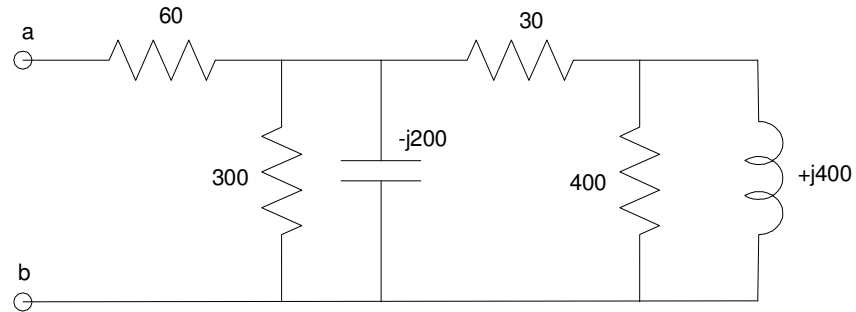


# ECE 320 - Homework #1

EE 206 Review, Phasors. Due Wednesday, January 18th

## Resistors in series and parallel

1) Determine the impedance:  $Z_{ab}$



$$400 \parallel j400 = 200 + j200$$

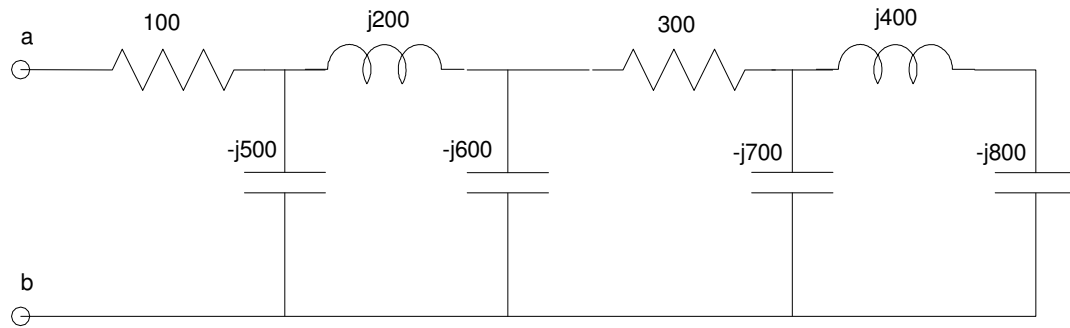
$$(200 + j200) + (30) = 230 + j200$$

$$(230 + j200) \parallel (-j200) \parallel (300) = 138.801 - j68.029$$

$$(138.801 - j68.029) + (60) = 198.801 - j68.029$$

**ans:  $Z_{ab} = 198.801 - j68.029$**

2) Determine the resistance  $R_{ab}$  (it will be a complex number)



