Active Filters with Real Poles

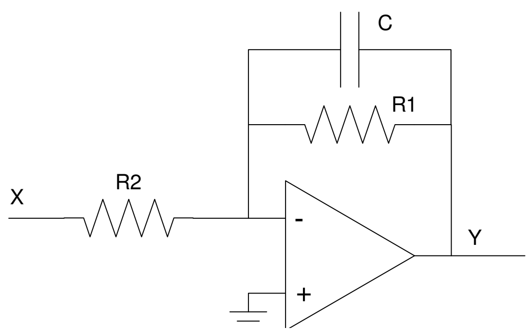
RLC filters work on paper. In practice, the inductors cause major problems due to their resisance. The 10H inductors from Digikey, for example, have a resistance of 278 Ohms. This is a problem with you want the resistance to be only 10 Ohms.

If you use op-amps, you can

- Get real and complex poles without using inductors, and
- The gain can be anything you want: more than one, less than one, positive, or negative.

Single-Pole Active Filter

There are several designs for a single-pole active low-pass filter. The one I like is:



Active Low-Pass Filter: $Y = -\left(\frac{\left(\frac{1}{R_2 C}\right)}{s+\left(\frac{1}{R_1 C}\right)}\right)X = -\left(\frac{a}{s+b}\right)X$

The gain can be found several ways. Using voltage nodes,

$$V_+ = V_- = 0V$$

$$\left(\frac{0-X}{R_2}\right) + \left(\frac{0-Y}{1/Cs}\right) + \left(\frac{0-Y}{R_1}\right) = 0$$

Grouping terms

$$\left(Cs + \frac{1}{R_1}\right)Y = -\left(\frac{1}{R_2}\right)X$$

$$(R_1 R_2 Cs + R_2)Y = -(R_1)X$$

$$Y = \left(\frac{-R_1}{R_1 R_2 Cs + R_2}\right)X$$

or

$$Y = -\left(\frac{\left(\frac{1}{R_2 C}\right)}{s+\left(\frac{1}{R_1 C}\right)}\right)X = -\left(\frac{a}{s+b}\right)X$$

If you forget the transfer function, just take the limits.

By inspection, this is a 1st-order low pass filter of the form

$$Y = -\left(\frac{a}{s+b}\right)X$$

At DC, the capacitor is an open circuit. The gain is then

$$\left(\frac{R_1}{R_2}\right) = \left(\frac{a}{b}\right)$$

The pole is when you switch from resistive to capacitive in the feedback

$$R_1 = \left(\frac{1}{Cs}\right)_{s=jb}$$

$$b = \left(\frac{1}{R_1 C}\right)$$

With this circuit, you can implement any transfer function with

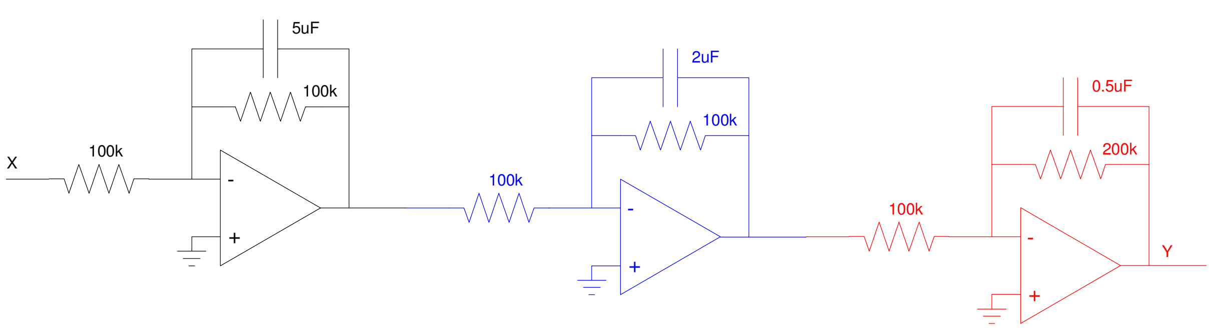
- Real poles, and
- No zeros.

