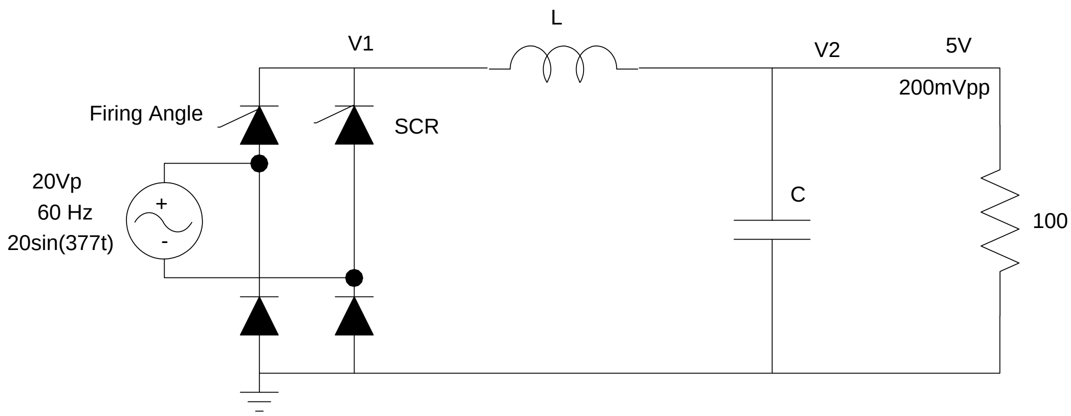
ECE 320 - Homework #6

DC to AC Converter, SCR. Due Monday, October 3rd

SCR (variation 1)



1) Determine the firing angle so that the mean of V1 is 5V. Time isn't important for an average, so let the period be pi:

$$\frac{1}{\pi}\int_{\theta}^{\pi+\theta}(20-1.4)\sin(t)\cdot dt = 5$$

$$(\cos(t))_{\theta}^{\pi+\theta} = \left(\frac{5\cdot\pi}{18.6}\right)$$

$$2\cos(\theta) = 0.8445$$

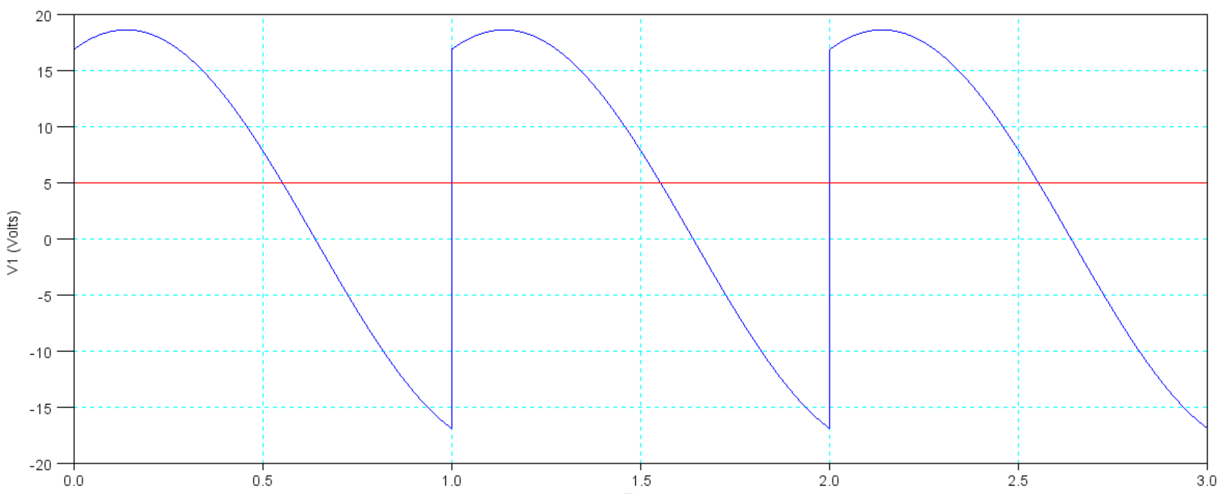
$$\theta = 65.02^{0}$$

Checking in Matlab:

