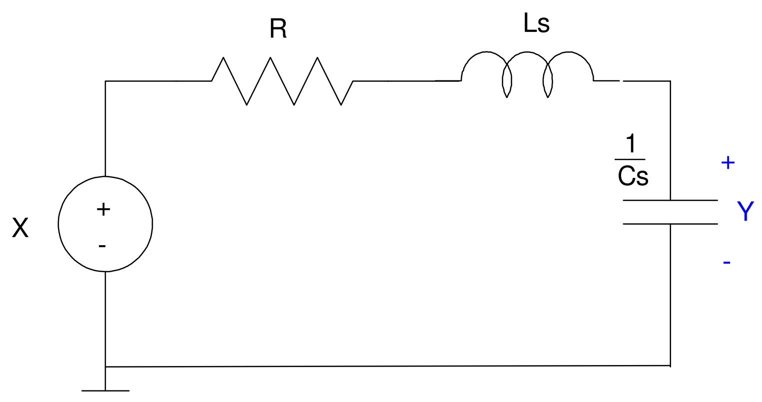
# Passive RLC Filters

With RC and RL filters, you're stuck with real poles. With RLC filters, you can have both real and complex poles. You can also built

- Low pass filters ( block high-frequency signals ),
- High-pass filters ( block DC terms ), and
- Band-pass filters ( block both high and low frequencies, only pass a select range of frequencies )

## Low Pass RLC Filter




Assume ideal components. The transfer function for the following RLC filter is

$$Y = \left(\frac{\frac{1}{Cs}}{R+Ls+\frac{1}{Cs}}\right)X$$

$$Y = \left(\frac{1}{LCs^2+RCs+1}\right)X$$

Note that

- At DC ( s = 0 ), the gain is one. This passes DC
- At high frequencies, the gain is proportional to $\frac{1}{s^2}$. This blocks high frequencies.
- Unlike an RC filter, the poles can be real or complex.

The poles of an RLC filter are the solutions to

$$LCs^2 + RCs + 1 = 0$$

or

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

If the roots are complex,

$$s = -a\angle \pm \theta$$

then

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = (s + a\angle\theta)(s + a\angle -\theta)$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + (2a\cos\theta)s + a^2$$

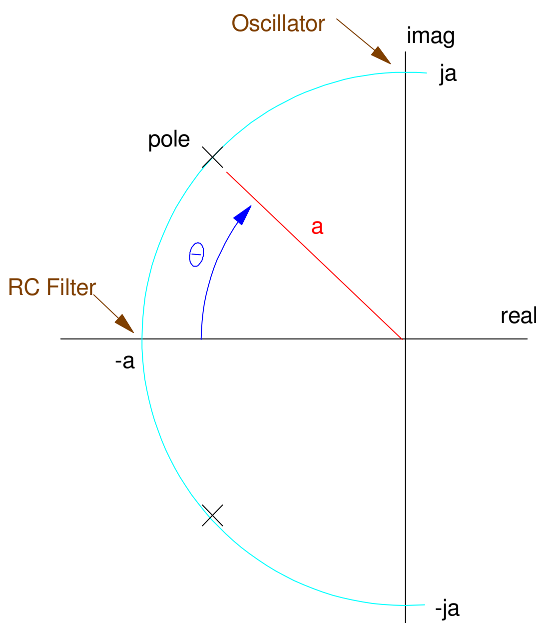
Matching terms

$$\frac{R}{L} = 2a\cos\theta$$

$$\frac{1}{LC} = a^2$$

With a Bode plot

- The amplitude of the poles tells you the corner frequency (in rad/sec),
- The angle of the poles tells you the damping ratio.



