ECE 311 - Homework #12

Natural Response with LaPlace Transforms

Find the solution to the following differential equations with initial conditions

1) $\frac{dy}{dt} + 7y = 0 \qquad y(0) = 10$

$(sY - 10) + 7Y = 0$

$(s+7)Y = 10$

$Y = \left(\frac{10}{s+7}\right)$

$\boxed{y(t) = 10e^{-7t}u(t)}$

2) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = 0 \qquad y(0) = 10 \qquad \frac{dy(0)}{dt} = 3$

$(s^2Y - s \cdot 10 - 3) + 6(sY - 10) + 5Y = 0$

$(s^2 + 6s + 5)Y = 10s + 63$

$Y = \left(\frac{10s+63}{s^2+6s+5}\right) = \left(\frac{10s+63}{(s+1)(s+5)}\right)$

This isn't in my table of LaPlace transforms, so use partial fractions

$Y = \left(\frac{13.25}{s+1}\right) + \left(\frac{-3.25}{s+5}\right)$

$\boxed{y(t) = (13.25e^{-t} - 3.25e^{-5t})u(t)}$

3) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = 0 \qquad y(0) = 10 \qquad \frac{dy}{dt}(0) = 3$

$(s^2Y - 10s - 3) + 2(sY - 10) + 10Y = 0$

$(s^2 + 2s + 10)Y = 10s + 23$

$Y = \left(\frac{10s+23}{s^2+2s+10}\right) = \left(\frac{10s+23}{(s+1+j3)(s+1-j3)}\right)$

$Y = \left(\frac{5.449\angle 23.4^0}{s+1+j3}\right) + \left(\frac{5.449\angle -23.4^0}{s+1-j3}\right)$

$\boxed{y(t) = 10.898 \cdot e^{-t} \cdot \cos(3t - 23.4^0) \cdot u(t)}$

4) $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = 0 \qquad y(0) = 10 \qquad \frac{dy}{dt}(0) = \frac{d^2y}{dt^2}(0) = 0$

$(s^3Y - 10s^2) + 6(s^2Y - 10s) + 11(sY - 10) + 6Y = 0$

$(s^3 + 6s^2 + 11s + 6)Y = 10s^2 + 60s + 110$

$Y = \left(\frac{10s^2+60s+110}{s^3+6s^2+11s+6}\right) = \left(\frac{10s^2+60s+110}{(s+1)(s+2)(s+3)}\right)$

$Y = \left(\frac{30}{s+1}\right) + \left(\frac{-30}{s+2}\right) + \left(\frac{10}{s+3}\right)$

$y(t) = (30e^{-t} - 30e^{-2t} + 10e^{-3t})u(t)$