

ECE 311 - Solution to Homework #29

Complex Fourier Transform

Note:

- If you don't mind complex numbers, the Complex Fourier Transform is easier to use.
- It's the same as assignment #28, only with $\text{real}(X) = \text{cosine}$, $-\text{imag}(X) = \text{sine}$

$$1) \quad x(t) = \begin{cases} 1 & \sin(5t) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the complex Fourier transform

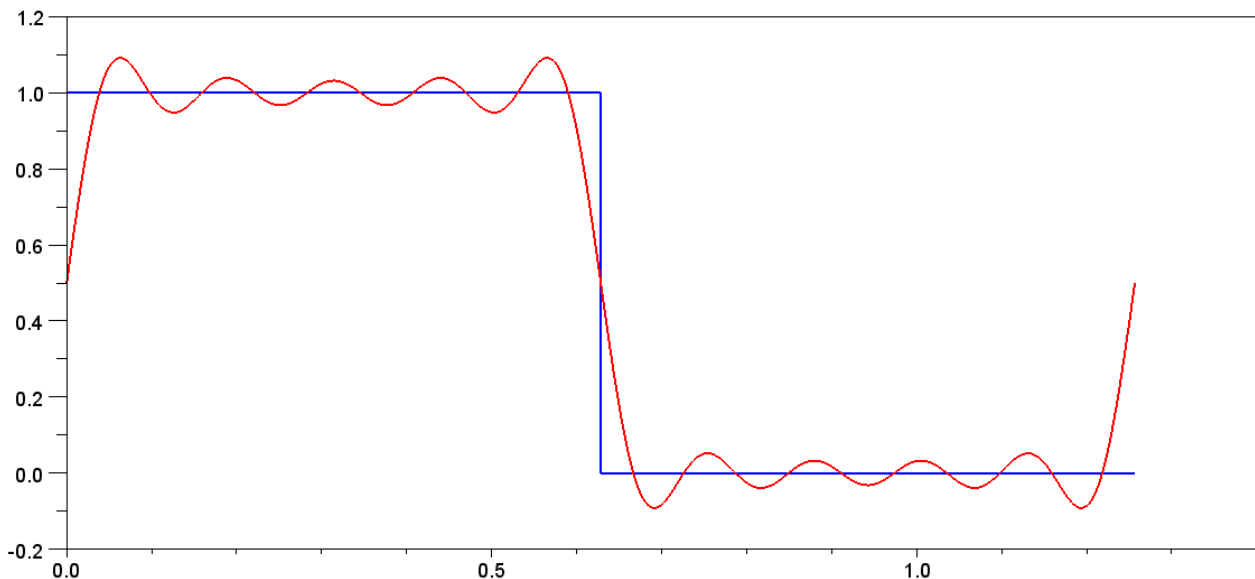
```
t = [1:10000]'/10000 * 2*pi/5;
w0 = 5;
x = 1 * (sin(5*t) > 0);
X0 = mean(x);
X = zeros(10,1);
for n=1:10
    X(n) = 2*mean(x .* exp(-j*5*n*t));
end
```

n	0	1	2	3	4	5	6	7	8	9	10
X(n)	0.5	-j0.637	0	-j0.212	0	-j0.127	0	-j0.091	0	-j0.071	0

Note that this is the same result as in homework #28 with $\text{real}(X) = \text{cosine terms}$, $-\text{imag}(X) = \text{sine terms}$

b) Plot $x(t)$ vs time along with it's Fourier transform approximation taken out to 10 terms

```
y = 0*t + X0;
for n=1:10
    y = y + real(X(n) * exp(j*5*n*t));
end
plot(t,x,t,y);
```



$$2) \quad x(t) = \begin{cases} \sin(5t) & \sin(5t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the complex Fourier transform

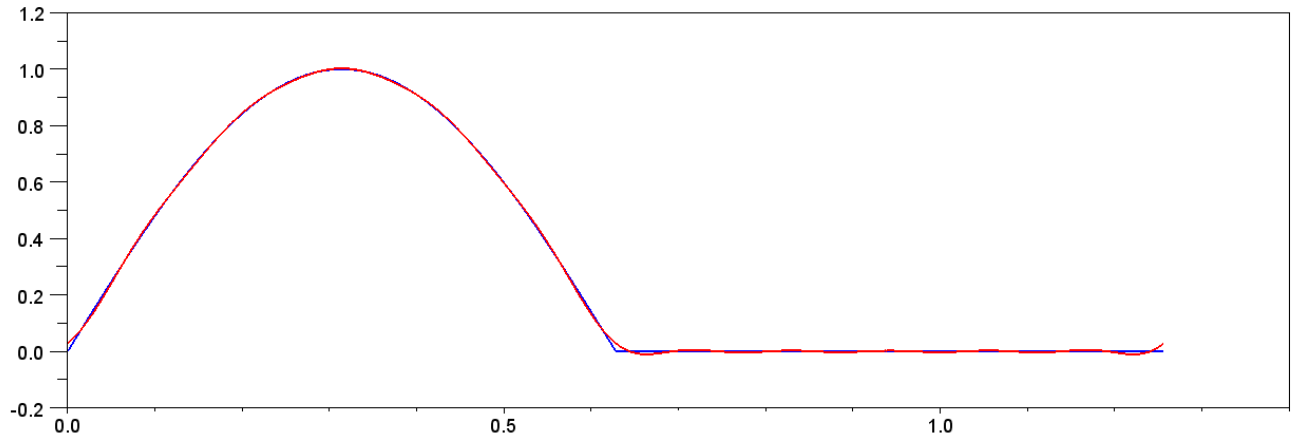
```
t = [1:10000]'/10000 * 2*pi/5;
w0 = 5;
x = max( 0, sin(5*t) );
X0 = mean(x);
X = zeros(10,1);
for n=1:10
    X(n) = 2*mean(x .* exp(-j*5*n*t));
end
```

n	0	1	2	3	4	5	6	7	8	9	10
X(n)	0.318	-j0.5	-0.212	0	-0.042	0	-0.018	0	-0.010	0	-0.006

Note that this is the same result as in homework #28 with $\text{real}(X) = \text{cosine terms}$, $-\text{imag}(X) = \text{sine terms}$

b) Plot $x(t)$ vs time along with it's Fourier transform approximation taken out to 10 terms

```
y = 0*t + X0;
for n=1:10
    y = y + real(X(n) * exp(j*5*n*t));
end
plot(t,x,t,y);
```



$$3) \quad x(t) = \begin{cases} \sin(5t) & \sin(5t) < 0.8 \\ 0.8 & \text{otherwise} \end{cases}$$

a) Find the complex Fourier transform

```
t = [1:10000]'/10000 * 2*pi/5;
w0 = 5;
x = min( 0.8, sin(5*t) );
X0 = mean(x);
X = zeros(10,1);
for n=1:10
    X(n) = 2*mean(x .* exp(-j*5*n*t));
end
```

n	0	1	2	3	4	5	6	7	8	9	10
X(n)	-0.027	-j0.948	0.046	-j0.037	-0.026	j0.016	0.007	0	0.003	-j0.005	-0.004

b) Plot x(t) vs time along with it's Fourier transform approximation taken out to 10 terms

```
y = 0*t + X0;
for n=1:10
    y = y + real(X(n) * exp(j*5*n*t));
end
plot(t,x,t,y);
```

