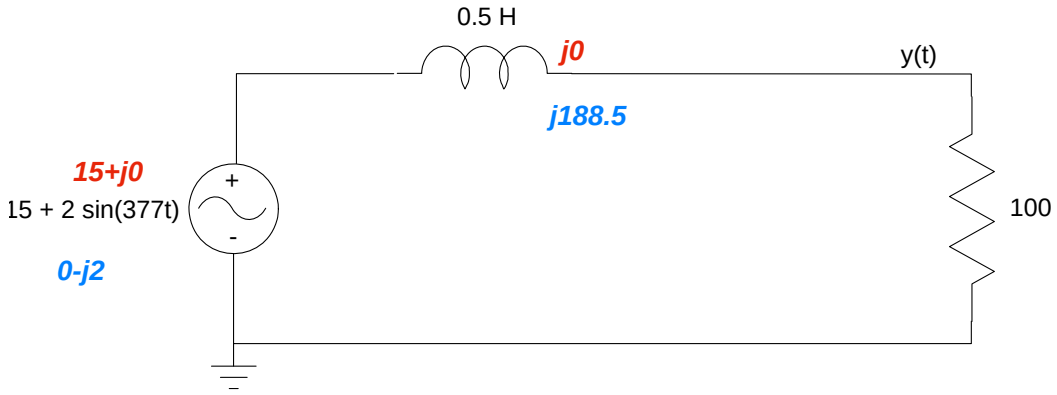
ECE 320 - Homework #2

Phasors Review. Due Monday January 23rd, 2017

1a) Convert the following circuit to phasor notation



Phasor Representation of Circuit at w = 0 (red) and w = 377 (blue)

1b) Solve for y(t) using phasors.

Vin = 15:

w = 0

$L \rightarrow j\omega L = j0$

By voltage division:

$$y = \left(\frac{100}{100+j0}\right) \cdot 15 = 15$$

Vin = 2 sin(377t)

w = 377

$L \rightarrow j\omega L = j188.5$

$V_{in} = 2\sin(377t) \rightarrow 0 - j2$

by voltage division

$$Y = \left(\frac{100}{100+j188.5}\right) \cdot (0 - j2)$$

$$Y = 0.9373\angle -152^0$$

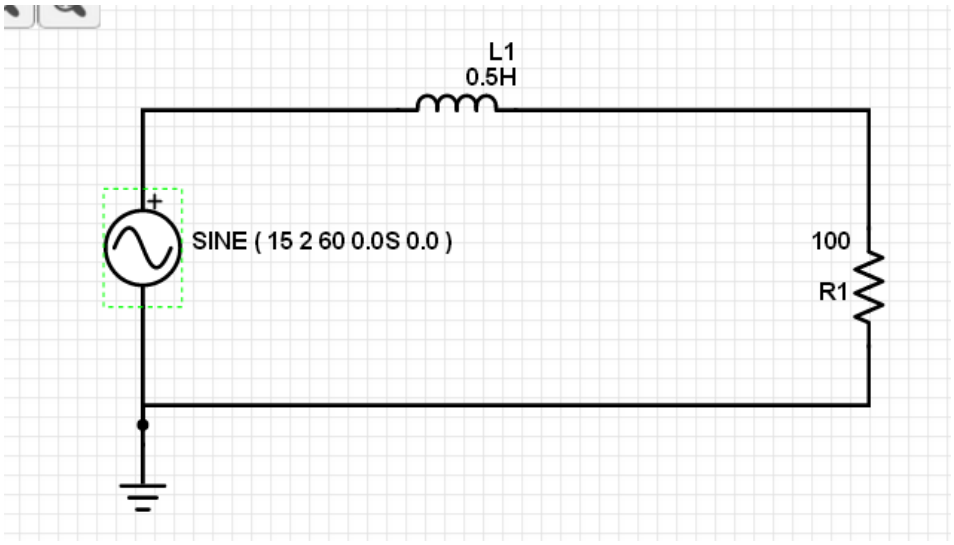
$$y(t) = 0.9373\cos(377t - 152^0)$$

To get the total input, add the two inputs together.