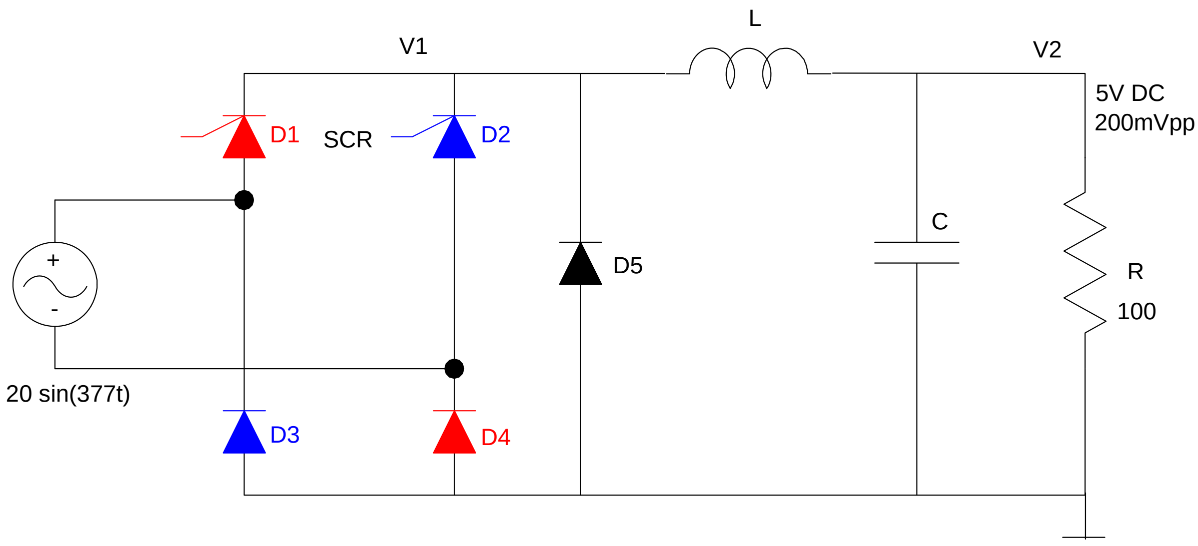
```
-->t = [0:0.001:1]';
-->V1 = 18.6*sin(pi*t + 65.0228*pi/180);
-->mean(V1)

    4.9950068

-->plot(t,V1)
```



SCR (variation 2): Add diode D5: This clips V1 at -0.7V



1) Determine the firing angle so that the mean of V1 is 5V. Time isn't important for an average, so let the period be pi:

$$5V = \frac{1}{\pi}\int_{\theta}^{\pi} 18.6 \cdot \sin(t) \cdot dt$$

$$5V = \frac{18.6}{\pi}(-\cos(t))_{\theta}^{\pi}$$

$$5V = \frac{18.6}{\pi}(1 + \cos\theta)$$

$$\theta = 98.94^0$$

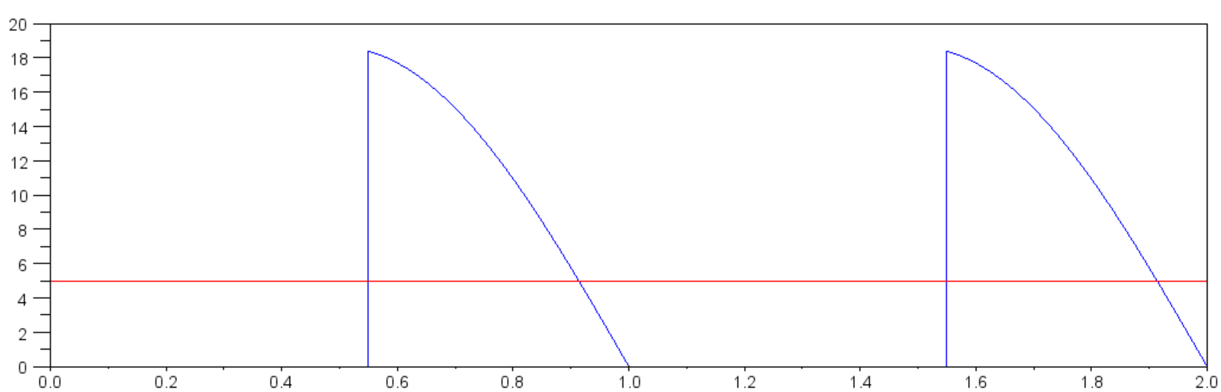
Time delay:

