
Voltage Nodes with Phasors

EE 206 Circuits I

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Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

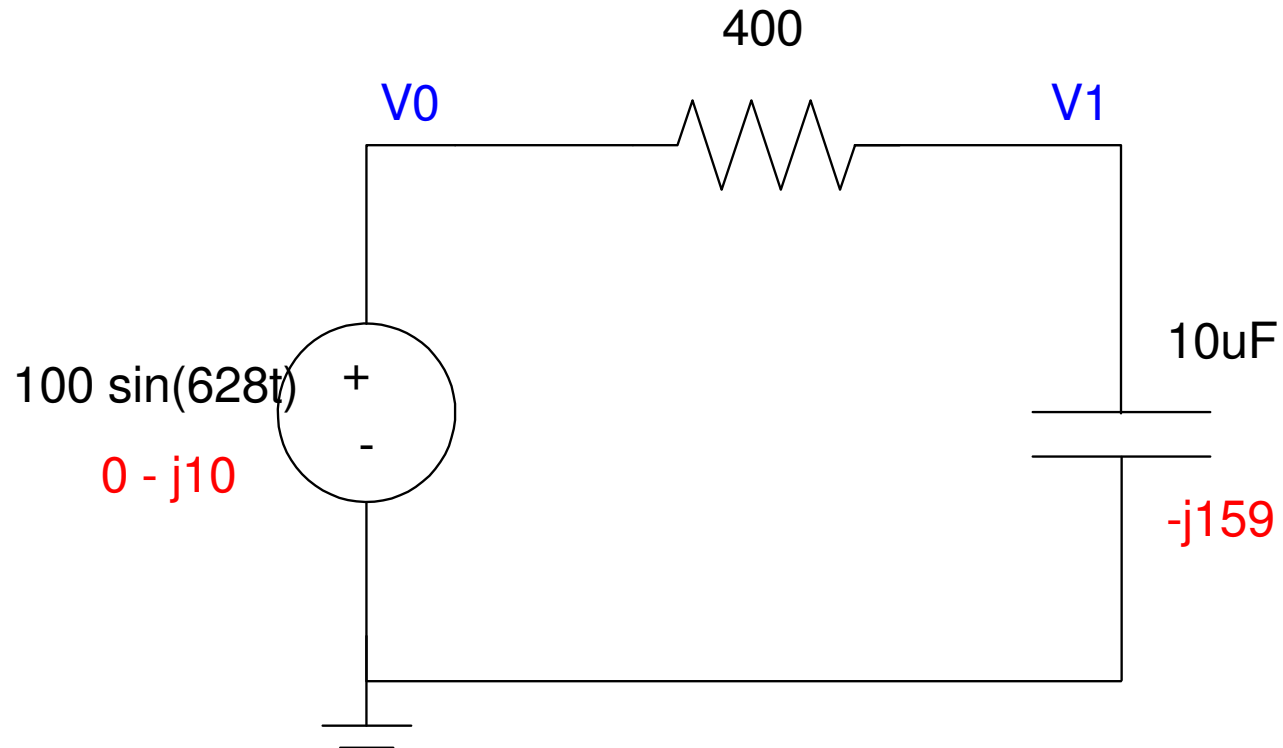
Voltage Nodes:

- The sum of the current from a node must sum to zero
- For AC, both the real part and complex part must sum to zero

	VI relationship	Phasor Notation
Voltage	$v(t) = a \cos(\omega t) + b \sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L \frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C \frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Example 1: RC Circuit

Determine the node voltages for the following circuit:



Step 1: Replace the elements with their phasor values (shown in red)