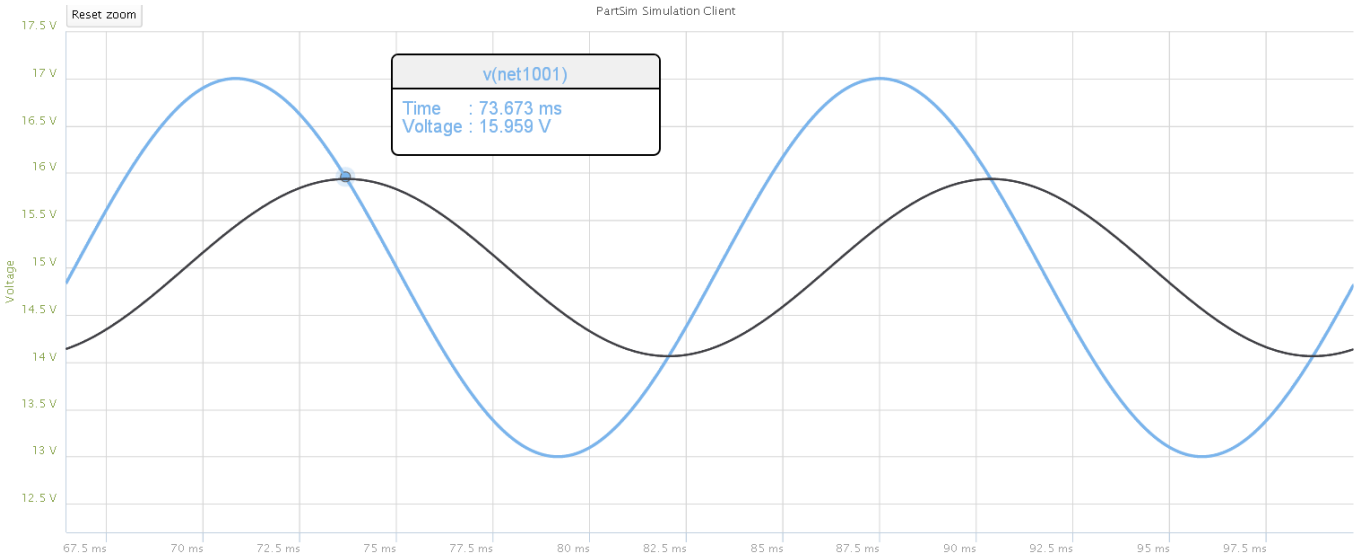
To get the total output, add the two outputs together

$$\boxed{y(t) = 15 + 0.9373\cos(377t - 152^0)}$$

2) Check your answer in PartSim.



PartSim Circuit



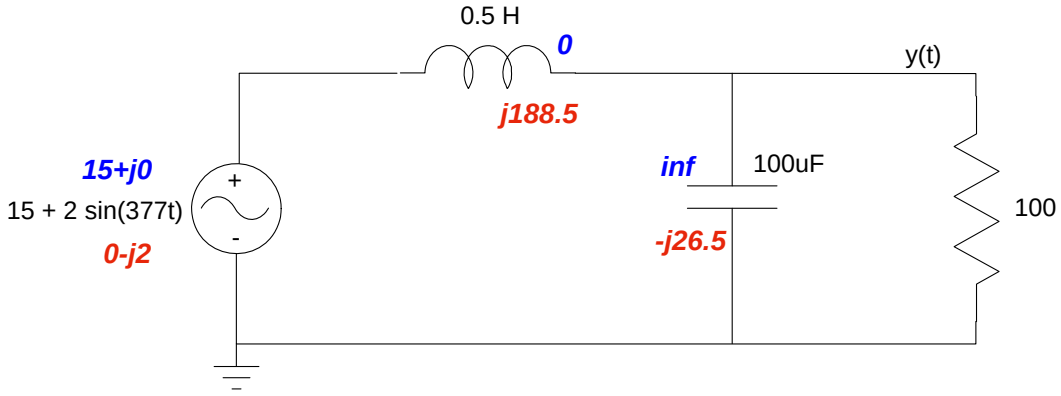
PartSim Transient Response. Vin (blue) and Y (black)

