Voltage Nodes

EE 206 Circuits I

Jake Glower - Lecture #5

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Voltage Nodes

To solve a circuit, we primarilly use three tools:

- Voltage Nodes (Kirchoff's current law)
- Current Loops (Kirchoff's voltage law)
- Thevenin Equivalents (stay tuned for these...)

The goal of the first two is to obtain N equations so solve for N unknowns.

Voltage Nodes

- The voltages at each node are such that the current from each node sums to zero

Idea:

- I know how to solve N equations for N unknowns
  - Math 129 Linear Algebra
- If I can redude the probem to this form, I can solve the problem

Note:

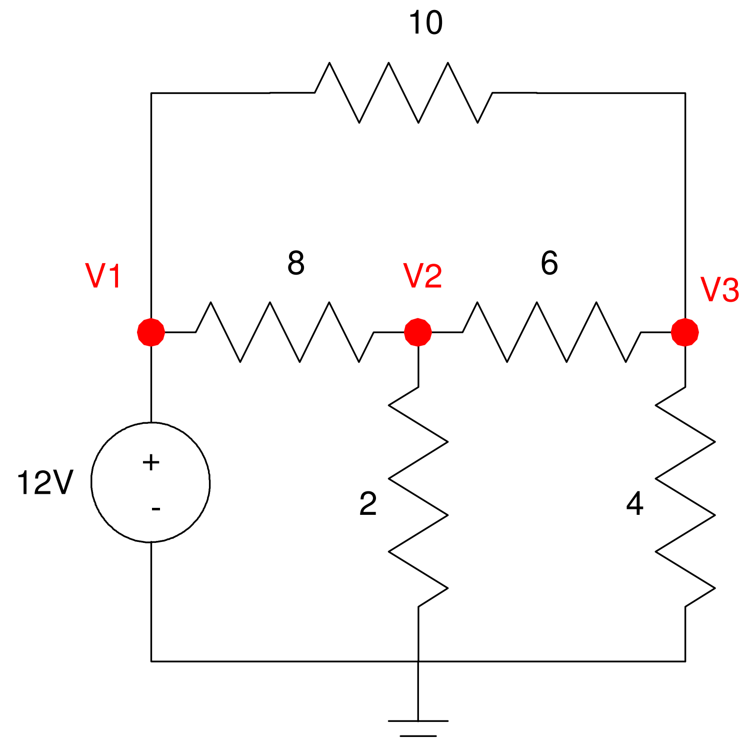
- This is a common theme throughout ECE
- What's important is getting the equations right
  - Matlab can solve 50 equations for 50 unknowns just as easily as 2 equations for 2 unkowns
  - Focus on getting the equations right.

Example: Find the voltages V1, V2, and V3

- Step 1. Define your ground reference. This is normally the bottom of the diagram.
- Step 2. Label the remaining voltage nodes

The number of unknowns tell you how many equations you need to write

- V1, V2, V3
- We need 3 equations for 3 unknowns



Step 3. Write 3 equations

- Start with the easy ones (the voltage sources)

$$V_1 - 0 = 12$$

- At V2, sum the current from the node to zero

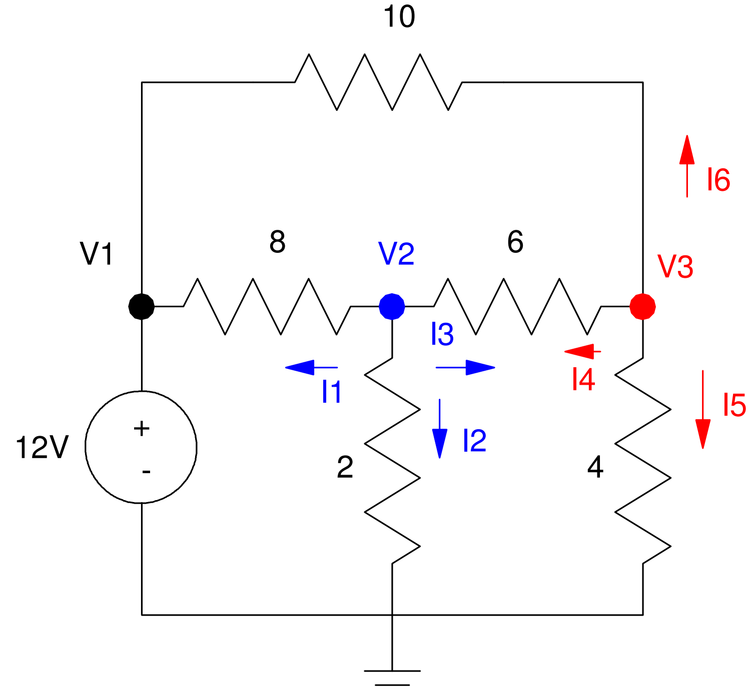
$$I_1 + I_2 + I_3 = 0$$

$$\left(\frac{V_2 - V_1}{8}\right) + \left(\frac{V_2 - 0}{2}\right) + \left(\frac{V_2 - V_3}{6}\right) = 0$$

