Circuits I Review

Current Loops, Voltage Nodes, Thevenin Equivalent

Introduction

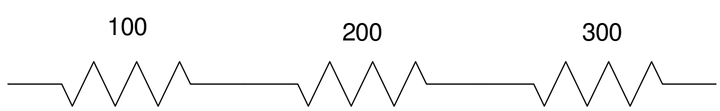
EE 206 Circuits I covers steady-state analysis of circuits with DC inputs. Three techniques are used:

- Current Loops
- Voltage Nodes
- Thevenin Equivalent

All of these techniques are used in Electronics I and II. There are shortcuts we'll introduce in this course - but you can analyze pretty much any circuit just using these techniques.

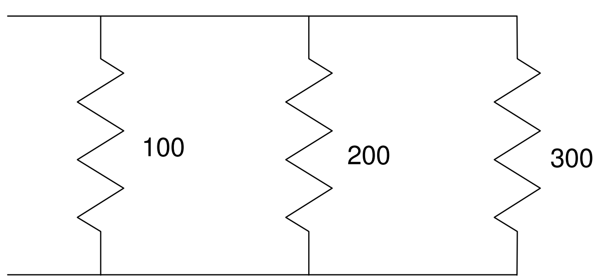
Resistors in Series and Parallel:

Resistors in series add:



Rtotal = 100 + 200 + 300 = 600 Ohms

Resistors in parallel add as the sum of the inverses inverted.



$$\text{Rotal} = \left(\frac{1}{100}+\frac{1}{200}+\frac{1}{300}\right)^{-1} = 54.54\Omega$$

Current Loops

The idea behind current loops is to set up N equations to solve for N unknowns.

If you take a volt meter and short the leads, you'll measure 0V. Likewise, if you measure the voltage around any closed path, the sum of the voltages must be zero.

The procedure for solving a circuit using Current Loops is

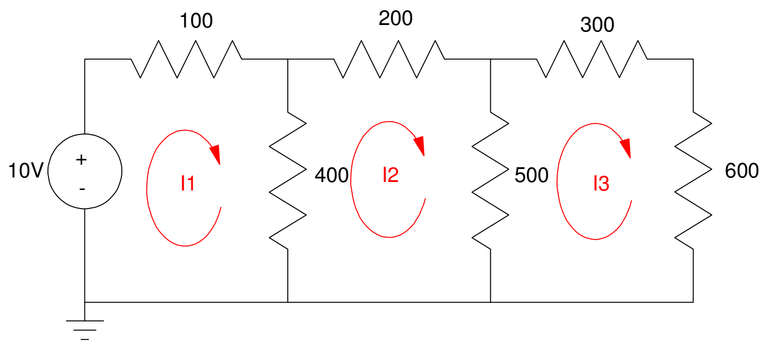
Step 1: Determine the number of "windows" the circuit has. This is how many loop equations you'll need to write.

Step 2: Define N current loops.

Step 3: Write N equations for N unknowns by summing the voltages around N closed loops to zero.

Step 4: Solve N equations for N unknowns.

Consider the following circuit as an example:



Example 1: Solve for the currents in the circuit

Step 1: This circuit has 3 windows

Step 2: Define the currents. I prefer to place a current in each window, each going clockwise. (shown in red).

Step 3) Write 3 equations for 3 unknowns:

Loop I1:

$$-10 + 100I_1 + 400(I_1 - I_2) = 0$$

Loop I2:

$$400(I_2 - I_1) + 200I_2 + 500(I_2 - I_3) = 0$$

Loop I3:

$$500(I_3 - I_2) + 300I_3 + 600I_3 = 0$$

Step 4: Solve. Here, Matlab helps. First group terms:

$$500I_1 - 400I_2 = 10$$

$$-400I_1 + 1100I_2 - 500I_3 = 0$$

$$-500I_2 + 1400I_3 = 0$$

Put in matrix form:

$$\begin{bmatrix} 500 & -400 & 0 \\ -400 & 1100 & -500 \\ 0 & -500 & 1400 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