$$(j400) + (-j800) = -j400$$

$$(-j400) \parallel (-j700) = -j254.545$$

$$(-j254.545) + (300) = 300 - j254.545$$

$$(300 - j254.545) \parallel (-j600) = 131.668 - j224.947$$

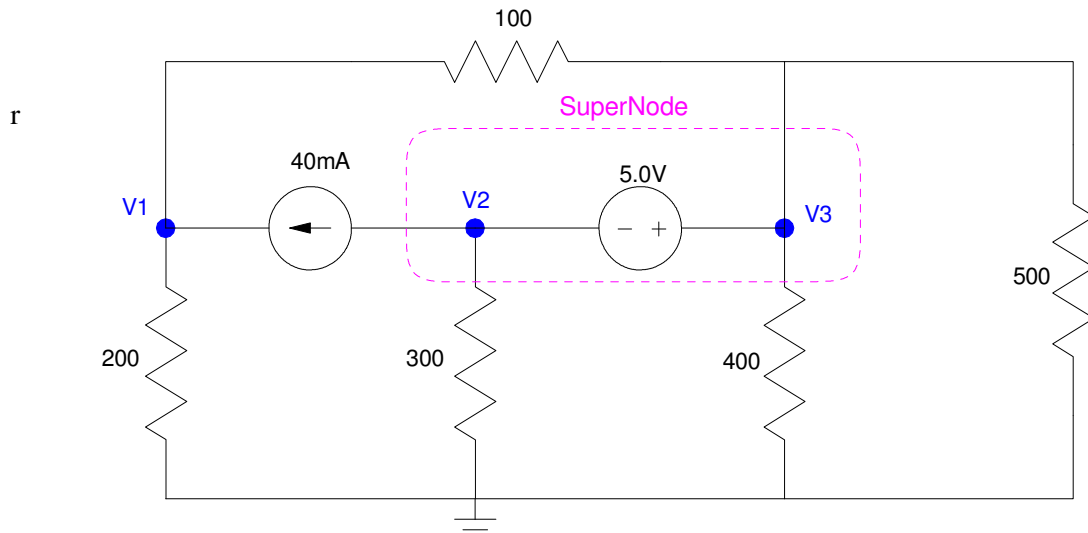
$$(131.668 - j224.947) + (j200) = 131.668 - j24.947$$

$$(131.668 - j24.947) \parallel (-j500) = 112.380 - j51.949$$

$$(112.380 - j51.949) + (100) = 212.380 - j51.949$$

**ans:  $Z_{ab} = 212.380 - j51.949$**

3) (Voltage Nodes): For the following circuit



a) Write the voltage node equations

Start with the easy ones (the voltage source)

$$V_3 - V_2 = 5.0$$

Add two more equations. At node V1

$$\left(\frac{V_1}{200}\right) - 40mA + \left(\frac{V_1 - V_3}{100}\right) = 0$$

Supernode (other supernodes also work)

$$40mA + \left(\frac{V_2}{300}\right) + \left(\frac{V_3 - V_1}{100}\right) + \left(\frac{V_3}{400}\right) + \left(\frac{V_3}{500}\right) = 0$$

b) Solve using Matlab (or similar program)

Group terms

$$V_3 - V_2 = 5$$

$$\left(\frac{1}{200} + \frac{1}{100}\right)V_1 - \left(\frac{1}{100}\right)V_3 = 40mA$$

$$-\left(\frac{1}{100}\right)V_1 + \left(\frac{1}{300}\right)V_2 + \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{500}\right)V_3 = -40mA$$

Place in matrix form

$$\begin{bmatrix} 0 & -1 & 1 \\ \left(\frac{1}{200} + \frac{1}{100}\right) & 0 & \left(\frac{-1}{100}\right) \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{300}\right) & \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{500}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0.04 \\ -0.04 \end{bmatrix}$$

Solve using Matlab

```
snip>> A = [0,-1,1 ; 1/200+1/100,0,-1/100 ; -1/100,1/300,1/100+1/400+1/500]
```

```
      0      -1.0000      1.0000
0.0150         0     -0.0100
-0.0100      0.0033      0.0145
```

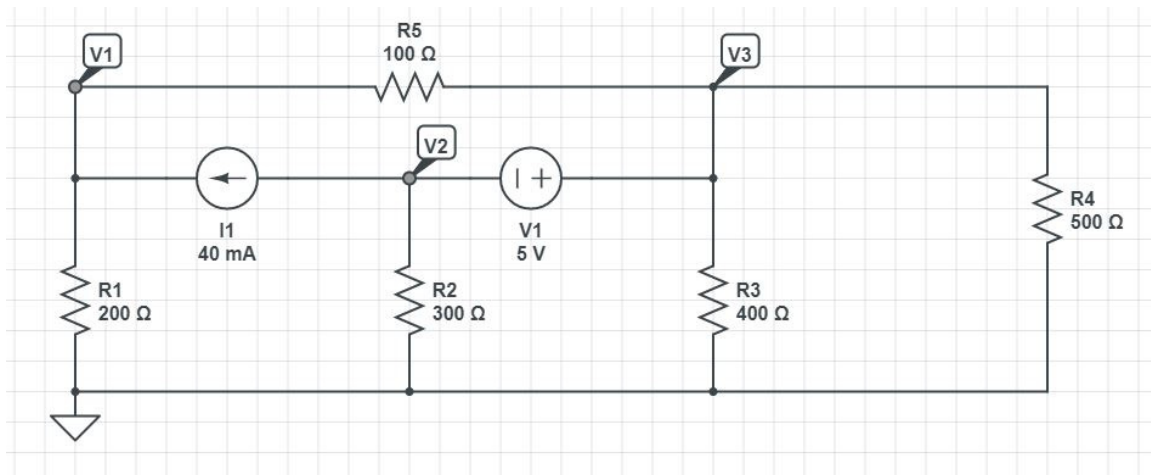
```
>> B = [5;0.04;-0.04]
```

```
      5.0000
      0.0400
     -0.0400
```

