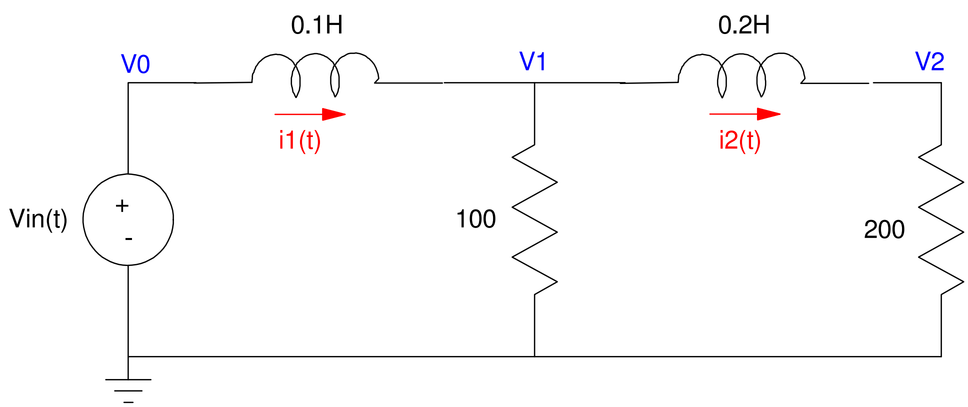
Current Loops in the LaPlace Domain

Note: The parallel model for inductors and capacitors work better when writing voltage node equations.

Example 1: Find V2(t) for the following circuit. Assume

$$v_{in}(t) = \begin{cases} 5V & t < 0 \\ 0V & t > 0 \end{cases}$$

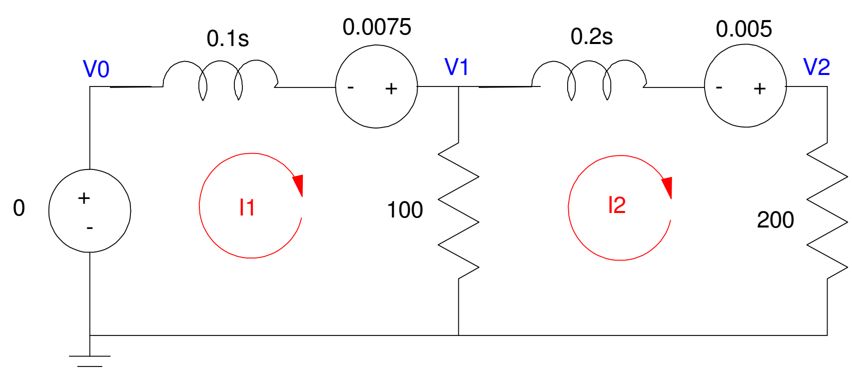


From before, the current at t=0 is

$$i_1(0) = 75mA$$

$$i_2(0) = 25mA$$

The series model (for current loops) would be



with the current loop equations being

$$0.1sI_1 - 0.0075 + 100(I_1 - I_2) = 0$$

$$100(I_2 - I_1) + 0.2sI_2 - 0.005 + 200I_2 = 0$$

Group terms and simplify

$$(0.1s + 100)I_1 - (100)I_2 = 0.0075 \qquad * \ 100$$

$$-(100)I_1 + (0.2s + 300)I_2 = 0.005 \qquad * \ (0.1s + 100)$$

Solve (using Gauss Elimination) add

$$(-100^2 + (0.2s + 300)(0.1s + 100))I_2 = 0.75 + 0.005(0.1s + 100)$$

$$I_2 = \left(\frac{0.0005s+1.25}{0.02s^2+50s+20000}\right)$$

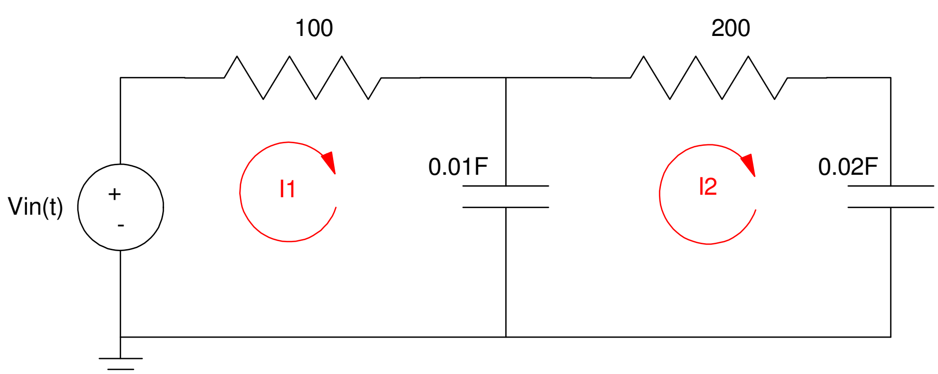
Taking the inverse LaPlace transform

$$I_2 = \left(\frac{0.025s+62.5}{(s+500)(s+2000)}\right) = \left(\frac{0.0333}{s+500}\right) + \left(\frac{-0.00833}{s+2000}\right)$$

$$i_2(t) = (0.0333e^{-500t} - 0.008333e^{-2000t})u(t)$$

Example 2: Capacitors: Find i2(t) assuming

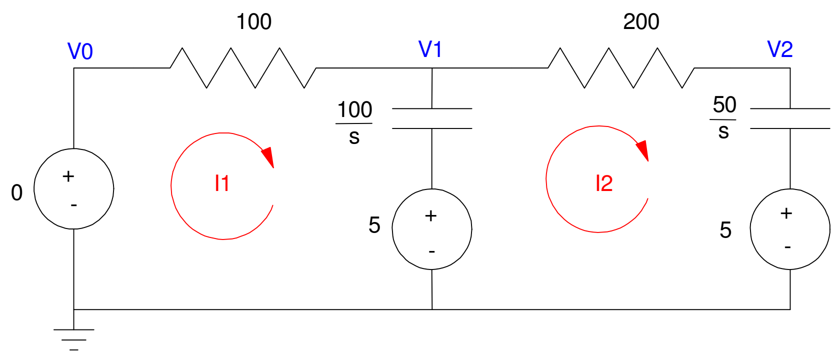
$$v_{in}(t) = \begin{cases} 5V & t < 0 \\ 0V & t > 0 \end{cases}$$



From before, the initial conditions are

$$v_1(t) = v_2(t) = 5V$$

Using the series model for a capacitor



Write the loop equations

$$100I_1 + \left(\frac{100}{s}\right)(I_1 - I_2) + 5 = 0 \quad (1)$$

$$-5 + \left(\frac{100}{s}\right)(I_2 - I_1) + 200I_2 + \left(\frac{50}{s}\right)I_2 + 5 = 0 \quad (2)$$

Group terms and simplify

$$(100s + 100)I_1 - 100I_2 = -5s \quad * \ 100$$

$$-100I_1 + (200s + 150)I_2 = 0 \quad * \ (100s + 100)$$

Solve for I2 add

$$(-100^2 + (200s + 150)(100s + 100))I_2 = -500s$$

$$I_2 = \left(\frac{-500s}{20000s^2+35000s+5000}\right)$$

$$I_2 = \left(\frac{-0.025}{s^2+1.75s+0.25}\right) = \left(\frac{-0.025}{(s+0.1569)(s+1.5931)}\right) = \left(\frac{-0.0174}{s+0.1569}\right) + \left(\frac{0.0174}{s+1.5931}\right)$$

Taking the inverse LaPlace transform

$$i_2(t) = (-0.0174e^{-0.1569t} + 0.0174e^{-1.5931t})u(t)$$