Step 2: Write the voltage node equations

- Same as DC

$$V_0 = 0 - j100$$

$$\left(\frac{V_1 - V_0}{400}\right) + \left(\frac{V_1}{-j159}\right) = 0$$

Solve:

$$V_1 = \left(\frac{-j159}{-j159 + 400}\right)(0 - j100)$$

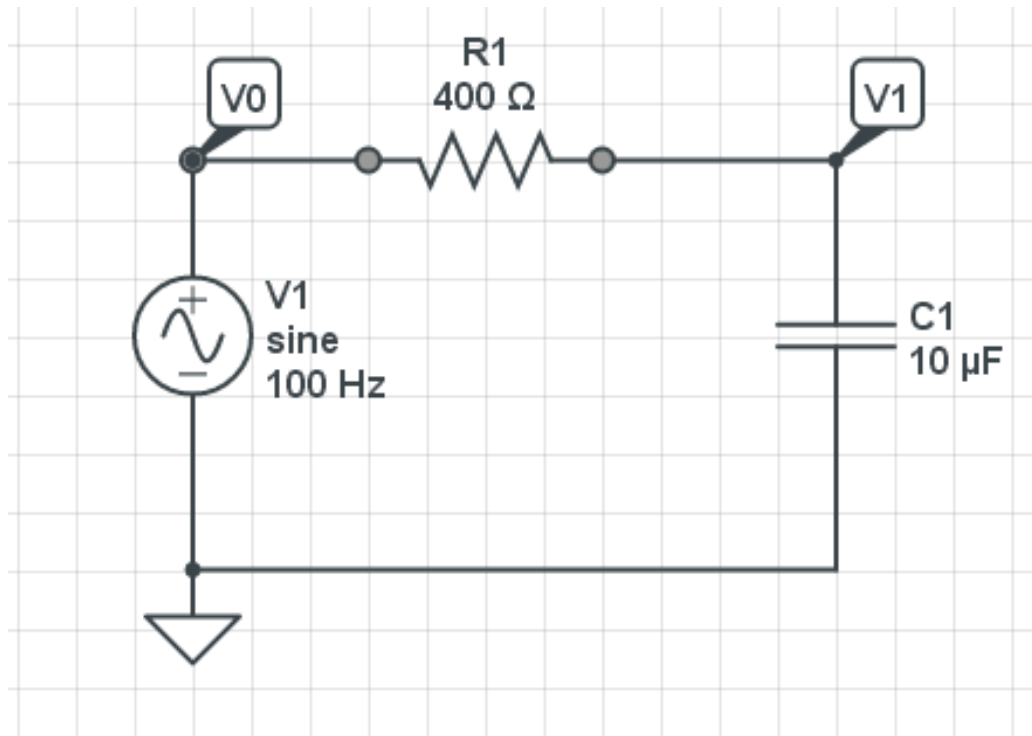