$$T_d = \left(\frac{98.94^0}{360^0}\right)\left(\frac{1}{60s}\right) = 4.581ms$$

You want to fire the SCR's 4.581ms after the zero crossing at Vin. Checking in Matlab:

```
-->t = [0:0.001:1]';
-->V1 = 18.6*sin(pi*t) .* (t > 0.5497);
-->plot(t,V1)
-->mean(V1)

  4.9985665
```



2) Find L so that the ripple at V2 is 2Vpp assuming C = 0. Use the second circuit:

First, find the peak-to-peak voltage at V1:

```
-->V1pp = max(V1) - min(V1)

     18.6
```

Pick L so that this is reduced by 9.3 times

$$\left(\frac{18.6V_{pp}}{2V_{pp}}\right) = 9.3$$

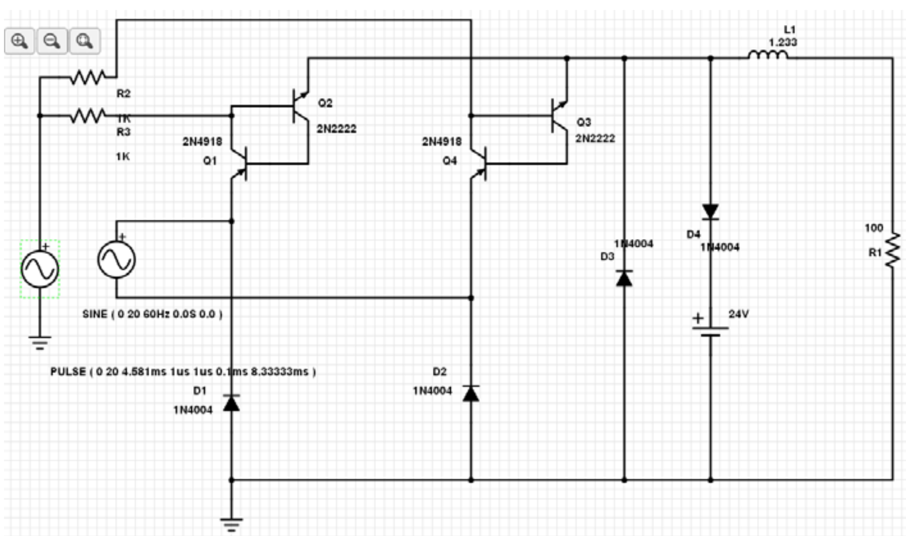
so

$$j\omega L = 9.3 \cdot R$$

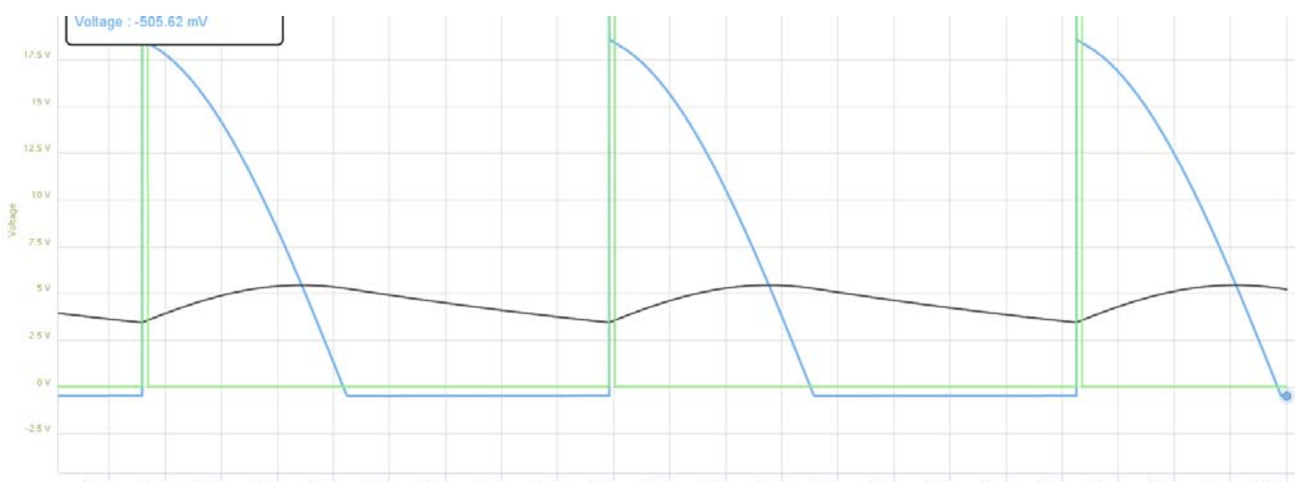
$$(2\pi \cdot 120Hz) \cdot L = 930$$

$$L = 1.233H$$

Checking in PartSim



PartSim Circuit with a firing angle of 98.43 degrees



PartSim Simulation: Firing (green), V1 (blue), V2 (black). 3.431V < V2 < 5.430 (1.999Vpp ripple vs. 2Vpp designed)

3) Find C so that the ripple at V2 is reduced to 200mVpp

The ripple is to be reduced by 10x from 2Vpp to 200mVpp. To do this, pick C so that Zc = 1/10 R

