Superposition with Phasors

EE 206 Circuits I

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Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

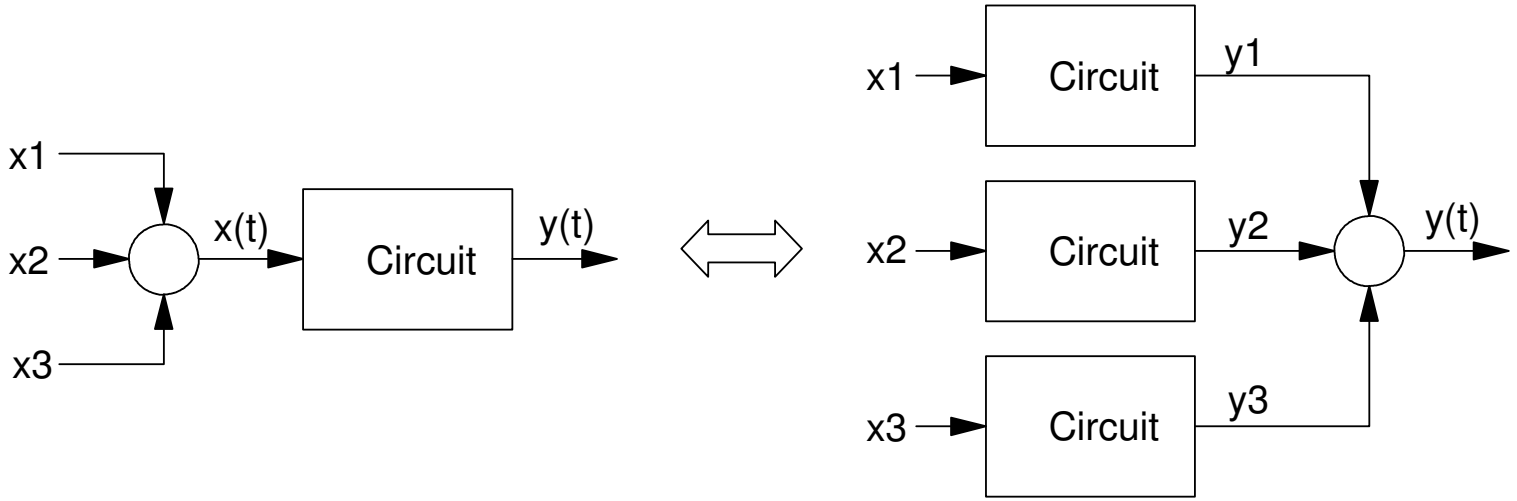
Superposition with Phasors

Op-Amp circutis with RLC components are linear circuits. *Linear* means

$$f(ax+by) = af(x) + bf(y)$$

In English, this means that