$$V_1 = 34.326 - j13.645$$

$$v_1(t) = 34.326 \cos(628t) + 13.645 \sin(628t)$$

Verify with CircuitLab

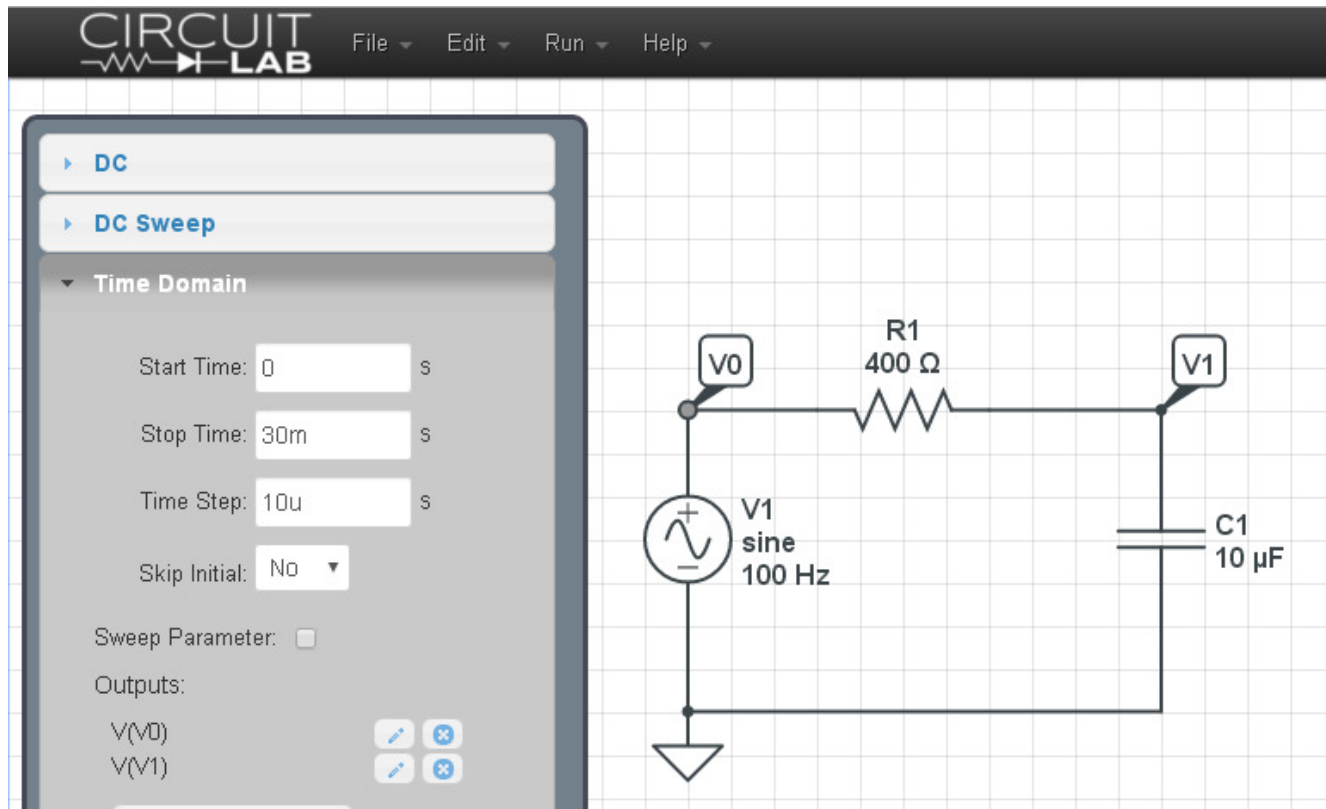
Make the input a sine wave with

- no DC offset
- 100V amplitude
- 100Hz (628 rad/sec)