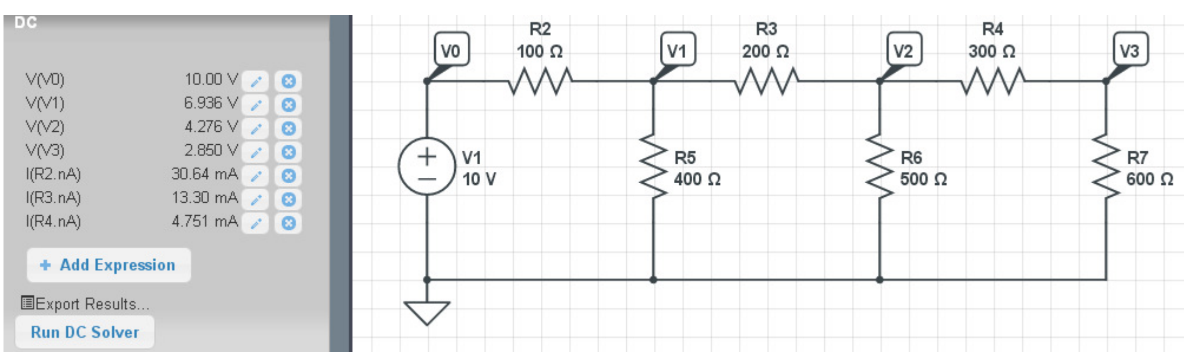
Solve in Matlab:

```
>> A = [500,-400,0 ; -400,1100,-500 ; 0,-500,1400]

         500         -400            0
        -400         1100         -500
           0         -500         1400

>> B = [10;0;0]

    10
     0
     0

>> I = inv(A)*B

I1 =     0.0306
I2 =     0.0133
I3 =     0.0048
```

Because of the way we set up the matrices, the result, I, is the current [I1, I2, I3]

Testing: Using CircuitLab, you can check your analysis using a circuit simulator. To display the currents, click on

- Add Expression
- The left side of each resistor.

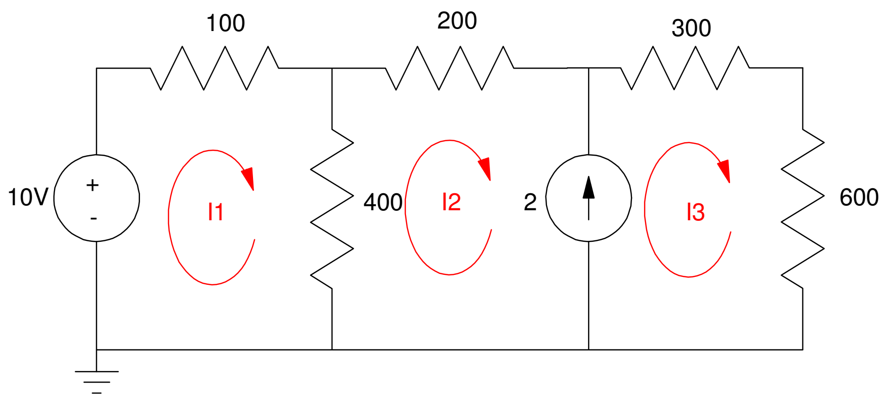


Circuitlab Simulation of the circuit to verify our calculations.

KCL with Current Sources

If you have a current source in the circuit, you still write N equations for N unknowns. One equation is the current source. The remaining equations are separate closed-paths that cover all the other elements.

For example: determine the currents in the following circuit:



Example 2: KCL with a current source

Step 1: There 3 windows in this circuit. We'll need to solve 3 equations for 3 unknowns.

Step 2: Define the current loops (shown in red).

Step 3: Write N equations for N unknowns.