- At V3, sum the current from the node to zero

$$I_4 + I_5 + I_5 = 0$$

$$\left(\frac{V_3 - V_2}{6}\right) + \left(\frac{V_3 - 0}{4}\right) + \left(\frac{V_3 - V_1}{10}\right) = 0$$



Step 4: Solve

- Group terms

$$V_1 = 12$$

$$-\left(\frac{1}{8}\right)V_1 + \left(\frac{1}{8}+\frac{1}{2}+\frac{1}{6}\right)V_2 - \left(\frac{1}{6}\right)V_3 = 0$$

$$-\left(\frac{1}{10}\right)V_1 - \left(\frac{1}{6}\right)V_2 + \left(\frac{1}{6}+\frac{1}{4}+\frac{1}{10}\right)V_3 = 0$$

- Place in matrix form

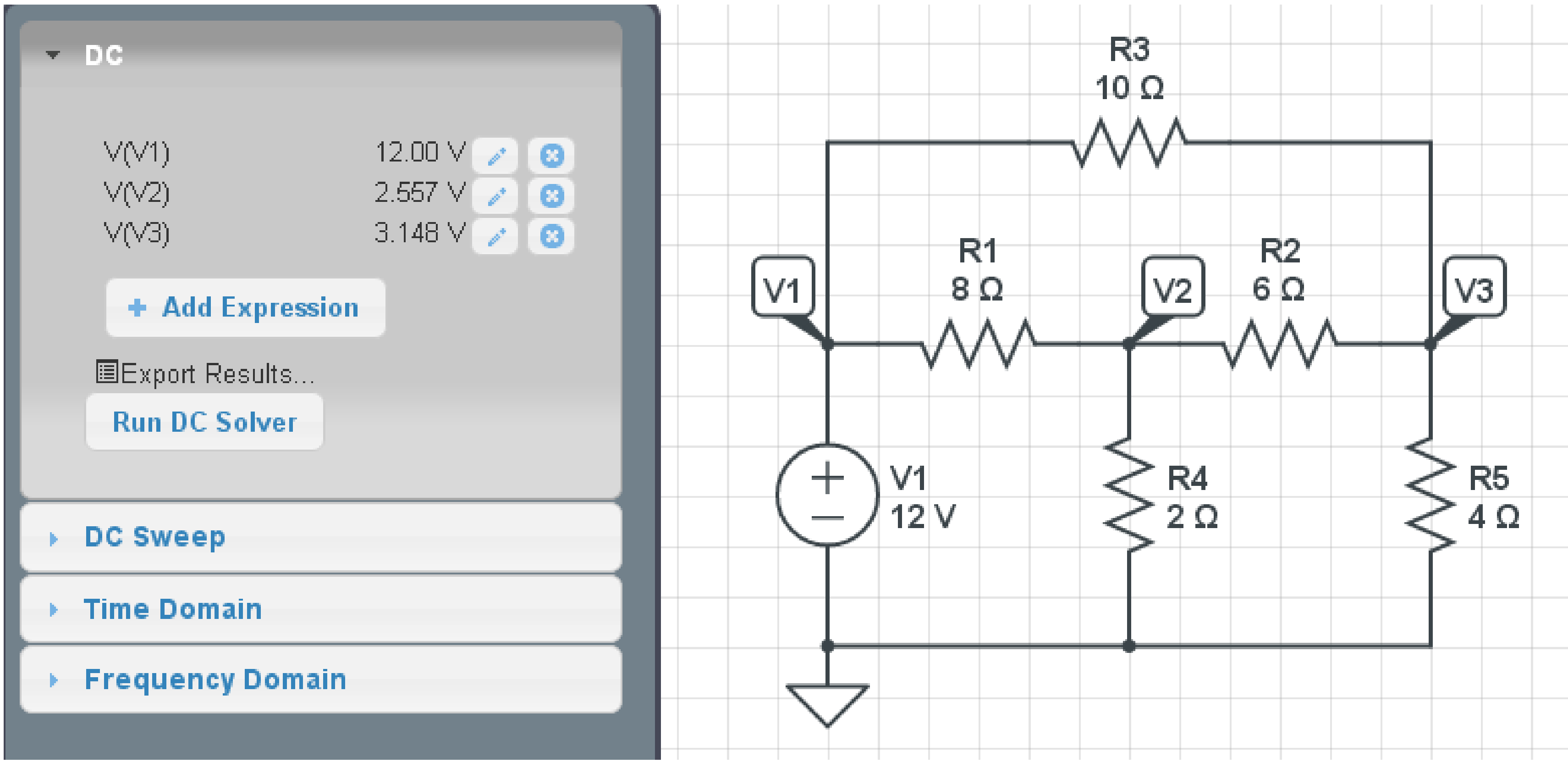
$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{8} & \left(\frac{1}{8}+\frac{1}{2}+\frac{1}{6}\right) & \left(\frac{-1}{6}\right) \\ \left(\frac{-1}{10}\right) & \left(\frac{-1}{6}\right) & \left(\frac{1}{6}+\frac{1}{4}+\frac{1}{10}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
A = [1,0,0;-1/8,1/8+1/2+1/6,-1/6;-1/10,-1/6,1/6+1/4+1/10]