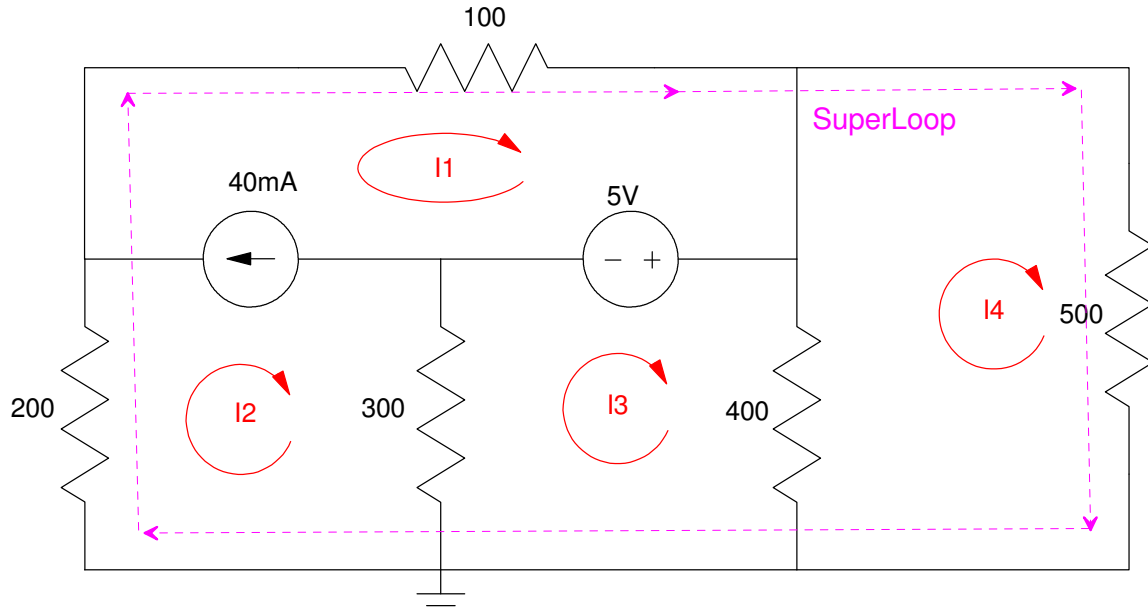
```
>> V = inv(A)*B
```

```
V1      2.8657
V2     -4.7015
V3      0.2985
```

c) Check your answers in CircuitLab (or similar circuit simulator)



4) (Current Loops) For the following circuit



a) Write the current loop equations

Start with the easy one (the current source)

$$I_1 - I_2 = 40mA$$

Loop I3

$$300(I_3 - I_2) - 5 + 400(I_3 - I_4) = 0$$

Loop I4

$$400(I_4 - I_3) + 500I_4 = 0$$

SuperLoop

$$200I_2 + 100I_1 + 500I_4 = 0$$

b) Solve using Matlab (or similar program)