Example: Design a circuit to implement

$$Y = \left(\frac{-200}{(s+2)(s+5)(s+10)}\right)X$$

Solution: Break this into three sections

$$Y = \left(\frac{-2}{s+2}\right)\left(\frac{-5}{s+5}\right)\left(\frac{-20}{s+10}\right)X$$



Stage 1: (black)

Let R1 = 100k

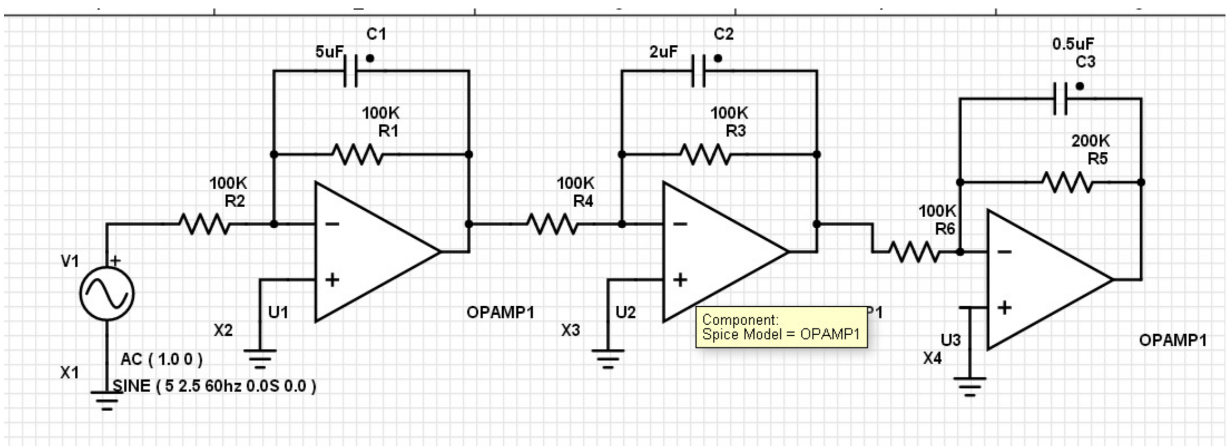
The DC gain is 1.000, so R2/R1 = 1. R2 = 100k.

The pole is at -2

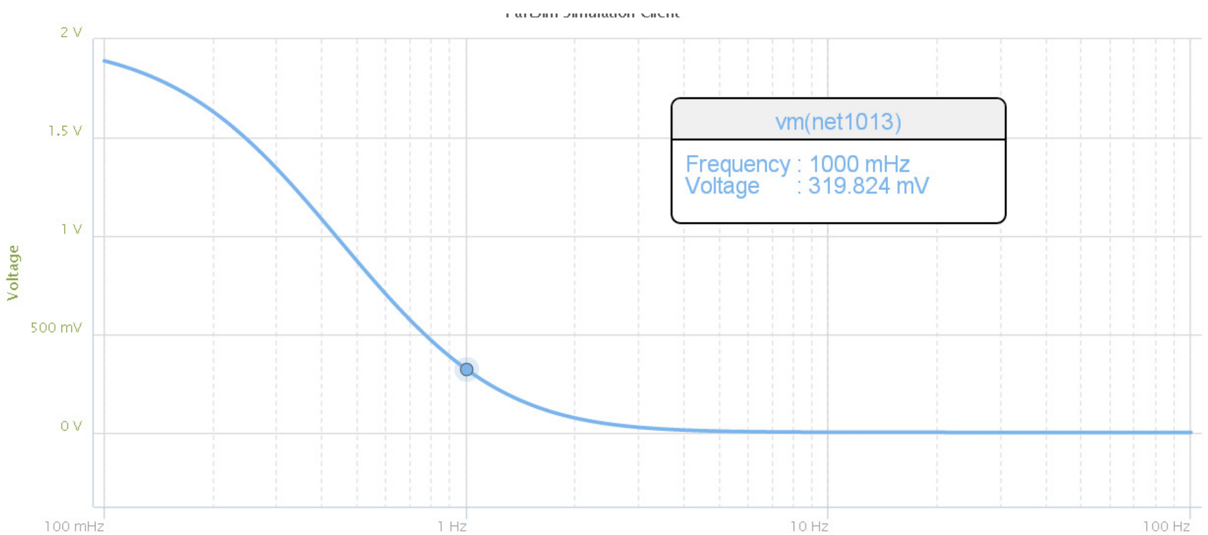
$$\frac{1}{R_1 C} = 2$$

$$C = 5\mu F$$

Checking this design in PartSim: first input the circuit:



then take the frequency response



Doing a point check vs. our calculations at 1Hz (selected somewhat arbitrarily)

$$\left(\frac{-200}{(s+2)(s+5)(s+10)}\right)_{s=j2\pi} = 0.3198\angle 24^0$$

This matches PartSim's result (319.824mV vs. 0.3198V)

Circuit 2:

Another solutionis to add an amplifier to a 3-stage RC filter:

$$Y \approx \left(\left(\frac{2}{s+2}\right)\left(\frac{5}{s+5}\right)\left(\frac{10}{s+10}\right) \cdot 2\right)X$$

To avoid loading, increase the impedance of each stage by 10x.

Pick C so that 1/RC is the pole