Start with the easy equation: the current source:

$$I_3 - I_2 = 2$$

We now need two more. Go around I1:

$$-10 + 100 I_1 + 400(I_1 - I_2) = 0$$

We need one more equation. We can't go around I2 or I3 since this includes the current source and we have no idea what the voltage drop across it is. Instead, find some other path. One that works is the outer loop:

$$-10 + 100 I_1 + 200 I_2 + 300 I_3 + 600 I_3 = 0$$

Step 4: Solve N equations for N unknowns.

Grouping terms:

$$I_3 - I_2 = 2$$

$$500 I_1 - 400 I_2 = 10$$

$$100 I_1 + 200 I_2 + 900 I_3 = 10$$

Put in matrix form:

$$\begin{bmatrix} 0 & -1 & 1 \\ 500 & -400 & 0 \\ 100 & 200 & 900 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 10 \end{bmatrix}$$

Solve in Matlab:

```
>> A = [0,-1,1 ; 500,-400,0 ; 100,200,900]

     0    -1     1
   500  -400     0
   100   200   900

>> B = [2;10;10]

     2
    10
    10

>> I = inv(A)*B

I1 =    -1.1949
I2 =    -1.5186
I3 =     0.4814
```

Voltage Nodes:

The idea with voltage nodes is conservation of current: the total current flowing to a node must be zero. Here, you solve N equations for N unknowns where N is the number of voltage nodes. The procedure is:

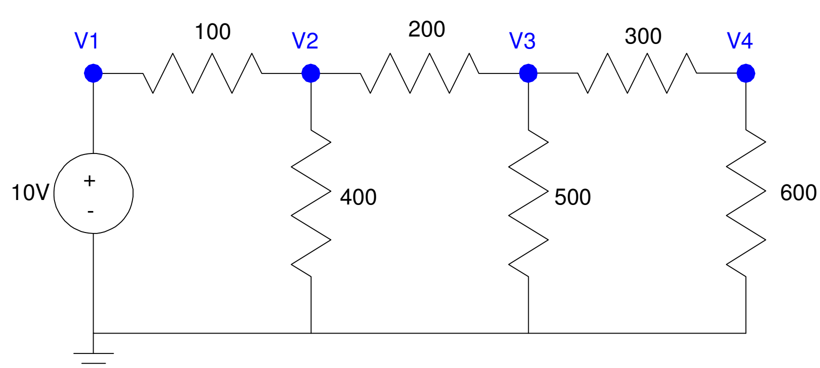
Step 1: Define one node as circuit ground.

Step 2: Define the remaining N voltage nodes.

Step 3: Write N equations for N unknowns by summing the current to (or from) any given node to zero.

Step 4: Solve N equations for N unknowns.

Example: Determine the voltages for the following circuit:



Example 3: Determine the voltages at each node

Step 1: Define ground to be the node at the bottom.

Step 2: Define the voltage at each node. Here there are four nodes (show in blue)

Step 3: Write N equations for N unknowns. For a voltage source

$$V_1 = 10$$

For the other nodes, sum the currents from the node to zero:

V2: $\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{400}\right) + \left(\frac{V_2 - V_3}{200}\right) = 0$

V3: $\left(\frac{V_3 - V_2}{200}\right) + \left(\frac{V_3}{500}\right) + \left(\frac{V_3 - V_4}{300}\right) = 0$

V4: $\left(\frac{V_4 - V_3}{300}\right) + \left(\frac{V_4}{600}\right) = 0$

Step 4: Solve N equations for N unknowns. Grouping terms:

$$V_1 = 10$$

$$\left(\frac{-1}{100}\right)V_1 + \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{200}\right)V_2 + \left(\frac{-1}{200}\right)V_3 = 0$$

$$\left(\frac{-1}{200}\right)V_2 + \left(\frac{1}{200} + \frac{1}{500} + \frac{1}{300}\right)V_3 + \left(\frac{-1}{300}\right)V_4 = 0$$

$$\left(\frac{-1}{300}\right)V_3 + \left(\frac{1}{300} + \frac{1}{600}\right)V_4 = 0$$

Place in matrix form:

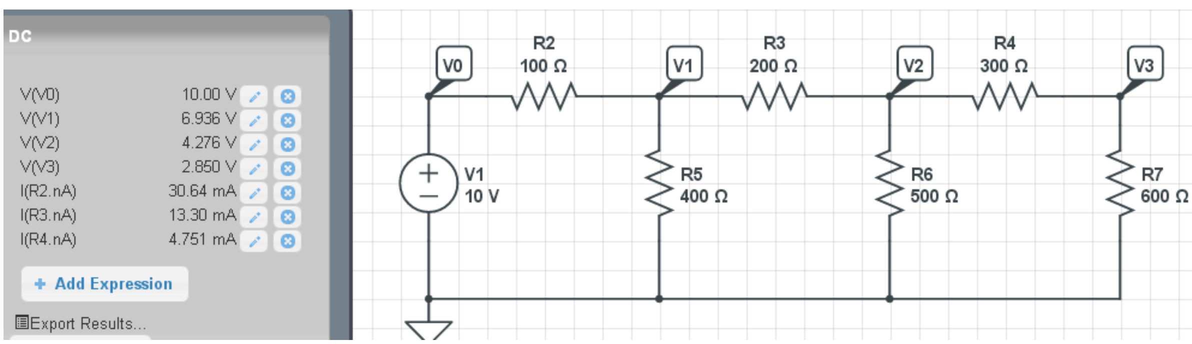
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{200}\right) & \left(\frac{-1}{200}\right) & 0 \\ 0 & \left(\frac{-1}{200}\right) & \left(\frac{1}{200} + \frac{1}{500} + \frac{1}{300}\right) & \left(\frac{-1}{300}\right) \\ 0 & 0 & \left(\frac{-1}{300}\right) & \left(\frac{1}{300} + \frac{1}{600}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> A = [1,0,0,0 ; -1/100, 1/100+1/400+1/200,-1/200,0];
>> A = [A ; 0,-1/200,1/200+1/500+1/300,-1/300 ; 0,0,-1/300,1/300+1/600]

A =

    1.0000         0         0         0
   -0.0100    0.0175   -0.0050         0
         0   -0.0050    0.0103   -0.0033
         0         0   -0.0033    0.0050
```

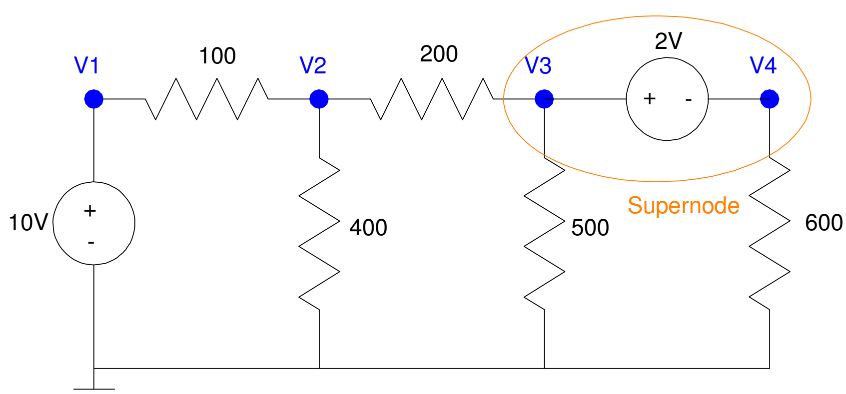
```
>> B = [10;0;0;0]