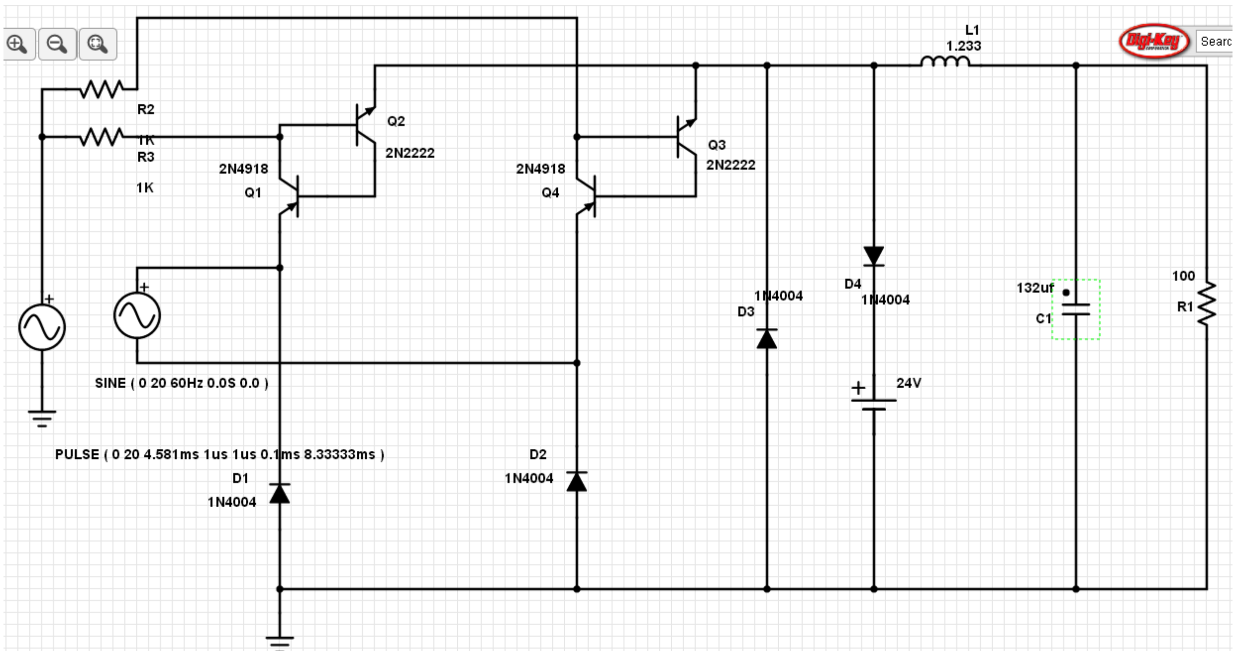
$$Z_c = \frac{1}{10} \cdot R$$

$$\frac{1}{j\omega C} = \frac{1}{10} \cdot 100$$

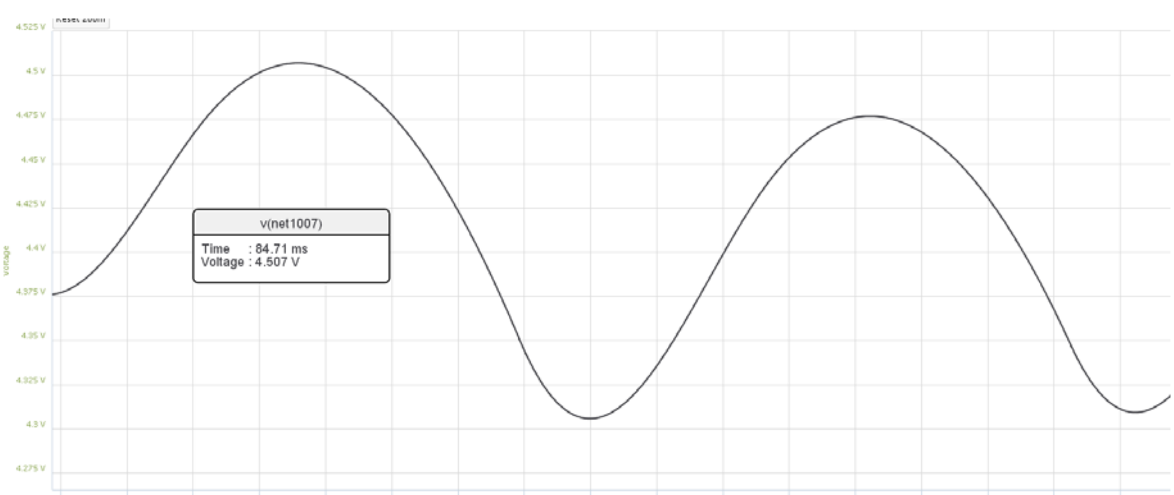
$$\frac{1}{C} = 10 \cdot 2\pi \cdot 120Hz$$

$$C = 132\mu F$$

Checking in PartSim (with a zero degree firing angle since I don't know how to insert a 65 degree firing angle)



PartSim Circuit with Diodes for the SCR's (meaning the firing angle is zero degrees)



Ripple at V2: 4.305 < V2 < 4.477V (172mVpp ripple vs. 200mVpp calculated)

4) Simulate this circuit with a firing angle of zero degrees (making the SCR just a diode).

Done previously.

What is the DC voltage at V2? Why isn't it 5V any more?

The DC voltage is 4.39V (vs. 5.00V computed).

The error is partially due to V1 being -0.7V when diode D5 is on (55% of the time).

This 0.7V offset when D5 is on will shift the DC voltage by 385mV

0.7V * (54.88% on time) = 385mV

If you remove this effect, the DC signal at V2 becomes 4.77V (vs. 5.00V)

V2 is still off slightly - probably due to the diodes D1 .. D4 having turn-on voltages slightly more than 0.7V.

