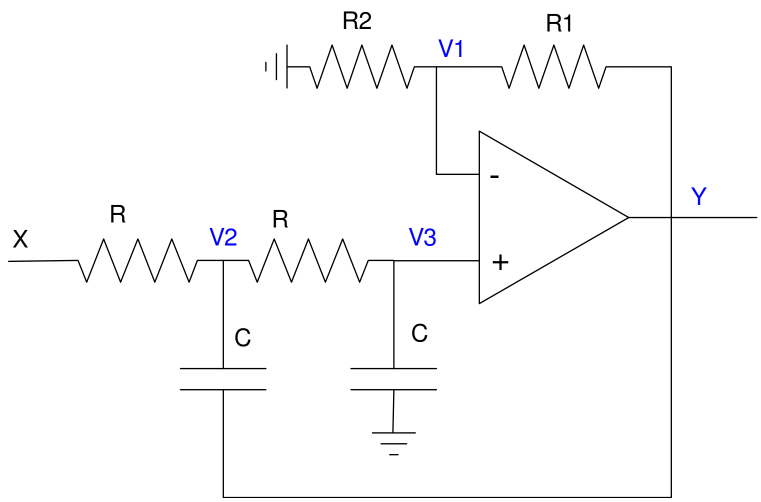
Active Filters with Complex Poles

Complex Poles, No Zeros



$$Y = \left(\frac{k \cdot \left(\frac{1}{RC}\right)^2}{s^2 + \left(\frac{3-k}{RC}\right)s + \left(\frac{1}{RC}\right)^2} \right) X \qquad k = 1 + \frac{R_1}{R_2}$$

To find the transfer function, write the voltage node equations

$$V_1 = V_2 \qquad \text{node equation at Y}$$

$$\left(\frac{V_1}{R_2}\right) + \left(\frac{V_1 - Y}{R_1}\right) = 0 \qquad \text{node equation at V1}$$

$$\left(\frac{V_2 - X}{R}\right) + \left(\frac{V_2 - V_3}{R}\right) + \left(\frac{V_2 - Y}{1/Cs}\right) = 0 \qquad \text{node equation at V2}$$

$$\left(\frac{V_3 - V_2}{R}\right) + \left(\frac{V_3}{1/Cs}\right) = 0 \qquad \text{node equation at V3}$$

Solving (about 40 minutes later) you get

$$Y = \left(\frac{k \cdot \left(\frac{1}{RC}\right)^2}{s^2 + \left(\frac{3-k}{RC}\right)s + \left(\frac{1}{RC}\right)^2} \right) X$$

where

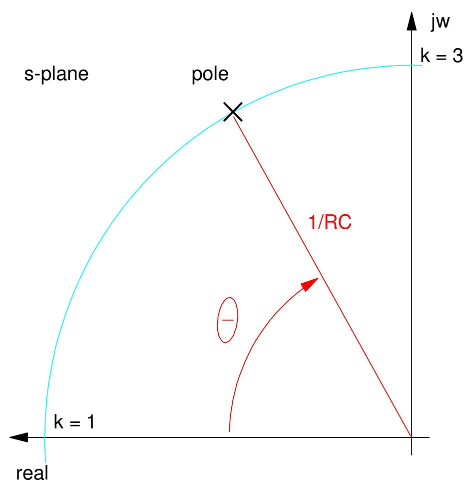
$$k = 1 + \frac{R_1}{R_2}$$

This filter has two complex poles with

- Amplitude = $\frac{1}{RC}$
- Angle: $3 - k = 2\cos\theta$
- DC gain $k = \left(1 + \frac{R_2}{R_1}\right)$

Note that the angle of the poles goes from

- 0 degrees when k =1
- 90 degrees when k = 3 (an oscillator)