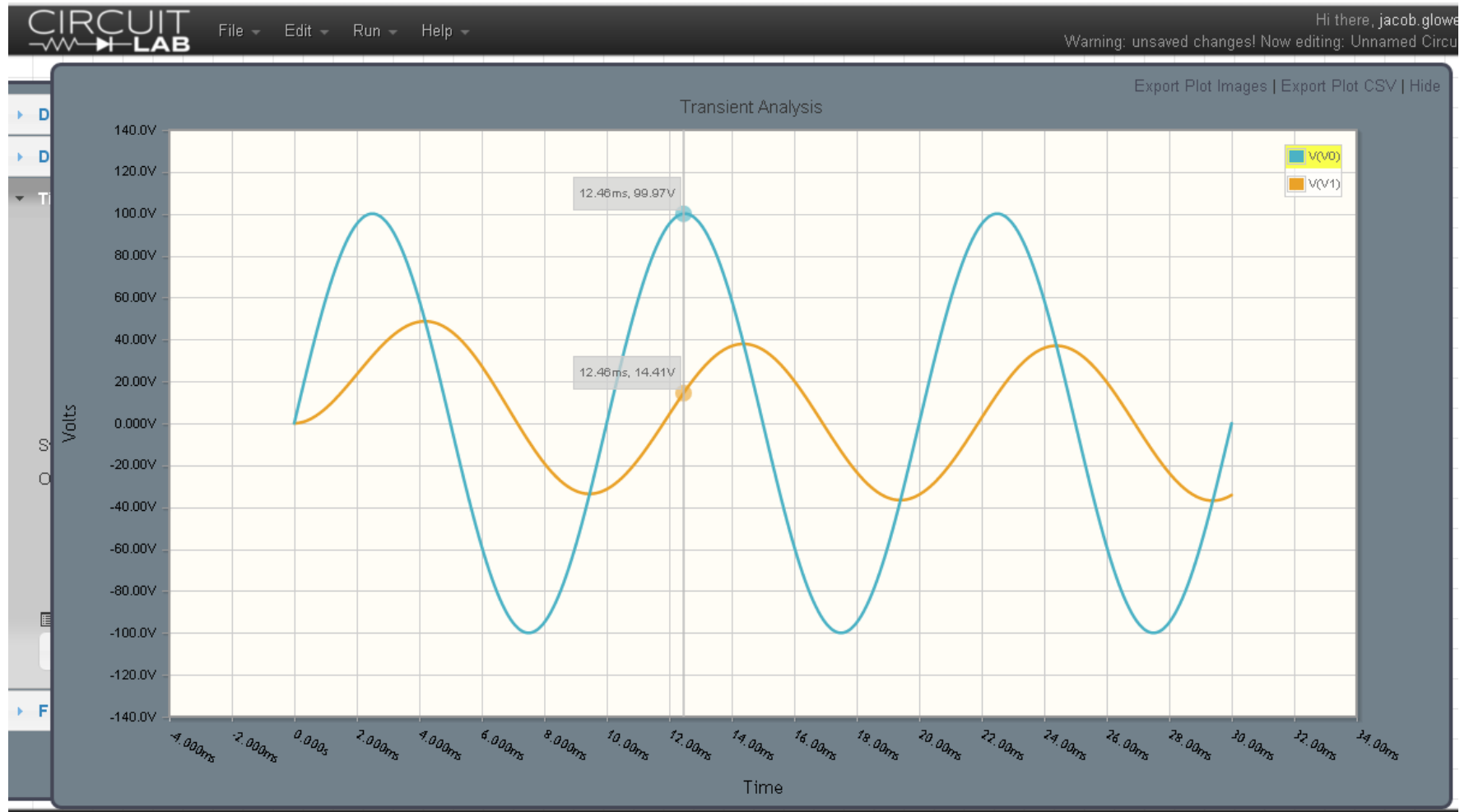


Run a transient response for

- 30ms (3 cycles)
- 10us step size



This results in a simulated input and output waveform:



The peak tells you the amplitude (V_p)

- $V_p = 37.87V$
- Delayed by 4.39ms from the zero crossing of V_0

The zero crossing is used as a reference since $V_0 = 100 \sin(628t)$ (V_0 is zero at $t=0$). The phase shift is thus

$$\theta_1 = -\left(\frac{4.39ms \text{ delay}}{10ms \text{ period}}\right) 360^\circ$$

