Complex Numbers and Phasors - January 26, 2017

1) For the following circuit, determine the phasor impedance for each element and the voltage, y(t)

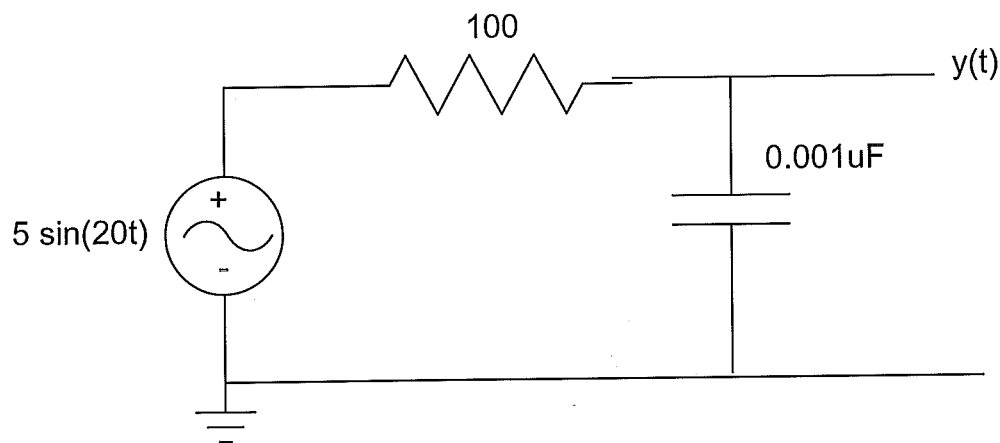| Phasor Representation for ...		
Vin = 5 cos(20t)	2H Inductor	Y
$5 + j0$	$j40$	$2.45\angle{-11°}$ $2.40 - j\,0.488$
y(t) = $2.45\cos(20t - 11°)$ $2.40\cos(20t) + 0.488\sin(20t)$		
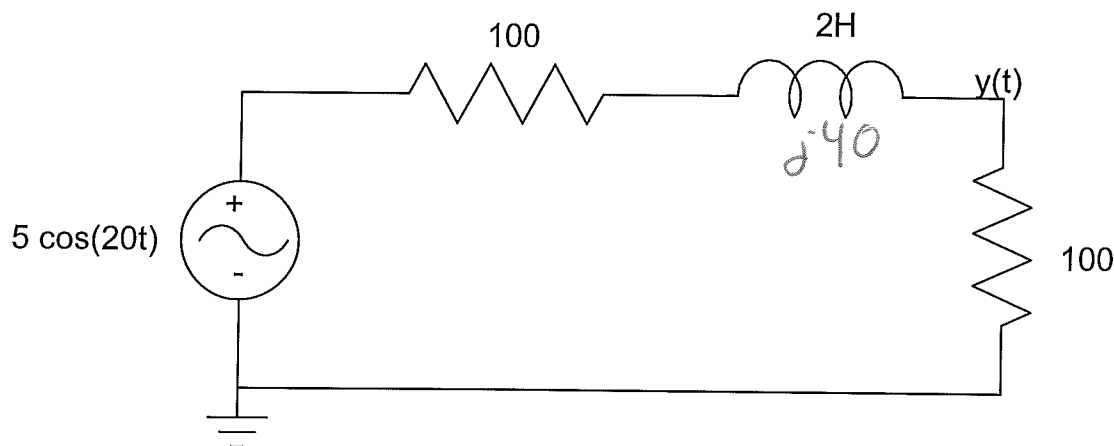
$$Y = \left(\frac{100}{100 + 100 + j40}\right) 5$$

$$= 2.45\angle{-11°}$$

2) For the following circuit, determine the phasor impedance for each element and the voltage, y(t)

