ECE 311 - Homework #13

Forced Response with LaPlace Transforms

Assume zero initial conditions. Find the solution to the following differential equations

Problem 1)

$$\frac{dy}{dt} + 7y = x \qquad\qquad x(t) = 10u(t)$$

convert to LaPlace

$$(s+7)Y = X$$

$$Y = \left(\frac{1}{s+7}\right)X$$

plug in the LaPlace transform for X

$$Y = \left(\frac{1}{s+7}\right)\left(\frac{10}{s}\right)$$

take the partial fraction expansion

$$Y = \left(\frac{1.4286}{s}\right) + \left(\frac{-1.4286}{s+7}\right)$$

take the inverse LaPlace transform

$$\boxed{y(t) = (\ 1.4286 - 1.4286e^{-7t}\ )u(t)}$$

Problem 2)

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = x \qquad\qquad x(t) = 10u(t)$$

convert to LaPlace

$$(s^2 + 6s + 5)Y = X$$

$$Y = \left(\frac{1}{s^2+6s+5}\right)X$$

plug in the LaPlace transform for X

$$Y = \left(\frac{1}{s^2+6s+5}\right)\left(\frac{10}{s}\right)$$

factor and take the partial fraction expansion

$$Y = \left(\frac{10}{s(s+1)(s+5)}\right) = \left(\frac{2}{s}\right) + \left(\frac{-2.5}{s+1}\right) + \left(\frac{0.5}{s+5}\right)$$

take the inverse LaPlace transform

$$\boxed{y(t) = (\ 2 - 2.5e^{-t} + 0.5e^{-5t}\ )u(t)}$$

Problem 3)

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = x \qquad x(t) = 10e^{-3t}u(t)$$

take the LaPlace transform

$$(s^2 + 2s + 10)Y = X$$

$$Y = \left(\frac{1}{s^2+2s+10}\right)X$$

Plug in the LaPlace transform for X

$$Y = \left(\frac{1}{s^2+2s+10}\right)\left(\frac{10}{s+3}\right)$$

Factor and take the partial fraction expansion

$$Y = \left(\frac{10}{(s+1+j3)(s+1-j3)(s+3)}\right)$$

$$Y = \left(\frac{0.7692}{s+3}\right) + \left(\frac{0.4623\angle 146^0}{s+1+j3}\right) + \left(\frac{0.4623\angle -146^0}{s+1-j3}\right)$$

take the inverse LaPlace transform

$$\boxed{y(t) = (\ 0.7692e^{-3t} + 0.9245e^{-t}\cos(3t - 146^0)\ )u(t)}$$

Problem 4)

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = x \qquad x(t) = 10\cos(3t)u(t)$$

take the LaPlace transform

$$(s^3 + 6s^2 + 11s + 6)Y = X$$

$$Y = \left(\frac{1}{s^3+6s^2+11s+6}\right)X$$

Plug in the LaPlace transform for X

$$Y = \left(\frac{1}{s^3+6s^2+11s+6}\right)\left(\frac{10s}{s^2+9}\right)$$

Factor and use partial fraction expansion

$$Y = \left(\frac{10s}{(s+1)(s+2)(s+3)(s+j3)(s-j3)}\right)$$

$$Y = \left(\frac{-0.5}{s+1}\right) + \left(\frac{1.5385}{s+2}\right) + \left(\frac{-0.8333}{s+3}\right) + \left(\frac{0.1034\angle 173^0}{s+j3}\right) + \left(\frac{0.1034\angle -173^0}{s-j3}\right)$$

Take the inverse LaPlace transform

$$\boxed{y(t) = (\ = 0.5e^{-t} + 1.5385e^{-2t} - 0.8333e^{-3t} + 0.2067\cos(3t - 173^0)\ )u(t)}$$