Group terms

$$I_1 - I_2 = 40mA$$

$$-300I_2 + 700I_3 - 400I_4 = 5$$

$$-400I_3 + 900I_4 = 0$$

$$100I_1 + 200I_2 + 500I_4 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -300 & 700 & -400 \\ 0 & 0 & -400 & 900 \\ 100 & 200 & 0 & 500 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0.04 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> A = [1,-1,0,0 ; 0,-300,700,-400 ; 0,0,-400,900 ; 100,200,0,500]
```

```
    1    -1     0     0
    0   -300    700   -400
    0     0   -400    900
   100    200     0    500
```

```
>> B = [0.04;5;0;0]
```

```
    0.0400
    5.0000
         0
         0
```

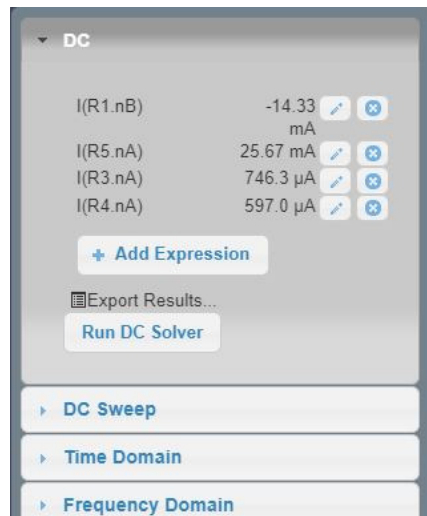
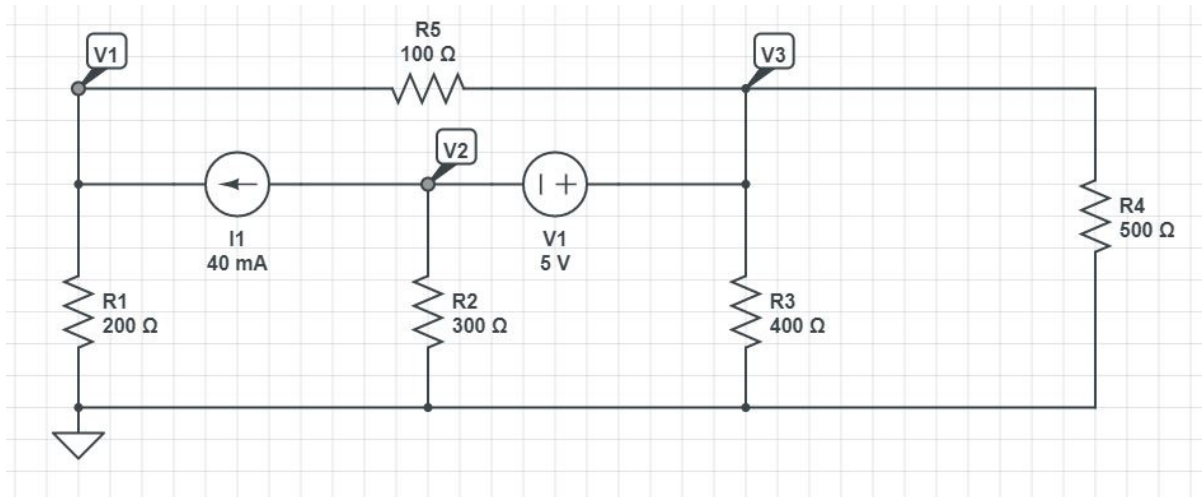
```
>> I = inv(A)*B
```