$$\theta_1 = -158^\circ$$

hence

$$V_1 = 37.87 \angle -158^\circ$$

This matches our calculations

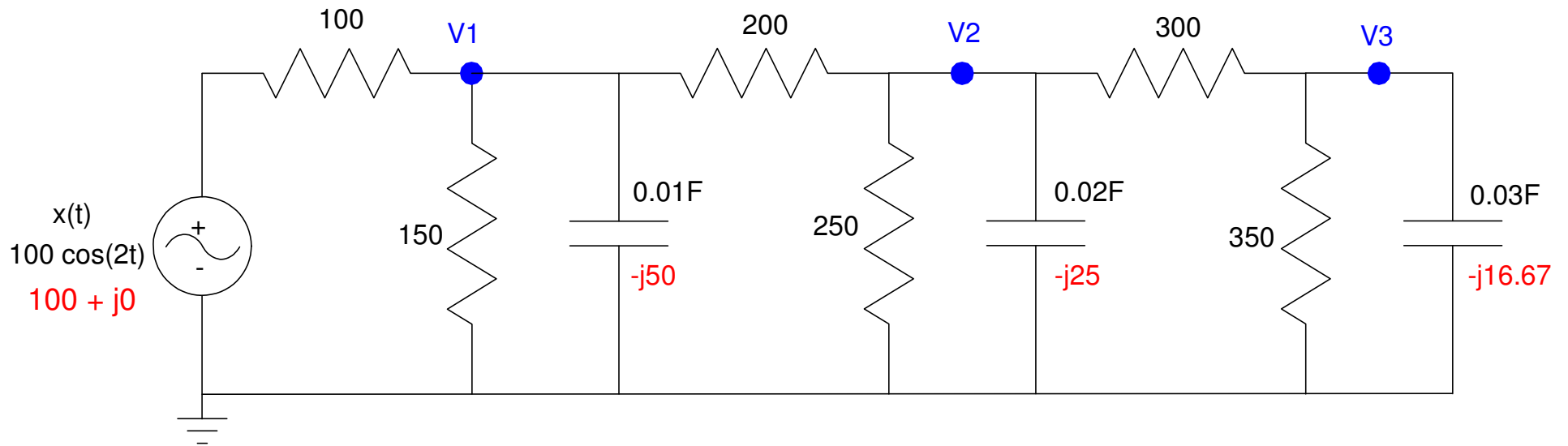
$$V_1 = 34.326 - j13.645$$

$$V_1 = 36.939 \angle -158.3^\circ$$

Example 2: 3-Stage RC Circuit

Find the voltages for the following circuit when the input is

$$x(t) = 100 \cos(2t)$$



Step 1: Express in Phasor form

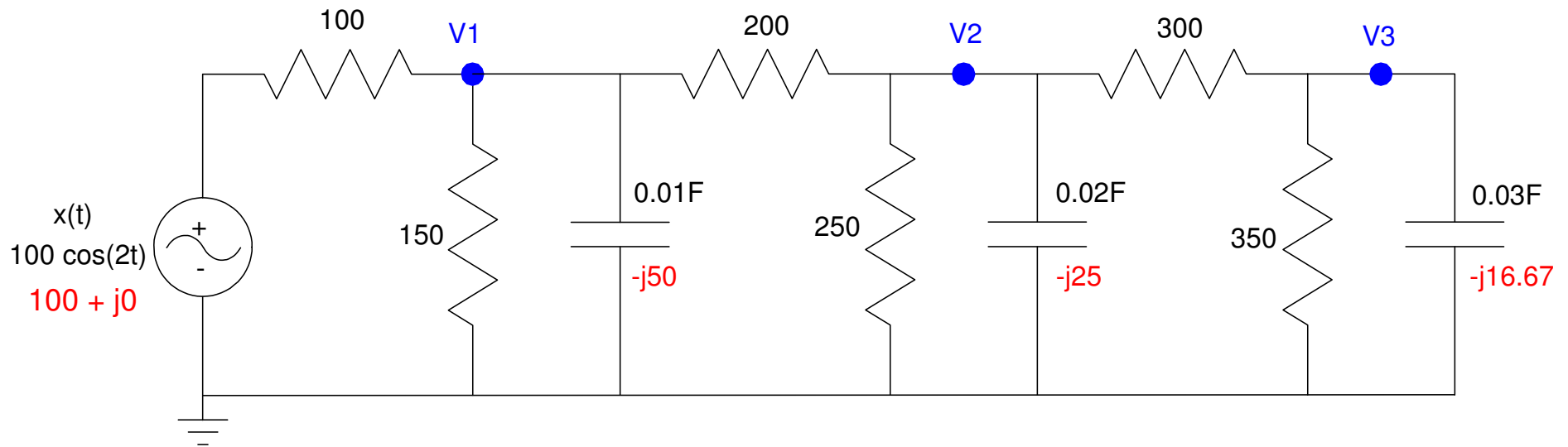
- (shown in red)
-

Step 2: Write N equations for N unknowns

$$\left(\frac{V_1 - X}{100}\right) + \left(\frac{V_1}{150}\right) + \left(\frac{V_1}{-j50}\right) + \left(\frac{V_1 - V_2}{200}\right) = 0$$

$$\left(\frac{V_2 - V_1}{200}\right) + \left(\frac{V_2}{250}\right) + \left(\frac{V_2}{-j25}\right) + \left(\frac{V_2 - V_3}{300}\right) = 0$$

$$\left(\frac{V_3 - V_2}{300}\right) + \left(\frac{V_3}{350}\right) + \left(\frac{V_3}{-j16.67}\right) = 0$$



Step 3: Solve.