What is the AC voltage at V2 (V2pp)?

The ripple is 172mVpp (rather than 200mVpp computed).

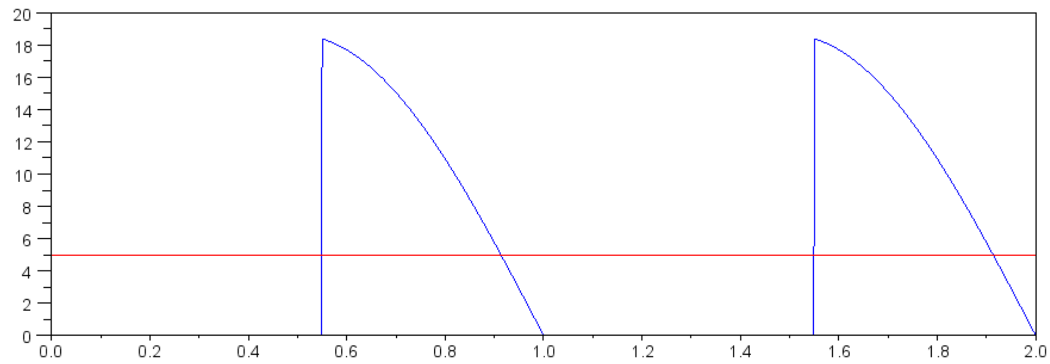
The AC component of V1 isn't exactly 18.6Vpp (see problem 5). Likewise, our computations will be a little off.

AC to DC

Problem 5) Find the Fourier transform for the signal at V1 with the firing angle you computed in problem 1.

$$V_1(t) \approx V_{DC} + \sum \left(a_n \sin(n\omega_o t) + b_n \cos(n\omega_o t)\right)$$

Note: The signal at V1 looks like the following:



Signal at V1 (blue) and its average (DC) value (red)

This signal makes analysis difficult. To simplify the problem we replaced V1(t) with a signal which has

- The same DC signal (5.00V)
- The same frequency (120Hz), and
- The same peak-to-peak value (18.6Vpp)

or

$$V_1(t) \approx 5.00 + 18.6 \cdot \left(\frac{\sin(754t)}{2}\right)$$

There is a more accurate way to do this: take the Fourier Transform of V1(t). Since V1(t) is periodic, we know that

$$V_1(t) = b_0 + \sum \left(a_n \sin(n\omega_0 t) + b_n \cos(n\omega_0 t)\right)$$

where

$$\omega_o = 120Hz = 754 \ \frac{rad}{\sec}$$

b0 is the DC term:

$$b_0 = mean(V_1) = 5.00V$$

an and bn can be computed using

$$a_n = \frac{\int (V_1(t) \cdot \sin(n\omega_o t))dt}{\int \left(\sin^2(n\omega_o t)\right) dt} \approx \frac{\sum V_1(t) \cdot \sin(n\omega_o t)}{\sum \sin^2(n\omega_o t)}$$

$$b_n = \frac{\int (V_1(t) \cdot \cos(n\omega_o t))dt}{\int \left(\cos^2(n\omega_o t)\right) dt} \approx \frac{\sum V_1(t) \cdot \cos(n\omega_o t)}{\sum \cos^2(n\omega_o t)}$$

To make computations easier, change the time axis so that $\omega_0 = 2\pi$ (meaning time goes form 0 to1). In Matlab

```
-->t = [0:0.001:1]';
-->V1 = 18.6*sin(pi*t) .* (t > 0.5488);          ( 98 degrees is 54.88% of 180 deg )
-->w0 = 2*pi;


-->for n=1:20
-->   a(n) = sum(V1 .* sin(w0*n*t)) / sum( sin(w0*n*t) .^ 2);
-->   b(n) = sum(V1 .* cos(w0*n*t)) / sum( cos(w0*n*t) .^ 2);
-->   end
```

