Sinusoidal Sources

EE 206 Circuits I

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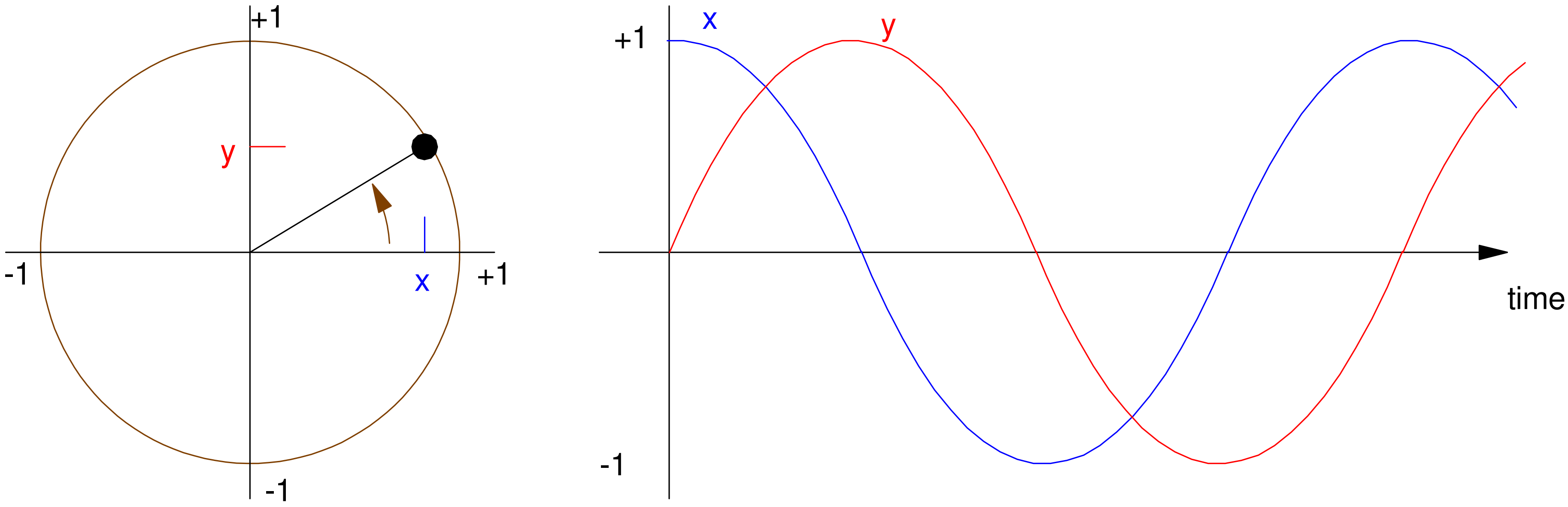
Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

What Is a Sine Wave?