B =

    10
     0
     0
     0

>> V = inv(A)*B

V =

V1   10.0000
V2    6.9359
V3    4.2755
V4    2.8504
```

Testing: You can check these results in Circuitlab



Super-Nodes: If you have a voltage source, you don't know the current from that source, so you can't in include it in any of your equations. Instead, use a super-node.

Example: Determine the node voltages:



Step 3: Here, the 4 equations for 4 unknowns would be as follows. Start with the 2 voltage sources - these give you two equations

$$V_1 = 10$$

$$V_3 - V_4 = 2$$

You need two more equations. At node V2:

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{400}\right) + \left(\frac{V_2 - V_3}{200}\right) = 0$$

At node V3 you don't know the current from the 2V source, so you can't write the voltage node equation at node 3. Instead, write it at the supernode (including nodes 3 and 4). The current flowing from this closed path (the supernode) is

$$\left(\frac{V_3 - V_2}{200}\right) + \left(\frac{V_3}{500}\right) + \left(\frac{V_4}{600}\right) = 0$$

Step 4: Solve. Group terms

$$V_1 = 10$$

$$V_3 - V_4 = 2$$

$$\left(\frac{-1}{100}\right)V_1 + \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{200}\right)V_2 + \left(\frac{-1}{200}\right)V_3 = 0$$

$$\left(\frac{-1}{200}\right)V_2 + \left(\frac{1}{200} + \frac{1}{500}\right)V_3 + \left(\frac{1}{600}\right)V_4 = 0$$

Place in matrix form:

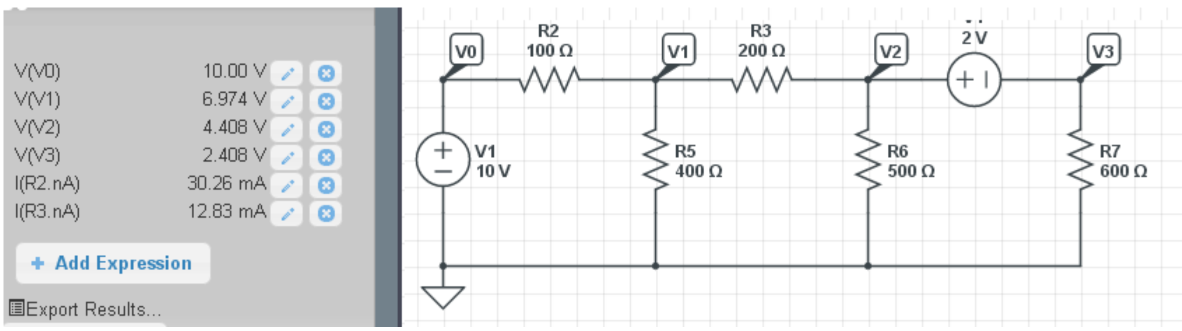
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{200}\right) & \left(\frac{-1}{200}\right) & 0 \\ 0 & \left(\frac{-1}{200}\right) & \left(\frac{1}{200} + \frac{1}{500}\right) & \left(\frac{1}{600}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab:

```
>> A = [1,0,0,0;0,0,1,-1;-1/100,1/100+1/400+1/200,-1/200,0;0,-1/200,1/200+1/500,1/600]

    1.0000         0         0         0
         0         0    1.0000   -1.0000
   -0.0100    0.0175   -0.0050         0
         0   -0.0050    0.0070    0.0017



>> B = [10;2;0;0]

