

# ECE 311 - Solution to Homework #30

## Properties of Fourier Transforms

Use properties of Fourier Transforms to determine the Fourier Transform for the following signals:

$$1) \quad x(t) = x(t + 2\pi)$$

$$x(t) = \begin{cases} t(\pi - t) & 0 < t < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$X = 0$$

$$x' = \begin{cases} \pi - 2t & 0 < t < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$jnX = 0$$

$$x'' = \begin{cases} \pi\delta(t) - 2 + \pi\delta(t - \pi) & 0 < t < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$(jn)^2 X = \pi + \pi e^{-jn\pi}$$

$$x''' = \begin{cases} -2\delta(t) + 2\delta(t - \pi) & 0 < t < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$(jn)^3 X = -2 + 2e^{-jn\pi}$$

Add it all up...

$$X_n = \left( \frac{\pi + \pi e^{-jn\pi}}{(jn)^2} \right) + \left( \frac{-2 + 2e^{-jn\pi}}{(jn)^3} \right)$$

$$2) \quad x(t) = x(t + \pi)$$

$$x(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$X = 0$$

$$x' = \begin{cases} \delta(t) - \delta(t-1) & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(jn)X = 1 - 1e^{-jn}$$

Adding it all up

$$X_n = \left( \frac{1 - e^{-jn}}{jn} \right)$$

$$3) \quad x(t) = x(t + \pi)$$

$$x(t) = \begin{cases} t & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$X = 0$$

$$x' = \begin{cases} 1 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(jn)X = 0$$

$$x'' = \begin{cases} \delta(t) - \delta(t-1) & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(jn)^2 X = 1 - e^{-jn}$$

Adding it all up

$$X_n = \left( \frac{1 - e^{-jn}}{(jn)^2} \right)$$