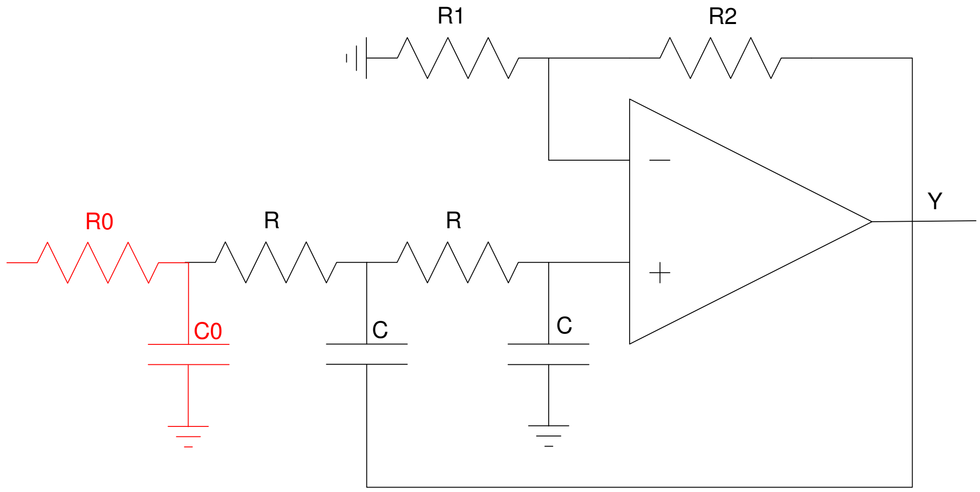
Example: 3rd Order LPF. Design a circuit to implement

$$Y = \left(\frac{1,244,485}{(s+85)(s+121\angle 69.5^0)(s+121\angle -69.5^0)}\right)X$$

Rewrite this as

$$Y = \left(\frac{85}{s+85}\right)\left(\frac{14,641}{(s+121\angle 69.5^0)(s+121\angle -69.5^0)}\right)X$$

Use the previous filters



$$\left(\frac{\left(\frac{1}{R_0 C_0}\right)}{s+\left(\frac{1}{R_0 C_0}\right)}\right)\left(\frac{k\cdot\left(\frac{1}{RC}\right)^2}{s^2+\left(\frac{3-k}{RC}\right)s+\left(\frac{1}{RC}\right)^2}\right)$$

To avoid loading, let

- R0 = 10k
- R = 100k

Matching terms in the denominator:

$$\left(\frac{1}{R_0 C_0}\right) = 85 \qquad C_0 = 1.17\mu F$$

$$\left(\frac{1}{RC}\right) = 121 \qquad C = 0.082\mu F$$

$$3 - k = 2\cos(69.5^0)$$

$$k = 2.3$$

$$1 + \frac{R_2}{R_1} = 2.3$$

R1 = 100k, R2 = 1.3k

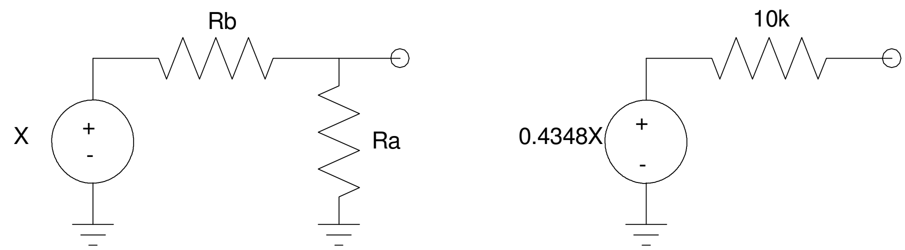
Note: This circuit has a DC gain of 2.3 (instead of 1.0).

- Option 1: Call the output 2.3Y
- Option 2: Reduce the gain by 2.3x to bring the DC gain back to 1.00

The former solution is usually the better solution. Presumably, your circuit will need some gain anyway - this filer provides 2.3x of the gain. The remaining gain comes from some other circuit.

The latter solution is usually better on tests where you want to answer the problem exactly to get full credit.

To attenuate the gain, use a Thevenin equivalent:



Actual Circuit Thevenin Equivalent

To find Ra and Rb:

$$R_a || R_b = 10k$$

$$\left(\frac{R_a R_b}{R_a + R_b}\right) = 10k$$

and

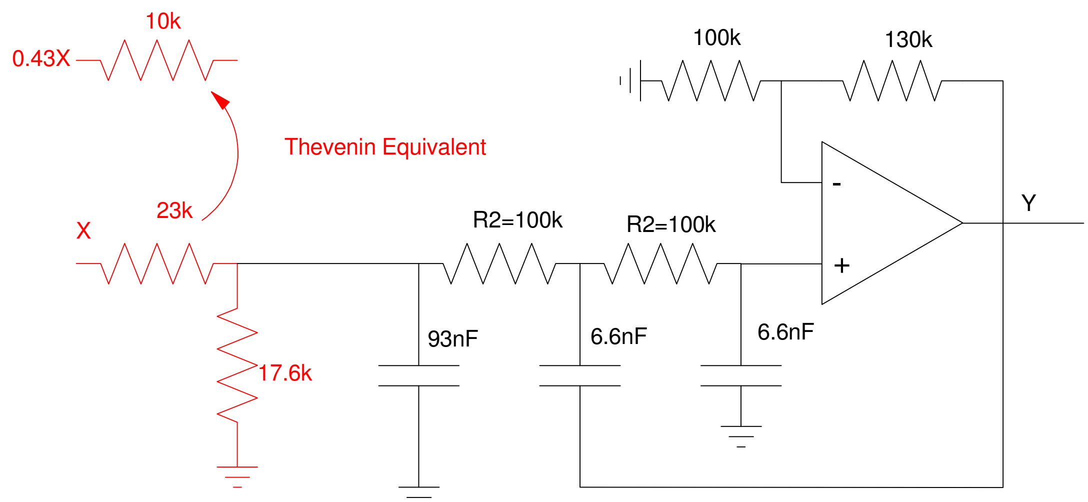
$$\left(\frac{R_a}{R_a + R_b}\right) X = 0.4348X$$

Solving

$$R_b = \left(\frac{1}{0.4348}\right) 10k = 23k$$

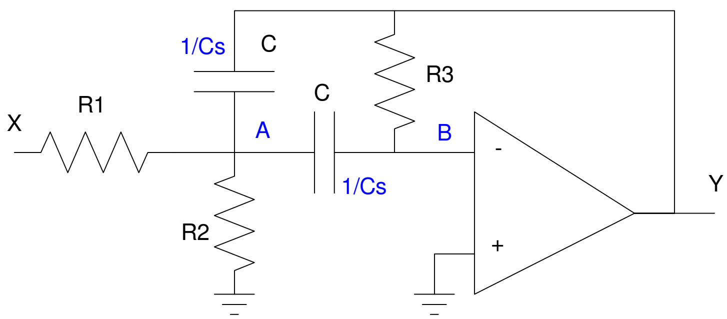
$$R_a = 17.69k$$

making the final circuit:



$$Y = \left(\frac{85}{s+85}\right)\left(\frac{14,641}{(s+121\angle 69.5^0)(s+121\angle -69.5^0)}\right) X$$

Complex Poles, One Zeros at s = 0: $Y = \left(\frac{as}{s^2+bs+c}\right)X$



$$Y = \left(\frac{-\left(\frac{1}{R_1 C}\right)s}{s^2+\left(\frac{2}{R_3 C}\right)s+\left(\frac{R_1+R_2}{R_1 R_2}\right)\left(\frac{1}{R_3 C^2}\right)} \right) X$$

To derive this transfer function, write the voltage node equations

$V_B = 0$ node equation at Y

$\left(\frac{V_A-X}{R_1}\right)+\left(\frac{V_A}{R_2}\right)+\left(\frac{V_A-V_B}{1/Cs}\right)+\left(\frac{V_A-Y}{1/Cs}\right)=0$ node equation at A

$\left(\frac{V_B-V_A}{1/Cs}\right)+\left(\frac{V_B-Y}{R_3}\right)=0$ node equation at B

Solving (about 40 minutes later) gives

$$Y = \left(\frac{-\left(\frac{1}{R_1 C}\right)s}{s^2+\left(\frac{2}{R_3 C}\right)s+\left(\frac{R_1+R_2}{R_1 R_2}\right)\left(\frac{1}{R_3 C^2}\right)} \right) X$$

Example: Design a filter to implement

$$Y = \left(\frac{4s}{s^2+2s+200}\right)X$$

Solution: There are 3 constraints and 4 degrees of freedom. Something is arbitrary, so let

$$C = 1\mu F$$

Now match terms

$\left(\frac{1}{R_1 C}\right) = 4$ R1 = 250k

$$\left(\frac{2}{R_3 C}\right) = 2$$

R3 = 1M

$$\left(\frac{R_1+R_2}{R_1 R_2}\right)\left(\frac{1}{R_3 C^2}\right) = 200$$

$$\left(\frac{R_1 R_2}{R_1+R_2}\right) = R_1 || R_2 = \left(\frac{1}{200 R_3 C^2}\right)$$

$$R_1 || R_2 = 5k$$

R2 = 5102