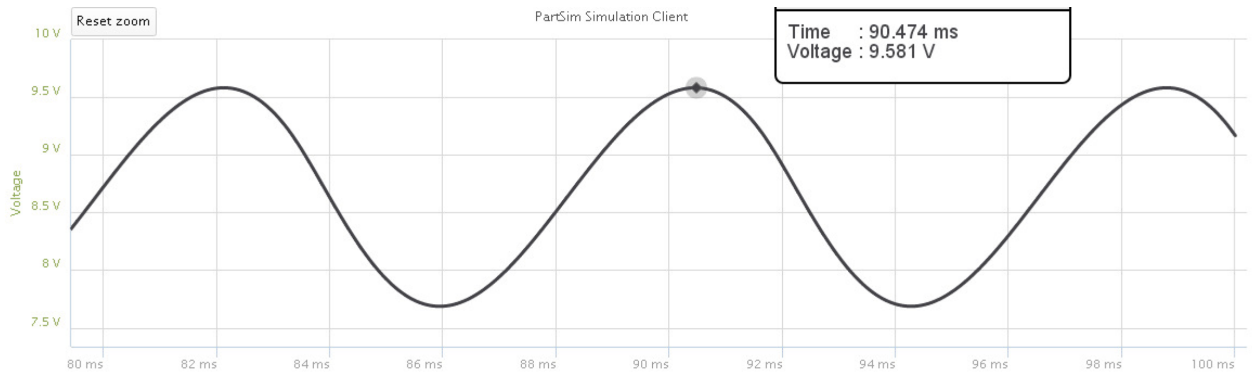
$$+0.0017\cos(3370t) - 0.0005\sin(3370t)$$

$$+\ldots$$

Comparing the calculated and PartSim signal at V2:




Calculate Signal At V2: Fourier Series Taken Out to the 5th Harmonic




Signal at V2 as computed by PartSim

| | Calculated V2 (lecture #30) | PartSim V2 | Calculated V2 (Fourier Series) |
|---|---|---|---|
| DC Value | 8.70 V | 8.644 V | 8.716 V |
| AC Value | 2.242 Vpp | 1.895 Vpp | 1.903Vpp |

Note the following:

- By taking the Fourier Series approximation for the signal at V1, our computed answer (column #4) matches up with the simulation results very well.

- The reason our previous answer from lecture #30 (column #2) was off is we overestimated the 1st harmonic. Since V1 is 20Vpp, we assumed that the 1st harmonic was also 20Vpp. Actually, it's 16.98Vpp (2 x 8.488V)

- In theory, you have to take the Fourier Series out to infinity. Actually, you can get a pretty good approximation just using the DC term and the 1st harmonic. The signal at V2 was

$$\begin{aligned} v_2(t) = 8.716 & \\ & -0.468\cos(754t) + 0.829\sin(754t) \\ & +0.052\cos(1508t) - 0.038\sin(1508t) \\ & +0.012\cos(2262t) - 0.006\sin(2262t) \\ & +0.004\cos(3016t) - 0.001\sin(3016t) \\ & +0.0017\cos(3370t) - 0.0005\sin(3370t) \\ & +\ldots \end{aligned}$$

It isn't that bad of an approximation to only include the first two terms of V2:

$$v_2(t) \approx 8.716 - 0.468\cos(754t) + 0.829\sin(754t)$$

meaning it isn't that bad of an approximation to take V1:

$$\begin{aligned} v_1(t) = \; & 11.331 - 8.488\cos(754t) - 1.698\cos(1508t) - 0.728\cos(2262t) \\ & -0.404\cos(3016t) - 0.257\cos(3770t) \end{aligned}$$

and only consider the first two terms:

$$v_1(t) \approx 11.331 - 8.488\cos(754t)$$

This is what we did back in lecture #30, only with a slightly less accurate way of determining the amplitude of the 1st harmonic. A better (and harder) way to do this is to use a Fourier Series approximation taken out to two terms (DC and AC fundamental).