Circles and Sine Waves: If you take a wheel and spin it counter clockwise,

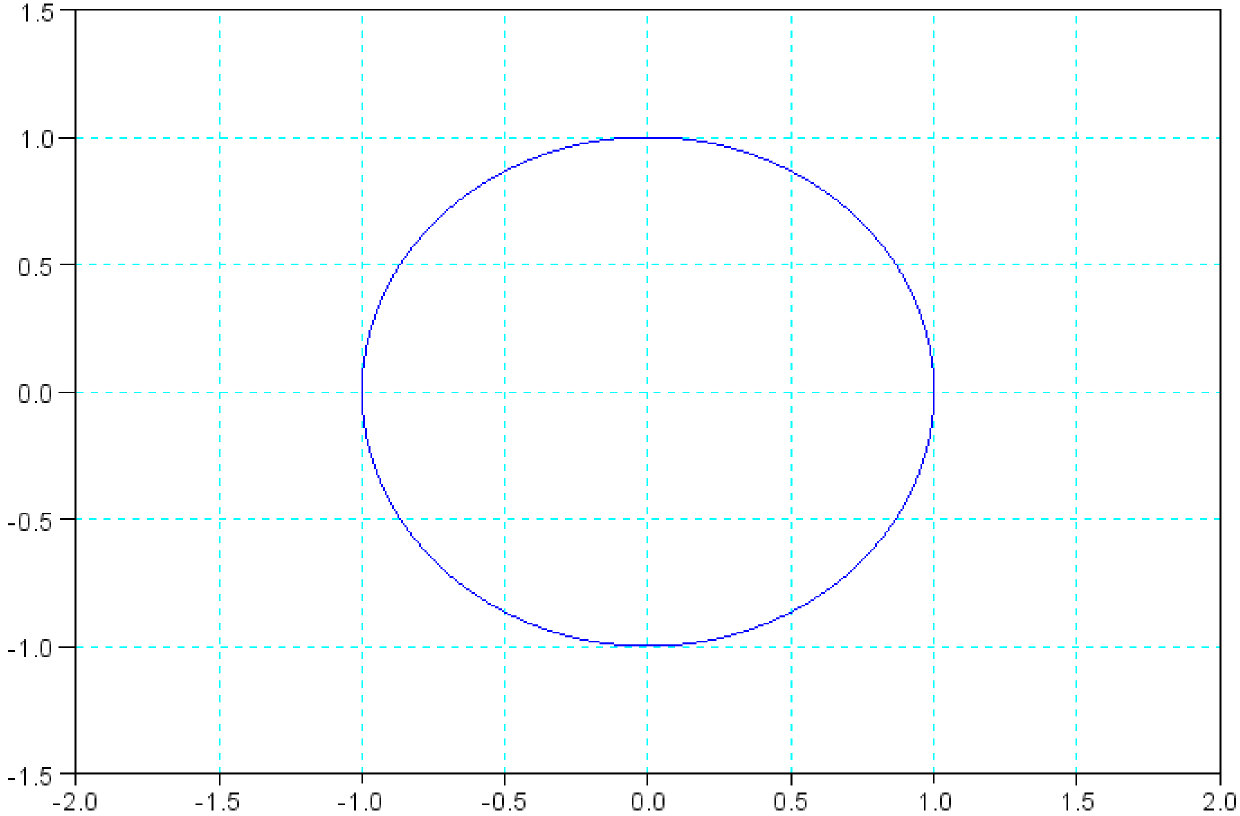
- The x-position of a point on the wheel maps out a cosine function, and
- The y-poisition of a point maps out a sine function



