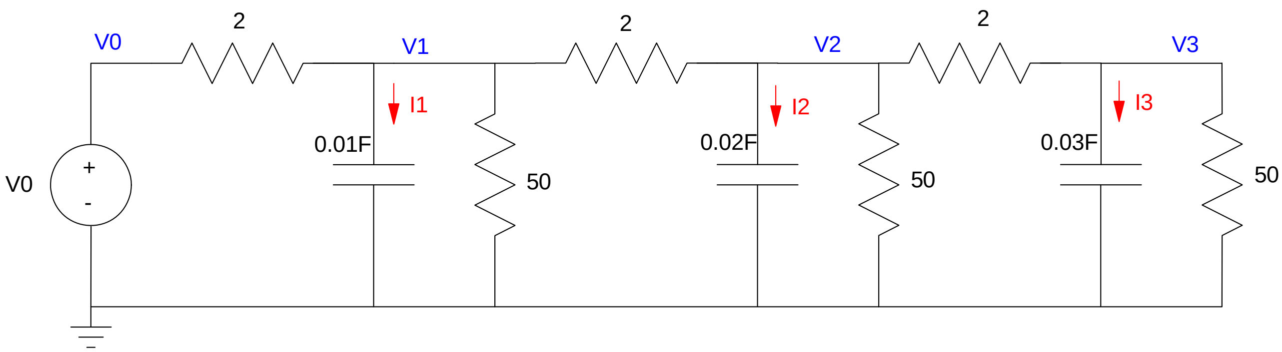
ECE 311 - Homework #19

Transfer Functions using State Variables

Problem 1 -3) For the following circuit, assume zero initial conditions.



Problem 1: Find the transfer function from Vo to V3

Express the dynamics in state-variable form. First, write the voltage node equations:

$$I_1 = 0.01sV_1 = \left(\frac{V_0 - V_1}{2}\right) + \left(\frac{V_2 - V_1}{2}\right) - \left(\frac{V_1}{50}\right)$$

$$I_2 = 0.02sV_2 = \left(\frac{V_1 - V_2}{2}\right) + \left(\frac{V_3 - V_2}{2}\right) - \left(\frac{V_2}{50}\right)$$

$$I_3 = 0.03sV_3 = \left(\frac{V_2 - V_3}{2}\right) + \left(\frac{V_3}{50}\right)$$

Solve for the highest derivative and group terms

$$sV_1 = 50V_0 - 102V_1 + 50V_2$$

$$sV_2 = 25V_1 - 51V_2 + 25V_3$$

$$sV_3 = 16.67V_2 - 17.33V_3$$

Place in matrix (state-variable) form

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \end{bmatrix} = \begin{bmatrix} -102 & 50 & 0 \\ 25 & -51 & 25 \\ 0 & 16.67 & -17.33 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix} V_0$$

$$y = V_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + [0]V_0$$

Find the transfer function from V0 to V3

```
>> A = [-102,50,0 ; 25,-51,25 ; 0,16.67,-17.33]

  -102.0000   50.0000         0
    25.0000  -51.0000   25.0000
          0   16.6700  -17.3300

>> B = [50;0;0]

    50
     0
     0

>> C = [0,0,1]

     0     0     1

>> D = 0;
>> G = ss(A,B,C,D);

                   2.084e004
V3 = ------------------------------------ V0
      s^3 + 170.3 s^2 + 6187 s + 2.598e004

>> zpk(G)


                20837.5
V3 = ---------------------------   V0
      (s+121) (s+44.55) (s+4.821)

>>
```

Problem 2: Find the transfer function from V0 to V2

The first set of equations remain unchanged:

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \end{bmatrix} = \begin{bmatrix} -102 & 50 & 0 \\ 25 & -51 & 25 \\ 0 & 16.67 & -17.33 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix} V_0$$

