ECE 311 - Solution to Homework #27

Superposition

1) A circuit has the following transfer funciton

$$Y = \left(\frac{20}{(s+1)(s+5)}\right) X$$

Find y(t) assuming

x(t) = 2 + 3 cos(4t) + 5 sin(6t)

Treat this as three separate problems:

a) x(2) = 2

s = 0

$$\left(\frac{20}{(s+1)(s+5)}\right)_{s=0} = 4$$

$$Y = (4) \cdot 2$$

$$y_a(t) = 8$$

b) x(t) = 3 cos(4t)

X = 3 + j0

s = j4

$$\left(\frac{20}{(s+1)(s+5)}\right)_{s=j4} = 0.7576\angle - 114.6^0$$

$$Y = (0.7576\angle - 114.6^0) \cdot (3 + j0) = 2.2727\angle - 114.6^0$$

$$y_b(t) = 2.2727 \cos(4t - 114.6^0)$$

c) x(t) = 5 sin(6t)

X = 0 - j5

s = j6

$$\left(\frac{20}{(s+1)(s+5)}\right)_{s=j6} = 0.4210\angle - 130.7^0$$

$$Y = (0.4210\angle - 130.7^0) \cdot (0 - j5)$$

$$Y = 2.1049\angle 139.3^0$$

$$y_c(t) = 2.1049 \cos(6t + 139.3^0)$$

The total answer is then

$$\boxed{y(t) = 8 + 2.2727 \cos(4t - 114.6^0) + 2.1049 \cos(6t + 139.3^0)}$$

2) A circuit has the following transfer funciton

$$Y = \left(\frac{20}{(s+1+j4)(s+1-j4)}\right)X$$

Find y(t) assuming

x(t) = 2 + 3 cos(4t) + 5 sin(6t)

Treat this as three separate problems

a) x(t) = 2

s = 0

$$\left(\frac{20}{(s+1+j4)(s+1-j4)}\right)_{s=0} = 1.1765$$

$$Y = 1.1765 \cdot 2 = 2.3529$$

$$y(t) = 2.3529$$

b) x(t) = 3 cos(4t)

X = 3 + j0

s = j4

$$\left(\frac{20}{(s+1+j4)(s+1-j4)}\right)_{s=j4} = 2.4807\angle -82.9^0$$

$$Y = (2.4807\angle -82.9^0) \cdot (3+j0) = 7.4421\angle -82.9^0$$

$$y(t) = 7.4421\cos(4t - 82.9^0)$$

c) x(t) = 5 sin(6t)

X = 0 - j5

s = j6

$$\left(\frac{20}{(s+1+j4)(s+1-j4)}\right)_{s=j4} = 0.8900\angle -147.7^0$$

$$Y = (0.8900\angle -147.7^0) \cdot (0-j5) = 4.4499\angle 122.3^0$$

$$y(t) = 4.4499\cos(6t + 122.3^0)$$

The total answer is then

$$\boxed{y(t) = 2.3529 + 7.4421\cos(4t - 82.9^0) + 4.4499\cos(6t + 122.3^0)}$$