This results in

harmonic	a(n)	b(n)
DC	-	5.0000000
1.	- 7.6117514	- 2.1423914
2.	2.6064825	- 2.507786
3.	- 1.2361068	1.2417294
4.	0.5043413	- 1.5855636
5.	- 0.0370832	1.0616346
6.	- 0.2734492	- 1.0257783
7.	0.4709152	0.6348440
8.	- 0.5784839	- 0.4984274
9.	0.6122396	0.1876071
10.	- 0.5860404	- 0.0527018

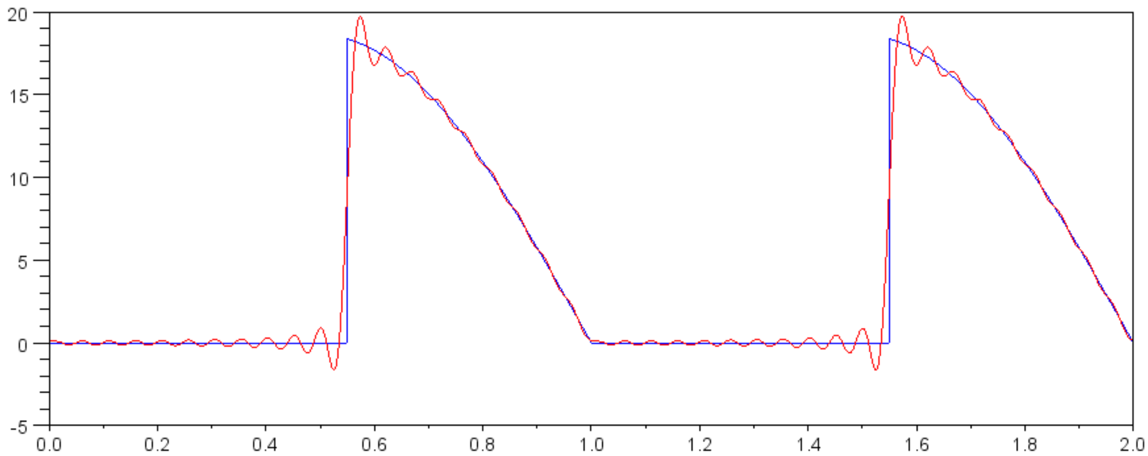
What this means is that a better approximation for V1(t) would be

$$\begin{array}{lcr} V_1(t) = & 5.00+ & \text{DC} \\ & -7.61\sin(754t) - 2.14\cos(754t) & 120\text{Hz} \\ & +2.60\sin(1508t) - 2.50\cos(1508t) & 240\text{Hz} \\ & -1.23\sin(2262t) + 1.24\cos(2262t) & 360\text{Hz} \\ & +0.504\sin(3016t) - 1.58\cos(3016t) & 480\text{Hz} \\ & \vdots & \end{array}$$

or if you prefer polar corrdinates:

$$\begin{array}{lcr} V_1(t) = & 5.00+ & \text{DC} \\ & +7.90\cos(754t + 105^0) & 120\text{Hz} \\ & +3.61\cos(1508t - 133^0) & 240\text{Hz} \\ & +1.75\cos(2262t + 44^0) & 360\text{Hz} \\ & +1.66\cos(3016t + 162^0) & 480\text{Hz} \\ & \vdots & \end{array}$$

If you plot V1(t) vs. it Fourier approximation, you can see that as you add more and more terms, the series expansion approaches V1(t):

