Transformers and AC Impedance

Objective:

- Present the characteristics of transformers
- Be able to reflect impedances through a transformer.

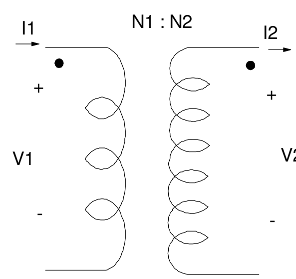
Characteristics of Transformers:

DC power has several advantages: transistors, microprocessors, lights, etc remain on all the time. They don't see their power source passing through zero 60 (or 120) times per second.

AC power has one huge advantage over DC power: you can increase or decrease the voltage with ease simply be using a transformer.

A transformer consists of two sets of wires, wound around a common core, typically made of iron. By being wound around a common core, any magnetic flux seen by one set of windings is seen by the other.

The symbol for a transformer is shown below. The dots denote the sign of the voltages: and AC voltage applied to V1 will produce a positive voltage at V2. The ratio of the number of turns between the two windings is denoted by N1 and N2 (for example, you might have 1:2 meaning winding 2 has twice as many turns as winding 1.)



From physics, the magnetic field produced by a changing current going around a loop is:

$$B = NI$$

where N is the number of turns and I is the current. Suppose you have two loops around the same iron core: one with N1 loops, the other with N2 loops. The current in the second loop is

$$B = N_1 I_1 = N_2 I_2$$

or

$$I_2 = \left(\frac{N_1}{N_2}\right) I_1$$

You can increase or decrease the current by varying the turns ratio. Energy has to balance (assuming a losses transformer). So

$$V_1 I_1 = V_2 I_2$$

$$V_1 I_1 = V_2 \left(\frac{N_1}{N_2}\right) I_1$$

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1$$

For example, assume the above transformer had N1:N2 = 1:10. If the input was

V1 = 1V

I1 = 1A,

then

$$V_2 = \left(\frac{10}{1}\right) V_1 = 10V$$

$$I_2 = \left(\frac{1}{10}\right) I_1 = 0.1A$$

The transformer has increased the voltage but reduced the current. Power remains unchanged.

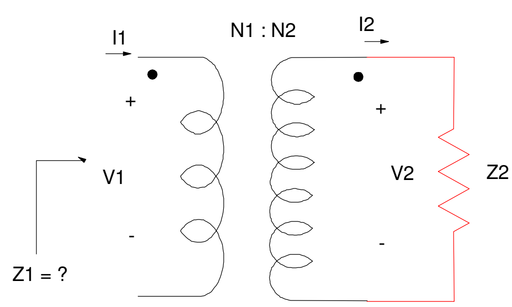
This reduction in current is what allows utilities to transmit power over large distances. Transmission line losses are related to current squared. By raising the voltage on the transmission lines to as high as 500,000V, the current is reduced (relative to 120V) by a factor of 4166. The transmission line losses are reduced by the square of this.

At the customer's house, the voltage is reduced from 500,000V down to 120V for obvious safety reasons. (It's actually reduced in several steps - the longer the transmission line, the higher the voltage in general however.) Transformers allow you to do this.

When you look through a transformer, you can see the load impedance. Its apparent value is changed by the transformer, however.

Impedance Seen Through a Transformer:

Suppose you have a load on the right side of a transformer. What load is seen on the left side?



By definition, Z2 sets the relationship between V2 and I2:

$$V_2 = Z_2 I_2$$

Substituting for V1 and I1:

$$\left(\left(\frac{N_2}{N_1}\right)V_1\right) = Z_2\left(\left(\frac{N_1}{N_2}\right)I_1\right)$$

Solving for V1:

$$V_1 = Z_2\left(\frac{N_1}{N_2}\right)^2 I_1$$

or

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

The impedance seen through a transformer is scaled by the square of the turns ratio.

Going from side 2 to side 1, the impedance is scaled by (where you're going to / where you're coming from) squared.

Example:

Suppose you have a 8 Ohm speaker connected to size Z1 with a turn ratio of N1:N2 = 1:10. Find the impedance as seen on side 2:

$$Z_2 = \left(\frac{10}{1}\right)^2 \cdot 8\Omega = 800\Omega$$

An amplifier on side 2 sees an 800 Ohm load. It's much easier for transistor amplifiers you cover in ECE321 to drive an 800 Ohm load. Likewise, several stereos use transformers between the power amplifier and the speaker.

Example 2: A utility's customer draws 10kW at 120VAC

- What is the resistance as seen by the customer?
- What is the resistance as seen at the utility's substation, which is at 13,200V
- What is the resistance as seen at the utility's power plant, which is at 300,000V

The customer's resistance is

$$R_1 = \frac{V}{I} = \frac{120V}{10kW} = 0.012\Omega$$

At the substation:

$$R_2 = \left(\frac{13{,}200V}{120V}\right)^2 0.012\Omega = 145.2\Omega$$

At the power plant:

$$R_2 = \left(\frac{300{,}000V}{120V}\right)^2 0.012\Omega = 75k\Omega$$

The customer doesn't look like much of a load at the power plant. But then, there are a *lot* of customers for each power plant.