    1.          0.             0.
  - 0.125       0.7916667    - 0.1666667
  - 0.1       - 0.1666667      0.5166667

B=[12;0;0]

    12.
    0.
    0.

V = inv(A)*B

V1     12.
V2     2.557377
V3     3.147541
```

CircuitLab (Validation):

You can check your answer using CircuitLab



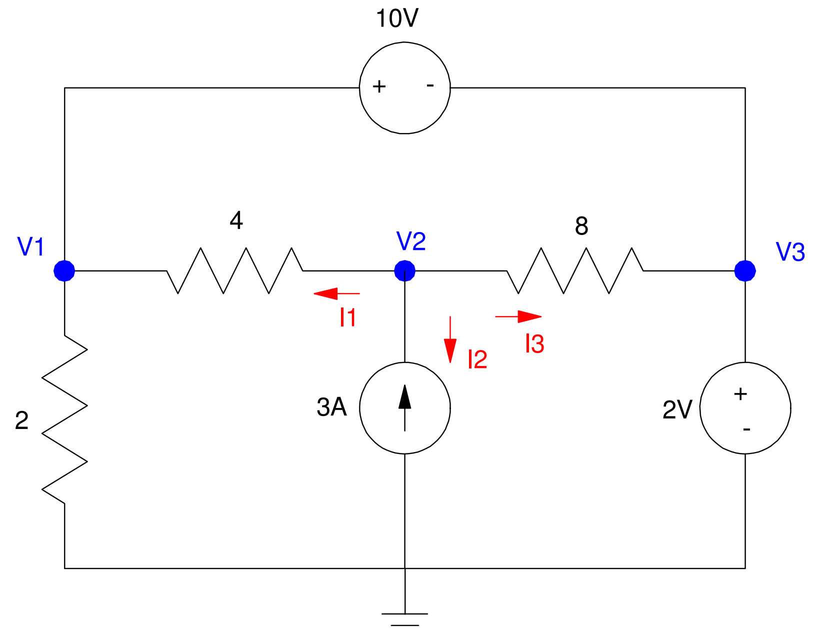
Example 2: Find V1, V2, and V3

3 unknowns, so write 3 equations for 3 unknowns

Start with the easy ones

$$V_3 - 0 = 2$$

$$V_1 - V_3 = 10$$



Now we need one more equation. Write the voltage node equation at V2

$$I_1 + I_2 + I_3 = 0$$

$$\left(\frac{V_2 - V_1}{4}\right) + (-3) + \left(\frac{V_2 - V_3}{8}\right) = 0$$

Group terms

$$V_3 = 2$$

$$V_1 - V_3 = 10$$

$$-\left(\frac{1}{4}\right)V_1 + \left(\frac{1}{4} + \frac{1}{8}\right)V_2 - \left(\frac{1}{8}\right)V_3 = 3$$

Place in matrix form:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ -0.25 & 0.375 & -0.125 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 3 \end{bmatrix}$$

Solving in Matlab:

```
A = [0,0,1 ; 1,0,-1 ; -0.25,0.375,-0.125]

         0         0    1.0000
    1.0000         0   -1.0000
   -0.2500    0.3750   -0.1250

B = [2;10;3]

     2
    10
     3

V = inv(A)*B
```

V1 12.0000
V2 16.6667
V3 2.0000

Check with CircuitLab