ECE 311 - Homework #10

LaPlace Transforms

Use the definition of LaPlace transforms

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \cdot dt$$

to find X(s)

1) $x(t) = 3u(t)$

$$X(s) = \int_{-\infty}^{\infty} 3u(t) \cdot e^{-st} \cdot dt$$

$$X(s) = \int_{0}^{\infty} 3 \cdot e^{-st} \cdot dt$$

$$X(s) = \left(\frac{-3}{s}\right) \cdot \left(e^{-st}\right)_0^{\infty}$$

$$X(s) = \left(\frac{-3}{s}\right) \cdot (0 - 1)$$

$$\boxed{X(s) = \left(\frac{3}{s}\right)}$$

2) $x(t) = 2e^{-3t}u(t)$

$$X(s) = \int_{-\infty}^{\infty} 2e^{-3t}u(t) \cdot e^{-st} \cdot dt$$

$$X(s) = \int_{0}^{\infty} 2e^{-3t} \cdot e^{-st} \cdot dt$$

$$X(s) = \int_{0}^{\infty} 2e^{-3t-st} \cdot dt$$

$$X(s) = \int_{0}^{\infty} 2e^{-(s+3)t} \cdot dt$$

$$X(s) = \left(\frac{-2}{s+3}\right)\left(e^{-(s+3)t}\right)_0^{\infty}$$

$$\boxed{X(s) = \left(\frac{2}{s+3}\right)}$$

3) $x(t) = \left(\frac{e^{j3t}+e^{-j3t}}{2}\right)u(t)$

$$X(s) = \int_{-\infty}^{\infty} \left(\frac{e^{j3t}+e^{-j3t}}{2}\right)u(t)\cdot e^{-st}\cdot dt$$

$$X(s) = \frac{1}{2}\int_{0}^{\infty} (e^{j3t}+e^{-j3t})\cdot e^{-st}\cdot dt$$

$$X(s) = \frac{1}{2}\int_{0}^{\infty} e^{j3t}\cdot e^{-st}\cdot dt + \frac{1}{2}\int_{0}^{\infty} e^{-j3t}\cdot e^{-st}\cdot dt$$

$$X(s) = \frac{1}{2}\int_{0}^{\infty} e^{j3t-st}\cdot dt + \frac{1}{2}\int_{0}^{\infty} e^{-j3t-st}\cdot dt$$

$$X(s) = \frac{1}{2}\int_{0}^{\infty} e^{-(s-j3)t}\cdot dt + \frac{1}{2}\int_{0}^{\infty} e^{-(s+j3)t}\cdot dt$$

$$X(s) = \frac{-1}{2(s-j3)}(e^{-(s-j3)t})_{0}^{\infty} + \frac{-1}{2(s+j3)}(e^{-(s+j3)t})_{0}^{\infty}$$

$$X(s) = \frac{1}{2(s-j3)} + \frac{1}{2(s+j3)}$$

$$X(s) = \frac{(s+j3)+(s-j3)}{2(s-j3)(s+j3)}$$

$$\boxed{X(s) = \left(\frac{1/2}{s+j3}+\frac{1/2}{s-j3}\right) = \frac{s}{(s-j3)(s+j3)}}$$

The LaPlace transfor for cosine is

$$\boxed{\cos(\omega t) \leftrightarrow \left(\frac{1/2}{s+j\omega}+\frac{1/2}{s-j\omega}\right) = \frac{s}{s^2+\omega^2}}$$

4) $x(t) = \left(\frac{e^{j3t} - e^{-j3t}}{2j}\right) u(t)$

$$X(s) = \int_{-\infty}^{\infty} \left(\frac{e^{j3t} - e^{-j3t}}{2j}\right) u(t) \cdot e^{-st} \cdot dt$$

$$X(s) = \left(\frac{1}{2j}\right) \int_0^{\infty} (e^{j3t} - e^{-j3t}) \cdot e^{-st} \cdot dt$$

$$X(s) = \left(\frac{-j}{2}\right) \int_0^{\infty} (e^{j3t-st} - e^{-j3t-st}) \cdot dt$$

$$X(s) = \left(\frac{-j}{2}\right) \int_0^{\infty} (e^{-(s-j3)t} - e^{-(s+j3)t}) \cdot dt$$

$$X(s) = \left(\frac{-j}{2}\right) \left(\left(\left(\frac{-1}{s-j3}\right) e^{-(s-j3)t} \right)_0^{\infty} - \left(\left(\frac{-1}{s+j3}\right) e^{-(s+j3)t} \right)_0^{\infty} \right)$$

$$X(s) = \left(\frac{-j}{2}\right) \left(\left(\frac{1}{s-j3}\right) - \left(\frac{1}{s+j3}\right) \right)$$

$$X(s) = \left(\frac{-j/2}{s-j3}\right) + \left(\frac{j/2}{s+j3}\right)$$

or

$$X(s) = \left(\frac{\left(\frac{-j}{2}\right)(s+j3) + \left(\frac{j}{2}\right)(s-j3)}{(s-j3)(s+j3)}\right) = \left(\frac{3}{s^2+9}\right)$$

$$\boxed{X(s) = \left(\frac{3}{s^2+9}\right)}$$

The LaPlace Transform for sin(t) is

$$\boxed{\sin(\omega t) \leftrightarrow \left(\frac{\omega}{s^2+\omega^2}\right) = \left(\frac{-j/2}{s-j\omega}\right) + \left(\frac{j/2}{s+j\omega}\right)}$$