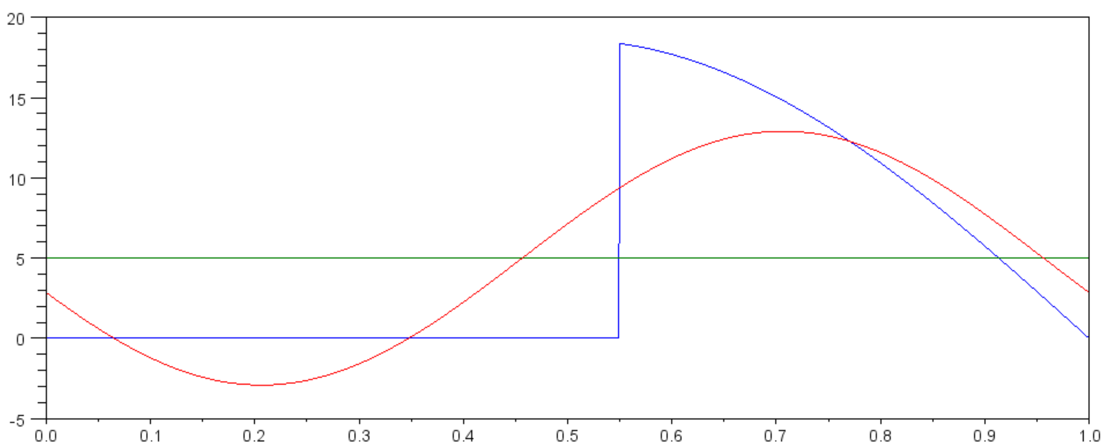


Signal at V1 (blue) and its Fourier Approximation out to its 20th harmonic (red)

If we only take two terms in this expansion (DC and 120Hz), a better approximation for V1(t) would be

$$V_1(t) \approx 5.00 + 7.90\cos(754t + 105^0)$$

which has a ripple of 15.8Vpp (vs. 18.6Vpp we assumed previously).



Signal at V1 (blue) and its Fourier Approximation using only two terms (DC and 120Hz)

Moral: There are methods which allow you to compute the peak-to-peak ripple at V2 more accurately than what we're doing (i.e. Fourier Transform)

- They're a lot more difficult to use, and
- The improvement in the results are not that much.

If you want to get a better estimate for the actual ripple at V2, use PartSim.

Problem 6) If you ignore the DC term, what percentage of the energy is in the 1st harmonic?

In other words, how good is this approximation (using only the DC and 1st harmonic)?

The energy in the signal at V1 for each harmonic is

$$E = \frac{1}{2}V_p^2 = \frac{1}{2}(a_n^2 + b_n^2)$$

N	V1(Watts)	% of total
0.	25.	36.85437
1.	31.2643	46.089044
2.	6.5413708	9.6431241
3.	1.534926	2.2627492
4.	1.384186	2.0405322
5.	0.5642216	0.8317613
6.	0.5634978	0.8306943
7.	0.3123940	0.4605234
8.	0.2915367	0.4297761
9.	0.2050169	0.3022307
10.	0.1731104	0.2551950

If you include the DC term and the 120Hz term (1st harmonic), you've captured 73% of the energy in the signal at V1.

ans: 73%

It's actually better than that. The RLC filter is a low-pass filter, attenuating the high-frequency terms with a gain of:

$$V_2 = \left(\frac{R || \frac{1}{j\omega C}}{R || \frac{1}{j\omega C} + j\omega L} \right) V_1$$

If you include the gain of the RLC filter to find V2, you get the following:

harmonic	gain	V2(Watts)	% total
DC	1.	25.	99.985023
1.	0.0108695	0.0036938	0.0147729
2.	0.0027057	0.0000479	0.0001915
3.	0.0012016	0.0000022	0.0000089
4.	0.0006757	0.0000006	0.0000025
5.	0.0004324	0.0000001	0.0000004
6.	0.0003003	5.080D-08	0.0000002
7.	0.0002206	1.520D-08	6.079D-08
8.	0.0001689	8.315D-09	3.325D-08
9.	0.0001334	3.650D-09	1.460D-08
10.	0.0001081	2.022D-09	8.087D-09

The first two terms (DC and 120Hz) contain 99.9998% of the energy in V2(t). That's usually good enough.