```
    0.0257
   -0.0143
    0.0013
    0.0006
```

```
>> mA = I * 1000
```

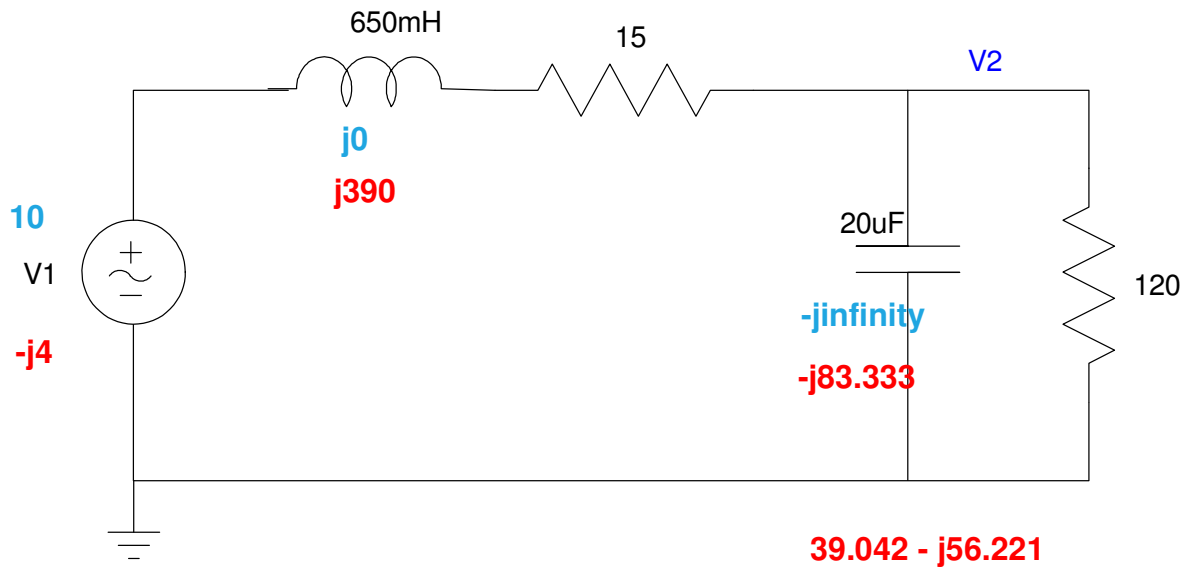
```
    25.6716
   -14.3284
     1.3433
     0.5970
```

c) Check your answers in CircuitLab (or similar circuit simulator)



5) Assume  $V_{in}$  contains a DC and AC signal:

$$V_1 = 10 + 4 \sin(600t)$$



DC (blue) & AC (red) circuit

a) Determine the voltage,  $V_2$ , using phasor analysis

DC:  $V_1 = 10$

$$V_2 = \left( \frac{120}{120+15} \right) 10$$

$$V_2 = 8.889V$$

AC:  $V_1 = 4 \sin(600t)$

$$V_1 = 0 - j4$$

$$C \rightarrow \frac{1}{j\omega C} = -j83.333\Omega$$

$$L \rightarrow j\omega L = j390\Omega$$

$$120\Omega || -j83.333\Omega = 39.042 - j56.221$$

$$V_2 = \left( \frac{39.042 - j56.221}{(39.042 - j56.221) + (15 + j390)} \right) (0 - j4)$$

$$V_2 = -0.562 + j0.583$$

meaning

$$v_2(t) = -0.562 \cos(600t) - 0.582 \sin(600t)$$

The total answer is DC + AC

$$v_2(t) = 8.889 - 0.562 \cos(600t) - 0.582 \sin(600t)$$



b) Check your answer using CircuitLab (or similar program)

Note: polar form is easier to see in lab. The AC term is

$$V_2 = -0.562 + j0.583 = 0.810 \angle 133.975^\circ$$

which has a peak-to-peak voltage of twice this

$$V_{2pp} = 1.619V_{pp}$$

From CircuitLab:

V2 (orange line)

- max = 9.693V
- min = 8.085V

DC = average(max, min)

$$DC = 8.889V \quad \text{vs. } 8.889V \text{ calculated}$$

AC = max - min

$$AC = 1.608V_{pp} \quad \text{vs. } 1.619V_{pp} \text{ calculated}$$

The time delay to V2 gives the phase shift (should be +134 degrees or -226 degrees)