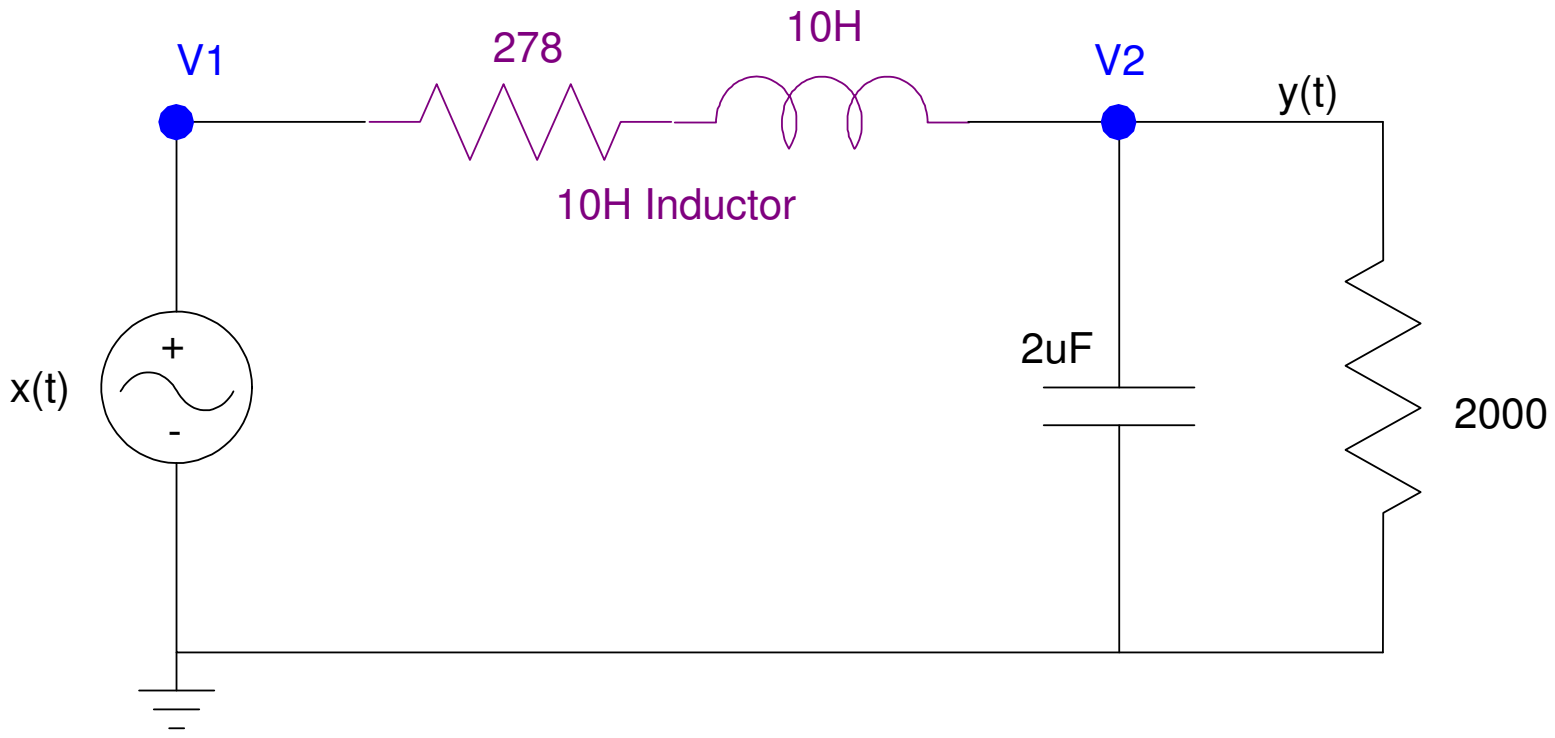
- If the input has N terms,
- You can treat this as N separate problems.



Example: RLC Filter

Determing y(t) for the following circuit assuming

$$x(t) = 10 + 9\sin(100t) + 8\cos(200t)$$



Solution: Use superposition and treat this as three separate problems.

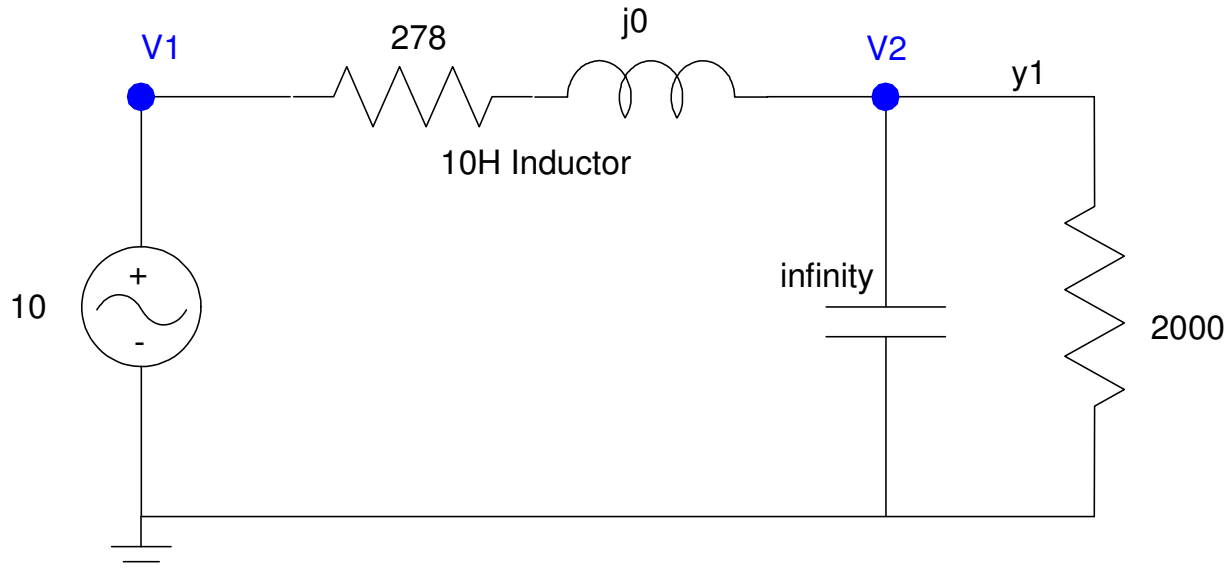
Analyze at w = 0 (DC)