As expected

- The DC level of Vout = 15V
- The peam if 0.973V above the DC level (0.9373 computed)
- The output is a cosine delayed by about 135 degres (-152 degrees computed)

3a) Convert the following circuit to phasor notation

3b) Solve for y(t) using phasors.



Phasor Representation at w = 0 (blue) and w = 377 (red)

Vin = 15

w = 0

$L \rightarrow j\omega L = j0$

$C \rightarrow \frac{1}{j\omega C} = -j\infty$

By voltage division

$$Y = \left(\frac{100}{100+j0}\right)15 = 15$$

Vin = 2sin(377t)

w = 377

$V_{in} = 2\sin(377t) \rightarrow 0 - j2$

$L \rightarrow j\omega L = j188.5$

$C \rightarrow \frac{1}{j\omega C} = -j26.5$

By voltage division

$$-j26.5 || 100 = 25.63\angle -75^0$$

$$Y = \left(\frac{25.63\angle -75^0}{25.63\angle -75^0 + j188.5}\right) \cdot (0 - j2) = 0.313\angle 107^0$$

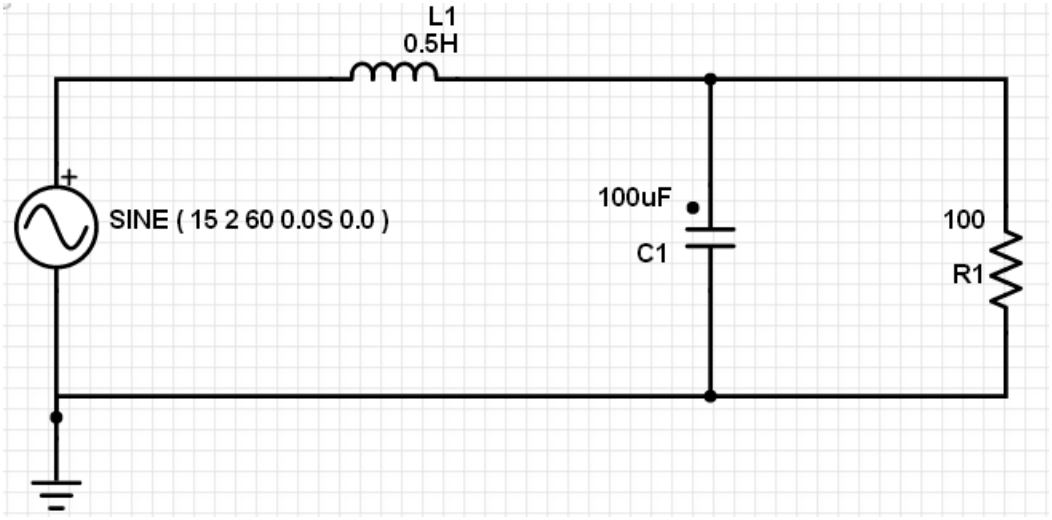
$$y(t) = 0.313\cos(377t + 107^0)$$

To get the total input, add the two inputs together.