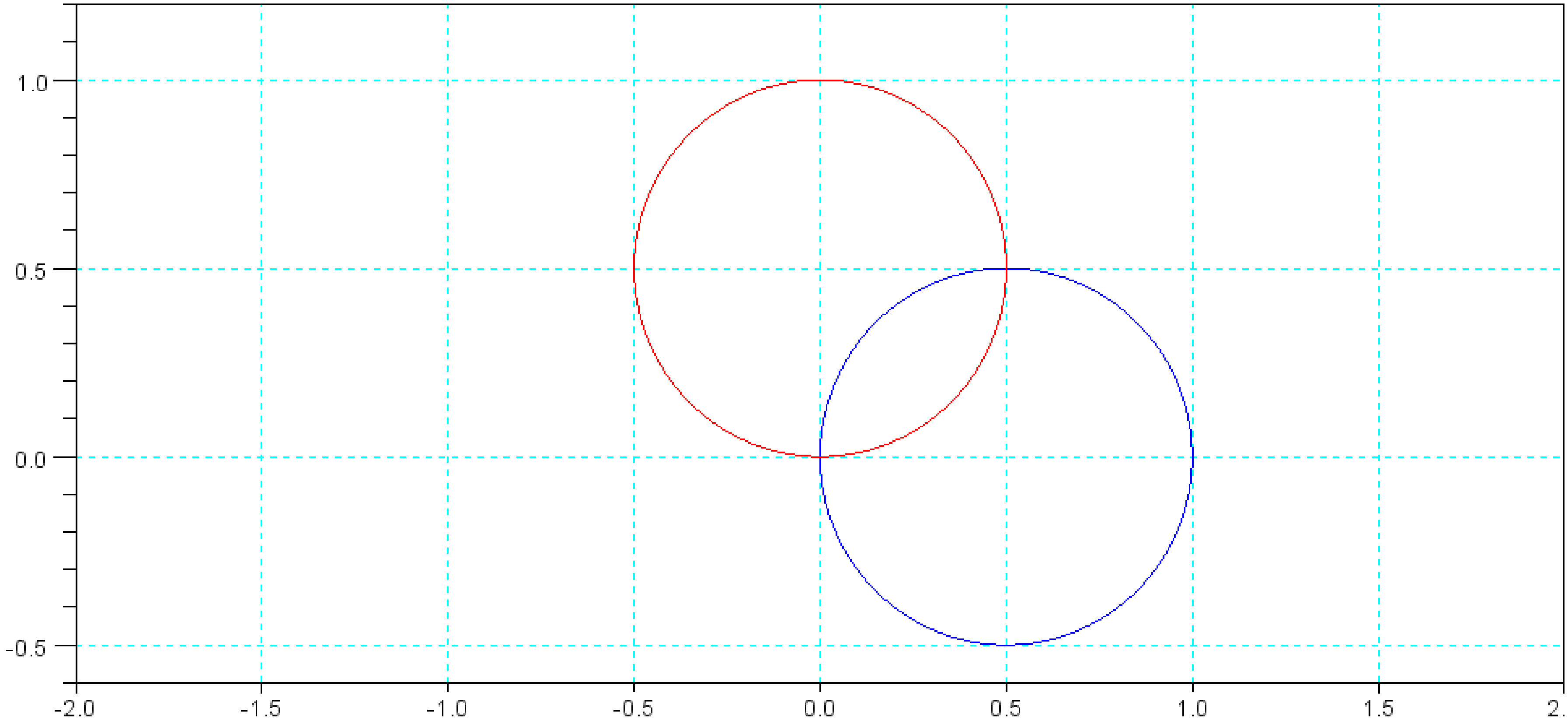
plotting x vs. y where x = cos(q) y = sin(q) produces the unit circle

If you plot cos(q) vs. sin(q), you get the unit circle

```
q = [0:0.001:1]' * 2 * pi;