The output equations do change though

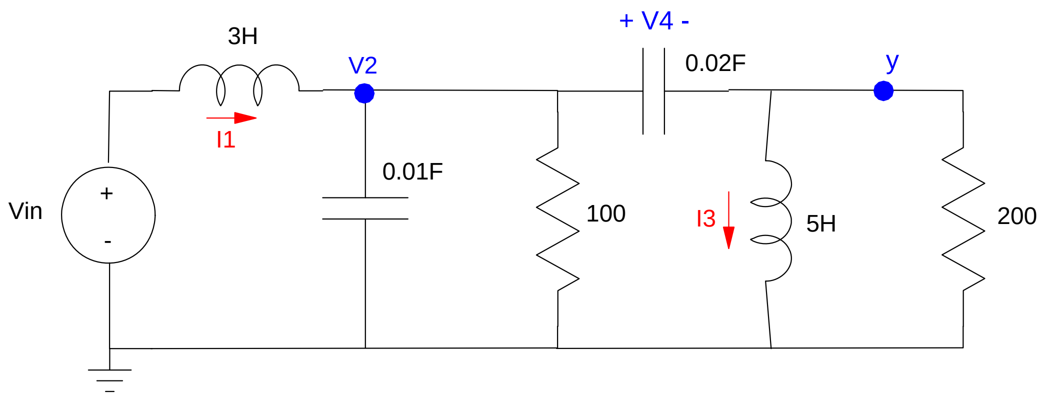
$$y = V_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + [0] V_0$$

```
>> C = [0,1,0];
>> D = 0;
>> G = ss(A,B,C,D);
>> tf(G)


         1250 s + 2.166e004
------------------------------------
s^3 + 170.3 s^2 + 6187 s + 2.598e004

>> zpk(G)


      1250 (s+17.33)
---------------------------
(s+121) (s+44.55) (s+4.821)
```

The poles remain unchanged (the denominator polynomial)

The zeros do change (the numerator polynomial)

Problem 3: Find the transfer function from Vin to y



i) Write the dynamics for this system (i.e. the voltage node equations using LaPlace notation

$$V_1 = 3sI_1 = V_{in} - V_2$$

$$I_2 = 0.01sV_2 = I_1 - \frac{V_2}{100} - 0.02sV_4$$

$$V_3 = 5sI_3 = V_2 - V_4$$

$$I_4 = 0.02sV_4 = I_3 + \left(\frac{V_2 - V_4}{200}\right)$$

ii) Place in matrix form. Group terms

$$3sI_1 = V_{in} - V_2$$

$$0.01sV_2 + 0.02sV_4 = I_1 - \frac{V_2}{100}$$

$$5sI_3 = V_2 - V_4$$

$$0.02sV_4 = I_3 + \left(\frac{V_2 - V_4}{200}\right)$$

Place in matrix form

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0.02 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0.02 \end{bmatrix} \begin{bmatrix} sI_1 \\ sV_2 \\ sI_3 \\ sV_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -0.01 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0.005 & 0 & -0.005 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \\ I_3 \\ V_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

Solve in Matlab

```
>> M = [3,0,0,0;0,0.01,0,0.02;0,0,5,0;0,0,0,0.02]

    3.0000         0         0         0
         0    0.0100         0    0.0200
         0         0    5.0000         0
         0         0         0    0.0200

>> A1 = [0,-1,0,0 ; 1,-0.01,0,0 ; 0,1,0,-1 ; 0,0.005,0,-0.005]

         0   -1.0000         0         0
    1.0000   -0.0100         0         0
         0    1.0000         0   -1.0000
         0    0.0050         0   -0.0050

>> A = inv(M)*A1

         0   -0.3333         0         0
  100.0000   -1.5000         0    0.5000
         0    0.2000         0   -0.2000
         0    0.2500         0   -0.2500

>> B = inv(M) * [1;0;0;0]

    0.3333
         0
         0
         0
```

meaning

$$\begin{bmatrix} sI_1 \\ sV_2 \\ sI_3 \\ sV_4 \end{bmatrix} = \begin{bmatrix} 0 & -0.333 & 0 & 0 \\ 100 & -1.5 & 0 & 0.5 \\ 0 & 0.2 & 0 & -0.2 \\ 0 & 0.25 & 0 & -0.25 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \\ I_3 \\ V_4 \end{bmatrix} + \begin{bmatrix} 0.3333 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

iii) Find y(t)

$$y = V_2 - V(4)$$

$$y = \begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \\ I_3 \\ V_4 \end{bmatrix} + [0]$$

```
>> C = [0,1,0,-1];
D = 0;
>>
>> G = ss(A,B,C,D);
>> tf(G)

                 33.33 s
y = -------------------------------- Vin
    s^3 + 1.75 s^2 + 33.58 s + 8.333
```