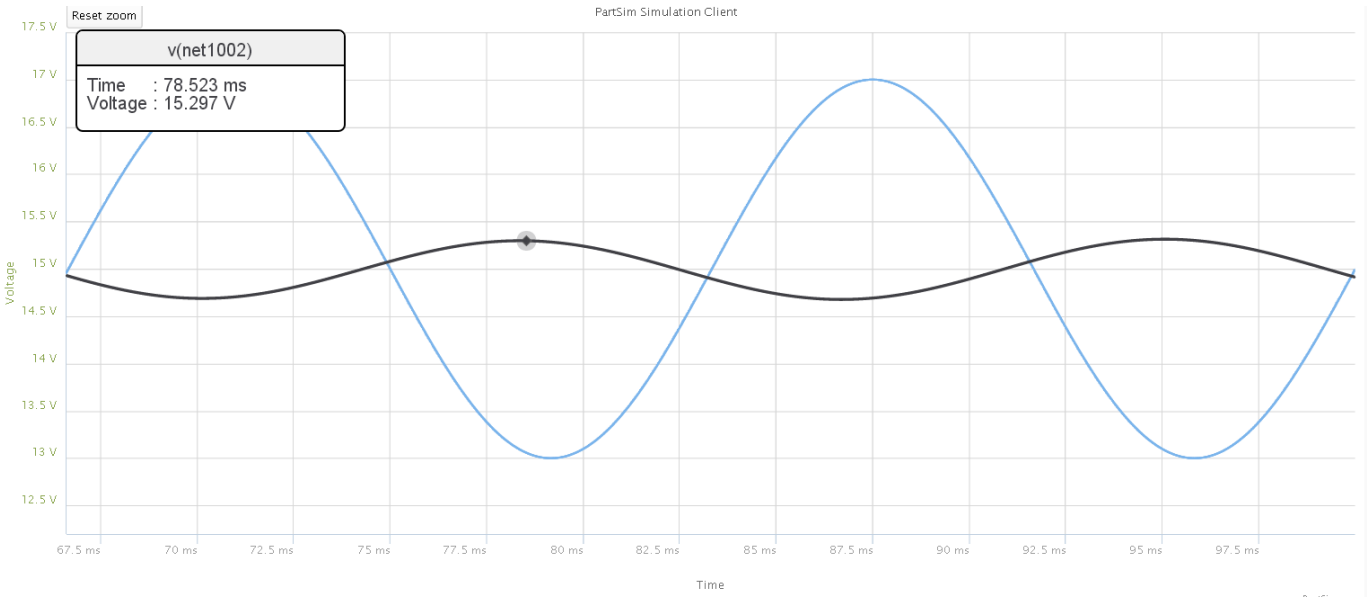
To get the total output, add the two outputs together

$$y(t) = 15 + 0.313\cos(377t + 107^0)$$

4) Check your answer in PartSim.



PartSim Circuit



PartSim Transient Response showing Vin (blue) and Y (black)

As expected

- The DC level of Vout = 15V
- The peam if 0.297V above the DC level (0.313 computed)
- The output is a cosine delayed by about 260 degres (-253 degrees = +107 degrees computed)

5) Determine the differential equation which satisfies the following transfer function

$$Y = \left(\frac{100}{(s+3)(s+10)}\right)X$$

Multiply out and cross multiply

$$(s^2 + 13s + 30)Y = 100X$$

meaning

$$\frac{d^2y}{dt^2} + 13\frac{dy}{dt} + 30y = 100x$$

or using prime notation

$$y'' + 13y' + 30y = 100x$$

6) Determine y(t) assuming x(t) is

$$x(t) = 3 + 4\cos(5t)$$

Use superposition

x(t) = 3

s = 0

$$\left(\frac{100}{(s+3)(s+10)}\right)_{s=0} = 3.333$$

Output = gain * input

$$Y = (3.333) \cdot 3 = 10$$

y(t) = 4 cos(5t)

s = j5

$$\left(\frac{100}{(s+3)(s+10)}\right)_{s=j5} = 1.533\angle -85.6^0$$

Output = gain * input

$$Y = (1.533\angle -85.6^0) \cdot (4\cos(5t))$$

$$y(t) = 6.13\cos(5t - 85.6^0)$$

Add the two inputs together to get the total input

Add the two outputs together to get the total output

$$y(t) = 10 + 6.13\cos(5t - 85.6^0)$$