x = cos(q);
y = sin(q);
plot(cos(q), sin(q))
```



$r = \cos\theta, \qquad r = \sin\theta$ also gives you circles



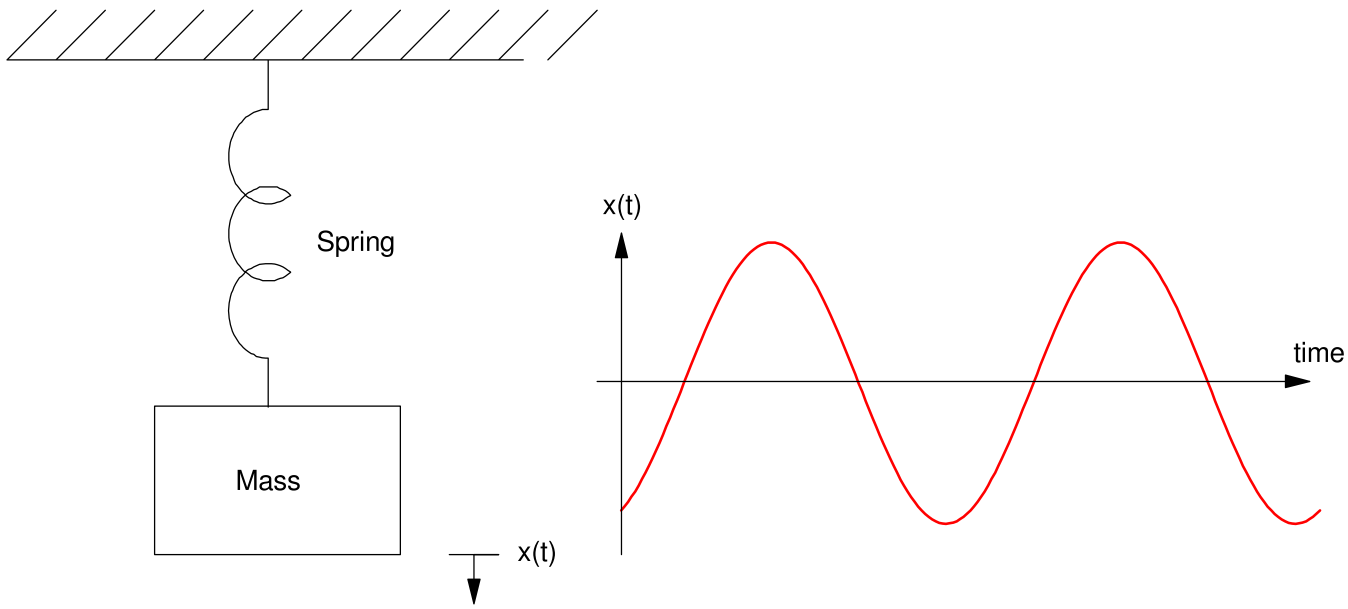
Natural Response to Differential Equations:

The natural response to a linear 2nd-order differential equation

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

is a sine wave:

$$y(t) = a\cos(\omega t) + b\sin(\omega t)$$



Why Use Sine Waves?

- Sine waves is sine waves are eigenfunctions
- You can decompose any periodic signal into a sum of sine waves

Eigenfunctions: The solution to a differential equation (i.e. a circuit) is the same as the forcing function.

Example:

$$y'' + 3y' + 2y = 2x$$

$$x(t) = \cos(2t)$$

Solution: (stay tuned...)

$$y(t) = -0.1\cos(2t) + 0.3\sin(2t)$$

Only sine waves (and exponentials) have this property.

- Square waves don't
- Triangle waves don't
- Sawtooth waves don't

Fourier Transform: (Covered in ECE 311 Circuits II)

If a function is periodic in time T

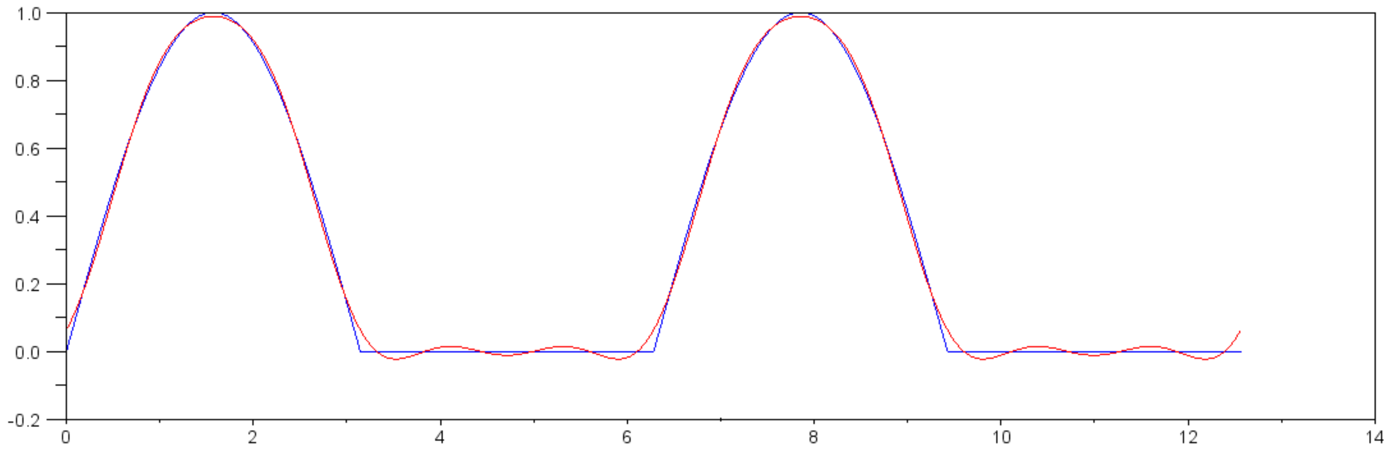
$$x(t) = x(t+T)$$

it can be expressed as a sum of sine waves. Example: a half-rectified sine wave

$$x(t) = \begin{cases} \sin(t) & \sin(t) > 0 \\ 0 & otherwise \end{cases}$$

can be expressed as the series

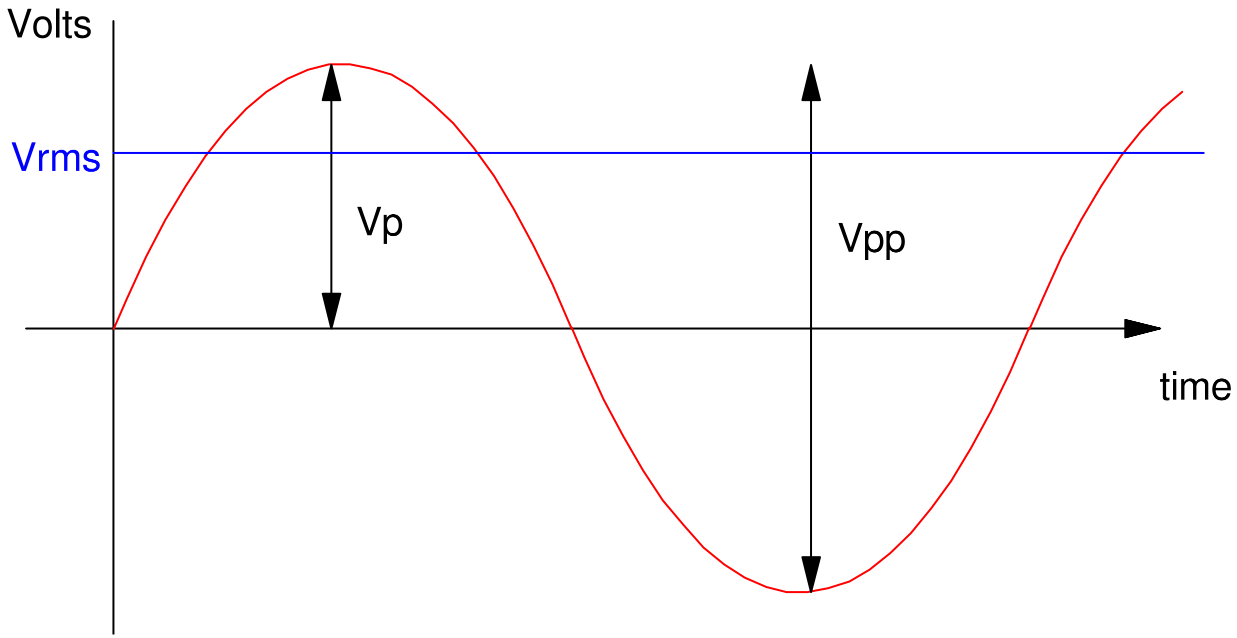
$$x(t) = \frac{1}{\pi} + \frac{1}{2}\sin(t) + \sum_{\text{n even}} \left(\frac{2}{\pi(n^2-1)}\right)\cos(nt)$$



Sine Wave Definitions

To alleviate some of the confusion, some definitions are needed.

- **Vp: Peak Voltage:** The amplitude of the sine wave from it's average voltage (usually zero).
- **Vpp: Peak to Peak Voltage:** The distance between the maximum and minimum voltage. Vpp = 2 Vp
- **Vrms: rms Voltage:** The DC votlage which would produce the same amount of heat through a 1 Ohm resistor.



- **Period (seconds):** Time time between zero crossings (or peak votlages)
- **Frequency (Hz):** One over the period
- **Frequency (rad/sec):** The natural frequency: 1Hz = 2π rad/sec.