$$x_1(t) = 10$$

Analyze the circuit for this input.

$$\omega = 0 \qquad L \rightarrow j\omega L = 0 \qquad C \rightarrow \frac{1}{j\omega C} = \infty$$

$$Y_1 = \left(\frac{2000}{2000+278}\right) \cdot 10 = 8.7796$$



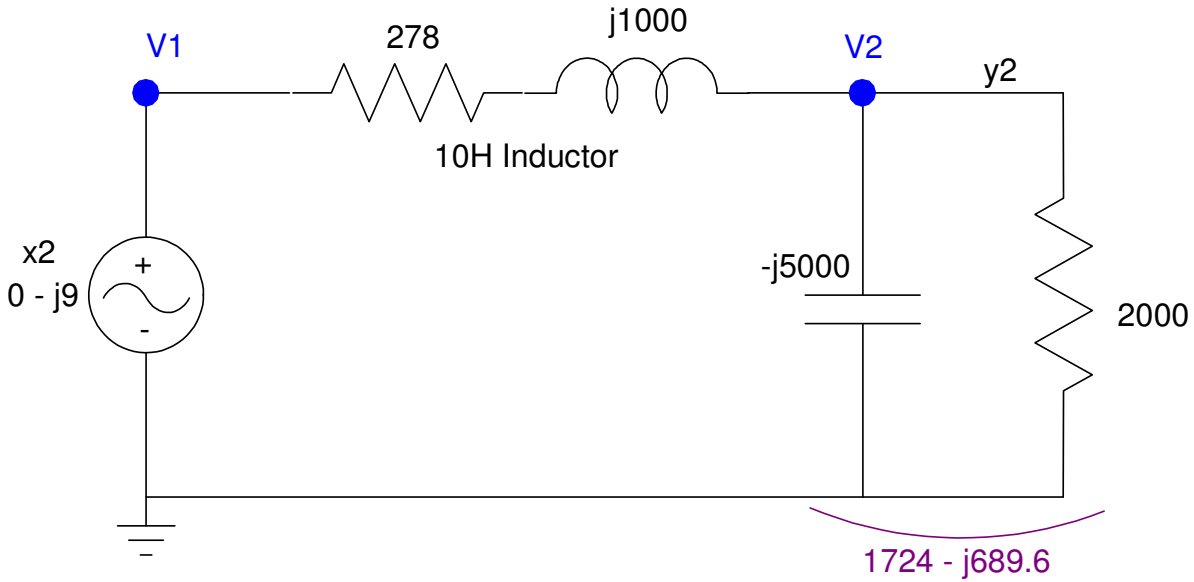
Analyze at w = 100 rad/sec

$$x_2(t) = 9\sin(100t)$$

$$\omega = 100 \qquad X = 0 - j9$$

$$L \rightarrow j\omega L = j1000 \qquad C \rightarrow \frac{1}{j\omega C} = -j5000$$

$$Y_2 = \left(\frac{(1724 - j689.6)}{(1724 - j689.6) + (278 + j1000)}\right) \cdot (0 - j9) = -4.201 - j7.099$$