First, group terms

$$\left(\frac{1}{100} + \frac{1}{150} + \frac{1}{-j50} + \frac{1}{200}\right) V_1 + \left(\frac{-1}{200}\right) V_2 = \left(\frac{1}{100}\right) X$$

$$\left(\frac{-1}{200}\right) V_1 + \left(\frac{1}{200} + \frac{1}{250} + \frac{1}{-j25} + \frac{1}{300}\right) V_2 + \left(\frac{-1}{300}\right) V_3 = 0$$

$$\left(\frac{-1}{300}\right) V_2 + \left(\frac{1}{300} + \frac{1}{350} + \frac{1}{-j16.67}\right) V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} \left(\frac{1}{100} + \frac{1}{150} + \frac{1}{-j50} + \frac{1}{200}\right) & \left(\frac{-1}{200}\right) & 0 \\ \left(\frac{-1}{200}\right) & \left(\frac{1}{200} + \frac{1}{250} + \frac{1}{-j25} + \frac{1}{300}\right) & \left(\frac{-1}{300}\right) \\ 0 & \left(\frac{-1}{300}\right) & \left(\frac{1}{300} + \frac{1}{350} + \frac{1}{-j16.67}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{100}\right) \\ 0 \\ 0 \end{bmatrix} X$$

Solve in Matlab

$$V = \text{inv}(A) * B * X$$

$$\begin{array}{l} 24.2853 \quad -23.2416i \\ -1.7971 \quad - 3.5726i \\ -0.2066 \quad + 0.0785i \end{array}$$

$$v_1(t) = 24.28 \cos(2t) + 23.24 \sin(2t)$$

$$v_2(t) = -1.79 \cos(2t) + 3.57 \sin(2t)$$

$$v_3(t) = 0.21 \cos(2t) + 0.08 \sin(2t)$$

$$v_1(t) = 33.61 \cos(2t - 43.7^\circ)$$

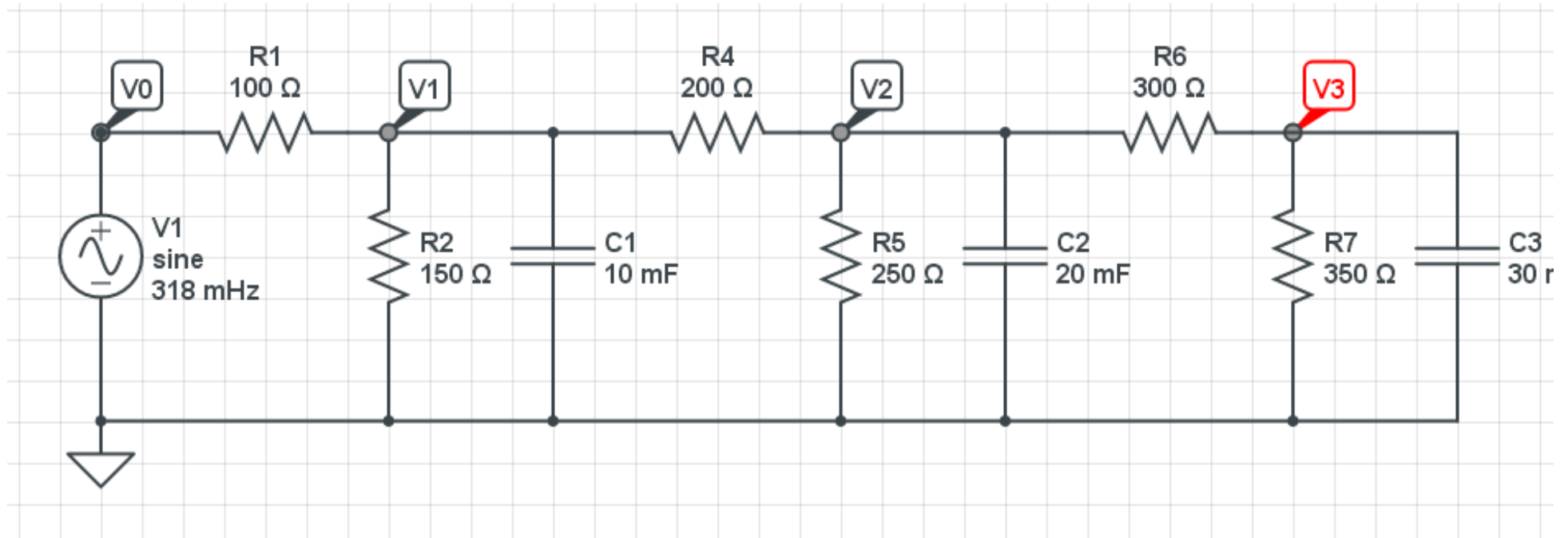
$$v_2(t) = 3.999 \cos(2t - 116.7^\circ)$$