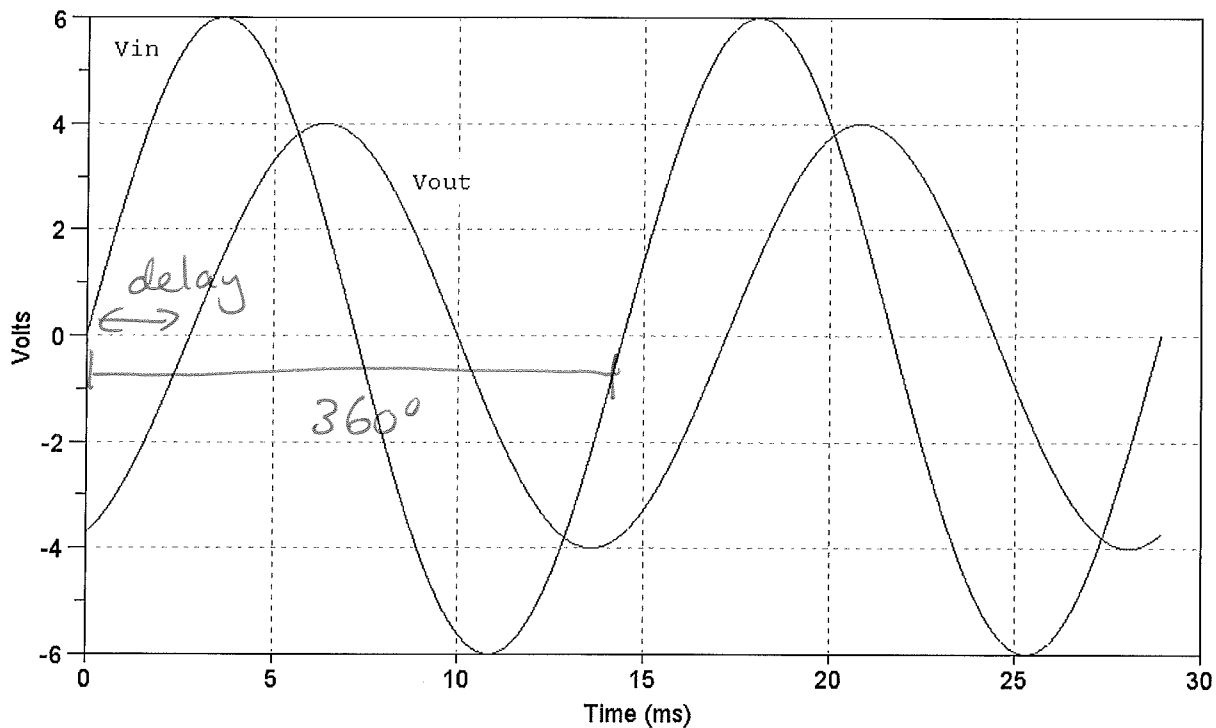| Phasor Representation for ...		
Vin = 5 sin(20t)	0.001F Capacitor	Y
$-j5$	$-j50$	$-2-j$ $2.236 \angle -153°$
y(t) =	$-2\cos(20t) + \sin(20t)$ $2.236 \cos(20t - 153°)$	

100

y(t)

0.001uF

5 sin(20t)   +
              −

$$Y = \left(\frac{-j50}{-j50 + 100} \right)(-j5)$$

$$Y = 2.236 \angle -153°$$

3) The input and output voltage as seen on an oscilloscope are shown below. Determine the following:

Gain from Vin to Vout	Phase Shift from Vin to Vout	Frequency (Hz)
$\dfrac{4}{6}$	$-61°$	$71 Hz$

$$period = 14 ms$$

$$f = \frac{1}{period} = 71 Hz$$

$$phase = -\left(\frac{12mm}{70mm}\right) 360° = -61°$$

4) A circuit has the following transfer function:

$$Y = \left(\frac{10s+3}{s^2+15s+100}\right)X$$

Find y(t) assuming

$$x(t) = 100 + 50\cos(10t)$$

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y(t) = $3 + 33.3 \cos\left(10t - 1.7°\right)$

$X(t) = 100$

$S = 0$

$\left(\frac{100+3}{- - \cdot}\right) = \frac{3}{100}$

$y = \left(\frac{3}{100}\right) 100$

$y = 3$

$X = 50 \cos(10t)$

$S = j10$

$\left(\frac{10s+3}{s^2+15s+100}\right)_{s=j10}$ $= .667\angle -1.7°$

$y = \left(.667\angle -1.7°\right)\left(50\cos(10t)\right)$

$y = 33.3 \cos(10t - 1.7°)$

5) A circuit has the following transfer function

$$Y = \left(\frac{10s+3}{s^2+15s+100} \right) X$$

What is the differential equation relating X and Y?

$$y'' + 15y' + 100y = 10x' + 3x$$

Bonus! It takes 109 million pounds of coal producing 327 million pounds of CO2 to power 300 million cell phones for a year.
- How many pounds of Uranium does it take?
- How much CO2 does this Uranium produce?

2-3 million : 1 ratio

36 to 54 pounds of uranium