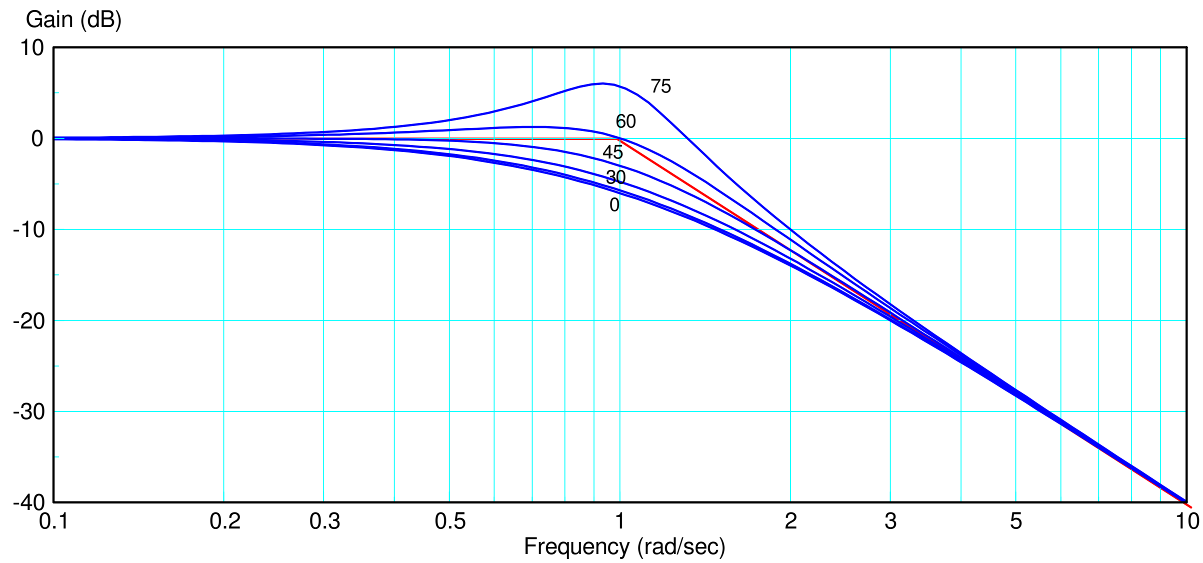

With an RLC filter, you can place the poles anywhere you want: real or complex.

The damping ratio determines the gain at the corner and how you go from one asymptote to another.

$$\zeta = \cos\theta$$

The damping ratio (and hence the angle) determines the gain at the corner

$$G(s = ja) = \left(\frac{a^2}{s^2 + 2\zeta a s + a^2}\right)_{s=ja} = \left(\frac{a^2}{-a^2 + 2j\zeta a^2 + a^2}\right) = \left(\frac{1}{j2\zeta}\right)$$




Gain vs. Frequency for poles at s = 1 and angles of { 0, 15, 30, 45, 60, 75} degrees

Note that by allowing the poles to be complex, you can build a better low-pass filter.

Example: Specify R, L, and C so that the filter has the transfer function of

$$Y = \left(\frac{100^2}{s^2+140s+100^2}\right)X = \left(\frac{100^2}{\left(s+100\angle 45^0\right)\left(s+100\angle -45^0\right)}\right)X$$

The transfer function of the RLC filter is

$$Y = \left(\frac{1}{LCs^2+RCs+1}\right)X = \left(\frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}\right)X = \left(\frac{100^2}{s^2+140s+100^2}\right)X$$

With 3 degrees of freedom and two constraints, something is arbitrary. Let

R = 100

Then

$$\frac{R}{L} = 140 \qquad L = 0.7143H$$

$$\frac{1}{LC} = 100^2 \qquad C = 140\mu F$$

## High-Pass RLC Filter

If the output is the voltage across the inductor, the gain becomes

$$Y = \left( \frac{Ls}{Ls+R+\frac{1}{Cs}} \right) X$$

$$Y = \left( \frac{LCs^2}{LCs^2+RCs+1} \right) X$$

For this filter

- The high frequency gain is one, and
- The DC gain is zero

## Band-Pass RLC Filter

If the output is the voltage across the resistor, the gain becomes

$$Y = \left( \frac{R}{Ls+R+\frac{1}{Cs}} \right) X$$

$$Y = \left( \frac{RCs}{LCs^2+RCs+1} \right) X$$

For this filter

- The DC gain is zero
- The high-frequency gain is zero
- At $\omega = \frac{1}{\sqrt{LC}}$, the gain is one