$$v_3(t) = 0.22 \cos(2t + 159.2^\circ)$$

CircuitLab Simulation

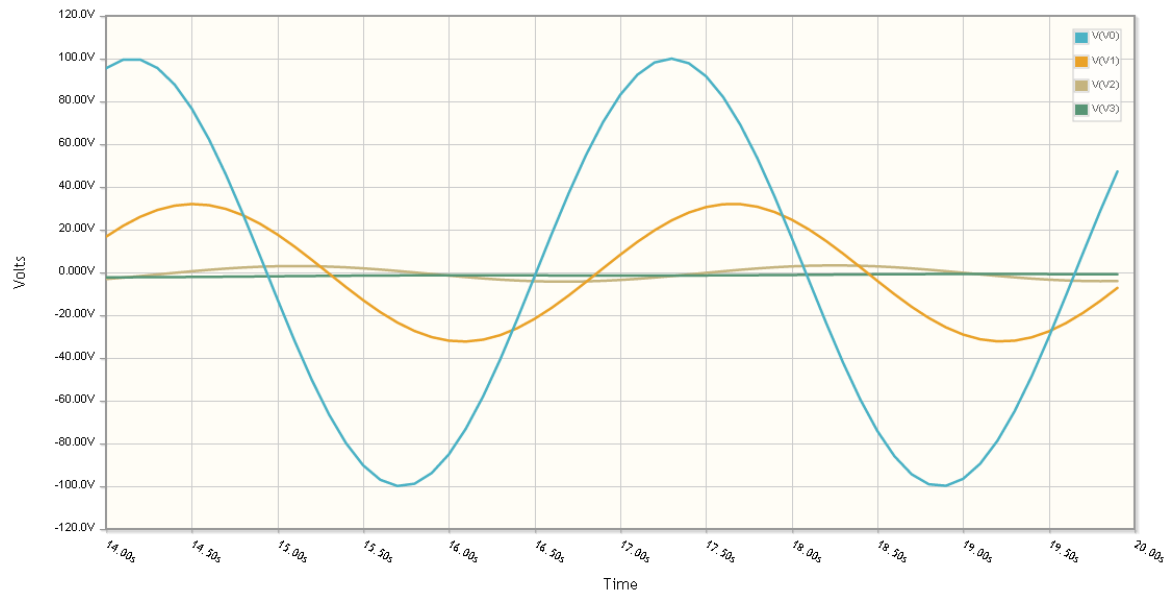
Build the circuit

- Note: $2 \text{ rad/sec} = 0.318 \text{ Hz}$



Run a transient simulation

- max time = 20 seconds (6 cycles)
- time step = 20ms (1000 points on the plot)



This matches our calculations:

$$v_1(t) = 33.61 \cos(2t - 43.7^\circ)$$

$$v_2(t) = 3.999 \cos(2t - 116.7^\circ)$$