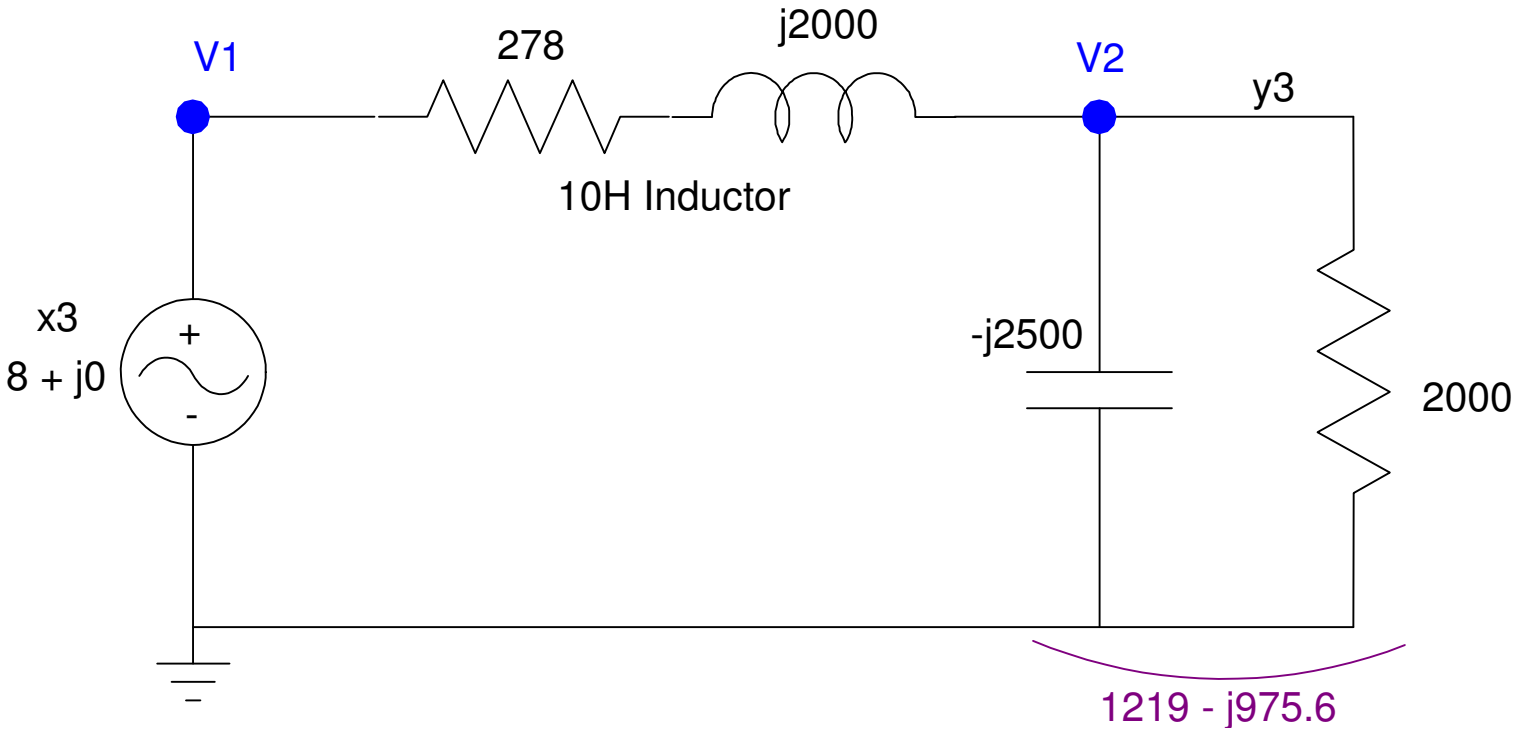


Analyze at w = 200 rad/sec

$$x_3(t) = 8\cos(200t)$$

$$X = 8 + j0 \qquad \omega = 200$$

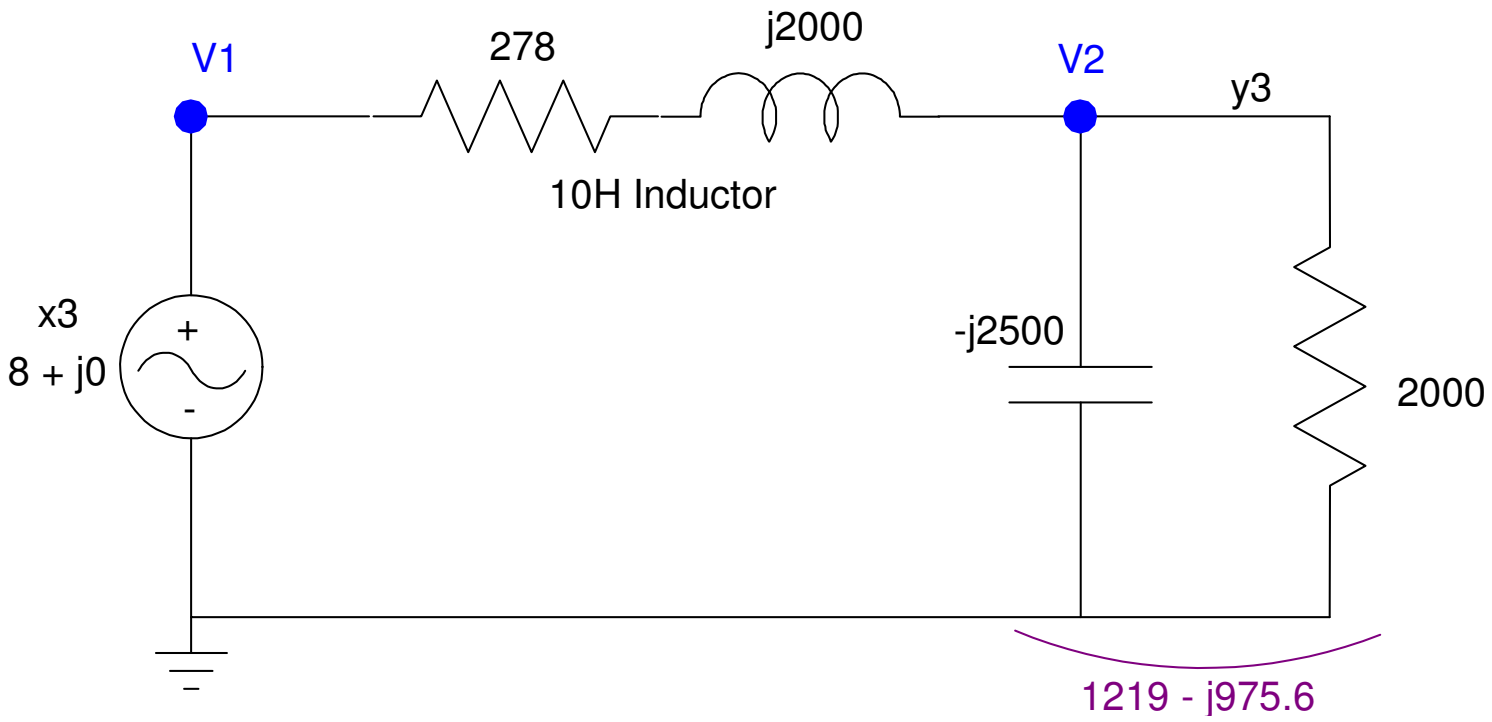
$$L \rightarrow j\omega L = j2000 \qquad C \rightarrow \frac{1}{j\omega C} = -j2500$$



$$\left(\frac{1}{2000}+\frac{1}{-j2500}\right)^{-1}=1219-j975.6$$

$$Y=\left(\frac{1219-j975.6}{(1219-j975.6)+(278-j2000)}\right)\cdot(8+j0)$$

$$Y=2.009-j6.586$$



The total answer is the sum of the three parts

$$y(t) = y_1 + y_2 + y_3$$

$$\omega = 0 \qquad Y_1 = 8.7796$$

$$\omega = 100 \qquad Y_2 = -4.201 - j7.099$$

$$\omega = 200 \qquad Y_3 = 2.009 - j6.596$$

$$\begin{aligned} y(t) = \; & 8.7796 \\ & -4.201\cos(100t) + 7.099\sin(100t) \\ & +2.009\cos(200t) + 6.596\sin(200t) \end{aligned}$$