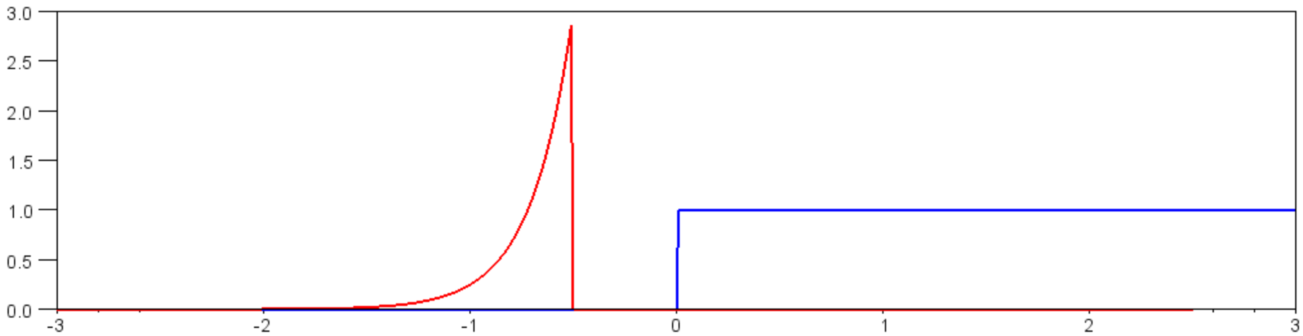
ECE 311 - Homework #9

Convolution

Find y(t) = x(t) ** h(t)

1) $x(t) = u(t)$

$h(t) = 3e^{-5t}u(t)$



x(t) (blue) and h(t-tau) (red) at t = -0.5
Sweep the red line to the right, multiply, and compute the area enclosed (integrate)

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$$

$$y(t) = \int_{-\infty}^{\infty} u(\tau) \cdot 3e^{-5(t-\tau)}u(t-\tau) \cdot d\tau$$

$$y(t) = \int_{0}^{t} 3e^{-5(t-\tau)} \cdot d\tau$$

$$y(t) = \int_{0}^{t} 3e^{-5t}e^{5\tau} \cdot d\tau \qquad t > 0$$

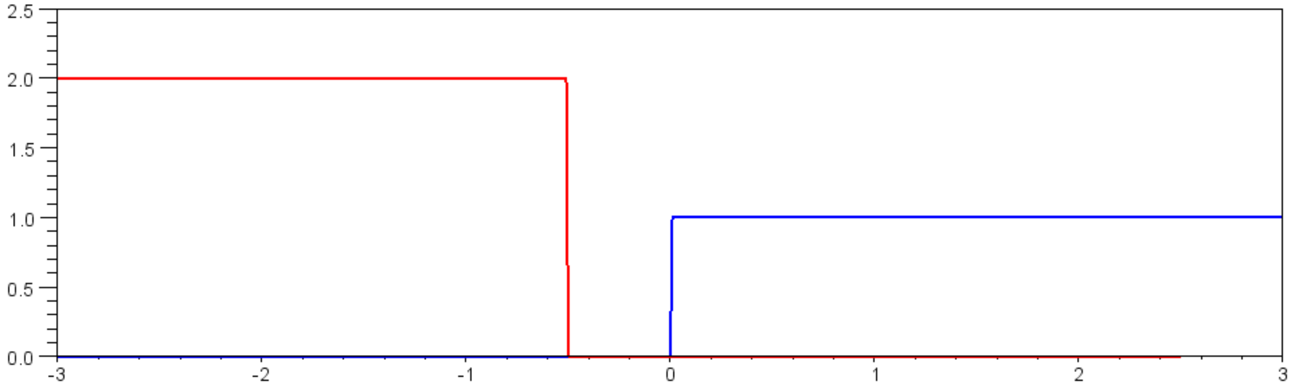
$$y(t) = 3e^{-5t} \cdot \left(\tfrac{1}{5}e^{5\tau}\right)_{0}^{t} \qquad t > 0$$

$$y(t) = 3e^{-5t} \cdot \left(\tfrac{1}{5}e^{5t} - \tfrac{1}{5}\right) \qquad t > 0$$

$$\boxed{y(t) = \tfrac{3}{5}(1 - e^{-5t}) \cdot u(t)}$$

2) $x(t) = u(t)$

$h(t) = 2u(t)$



x(t) (blue) and h(t-tau) (red) at t = -0.5
Sweep the red line to the right, multiply, and compute the area enclosed (integrate)

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$$

$$y(t) = \int_{-\infty}^{\infty} u(\tau) \cdot 2u(t-\tau) \cdot d\tau$$

$$y(t) = \int_{0}^{t} 2 \cdot d\tau \qquad t > 0$$

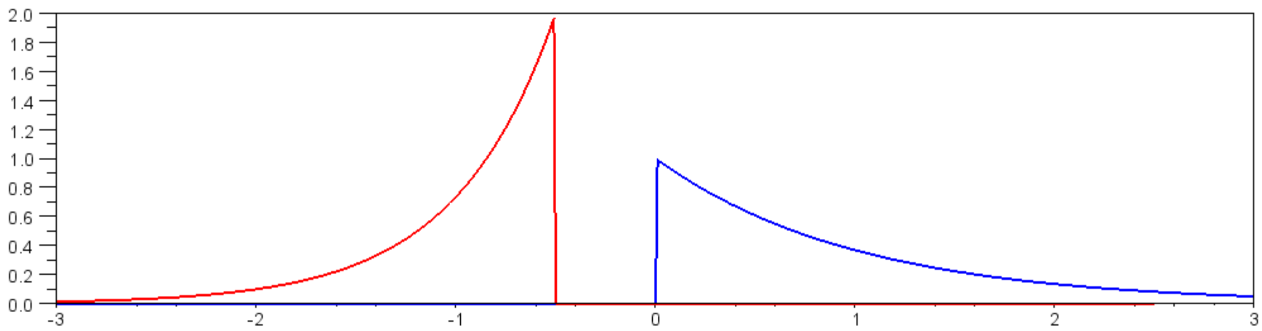
$$y(t) = (2\tau)_0^t \qquad t > 0$$

$$y(t) = 2t - 0 \qquad t > 0$$

$$\boxed{y(t) = 2t \cdot u(t)}$$

3) $x(t) = e^{-t}u(t)$

$h(t) = 2e^{-2t}u(t)$



x(t) (blue) and h(t-tau) (red) at t = -0.5
Sweep the red line to the right, multiply, and compute the area enclosed (integrate)

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau) \cdot 2e^{-2(t-\tau)}u(t-\tau) \cdot d\tau$$

$$y(t) = \int_{0}^{t} e^{-\tau} \cdot 2e^{-2(t-\tau)} \cdot d\tau \qquad t > 0$$

$$y(t) = \int_{0}^{t} e^{-\tau} \cdot 2e^{-2t} \cdot e^{2\tau} \cdot d\tau \qquad t > 0$$

$$y(t) = \int_{0}^{t} 2e^{-2t} \cdot e^{\tau} \cdot d\tau \qquad t > 0$$

$$y(t) = 2e^{-2t} \cdot (e^{\tau})_0^t \qquad t > 0$$

$$y(t) = 2e^{-2t} \cdot (e^{t} - 1) \qquad t > 0$$

$$\boxed{y(t) = (2e^{-t} - 2e^{-2t})u(t)}$$