    10
     2
```

```
       0
       0

>> V = inv(A)*B

V1    10.0000
V2     6.9737
V3     4.4079
V4     2.4079
```

Checking in Circuitlab

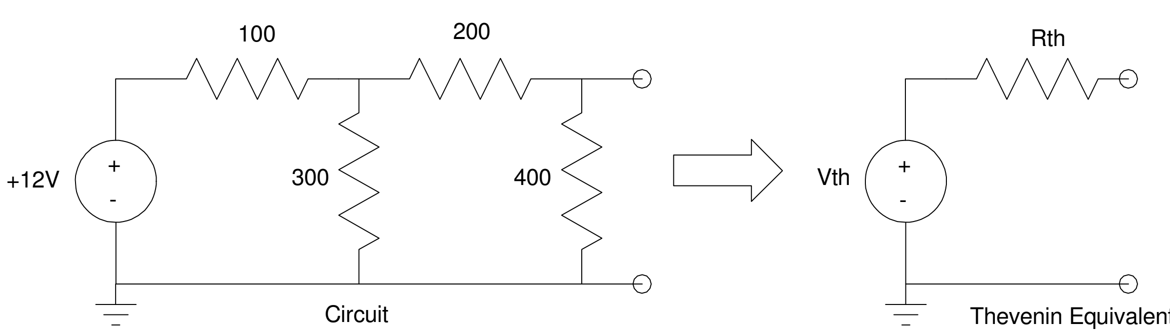


Thevenin Equivalents

The third technique taught in EE 206 Circuits I is Thevenin equivalents. The idea here is that

- If a circuit is linear (i.e. only contains sources, resistors, capacitors, and inductors), its voltage / current relationship will follow a straight line, termed the load line.
- Any circuit which has the same load line is indistinguishable from the original circuit.
- The simplest circuits which produce a load line are a voltage source and resistor (Thevenin equivalent) and a current source and a resistor (Norton equivalent).

For example, find the Thevenin equivalent for the following ciruit.

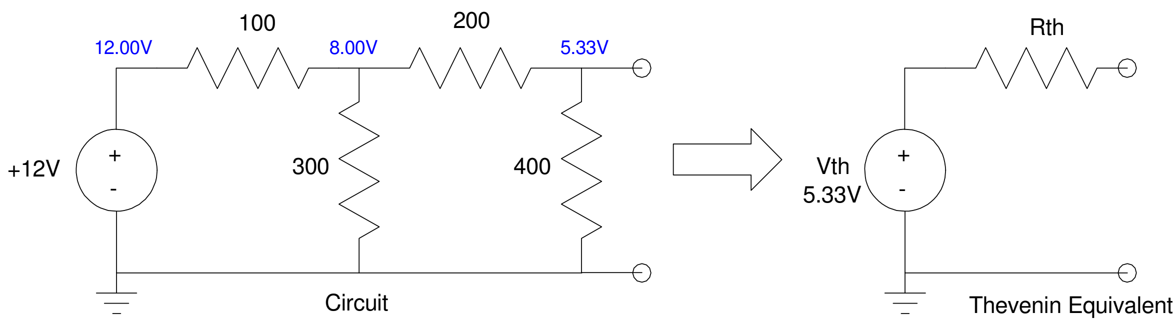


Since the two circuits are equivalent, whatever test you do on the circuit to the right to find Vth and Rth, do the same test to the original circuit.

Vth: Measure the open-circuit votlage.

- The original circuit gives 5.33V
- The Thevenin equivalent gives Vth

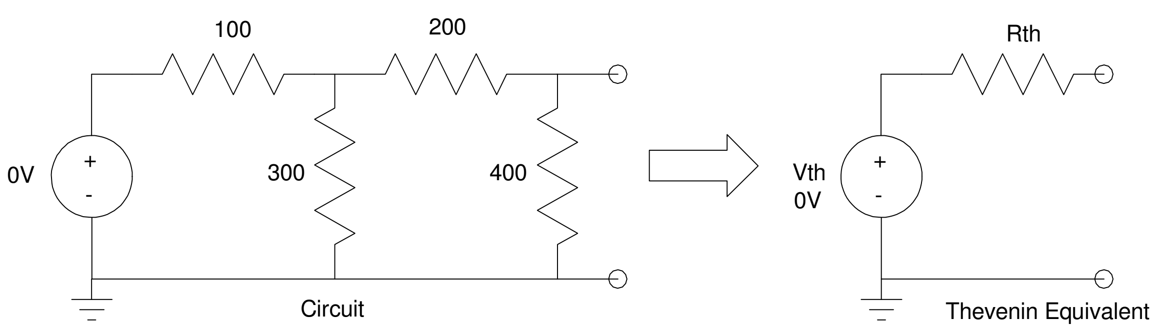
So Vth = 5.33V



Rth: Turn off the source and measure the resistance looking in. This gives

- R = 162.96 Ohms (original circuit)
- R = Rth (circuit to the right)

so Rth = 162.96 Ohms.



Rth (take 2): Sometimes the resistance looking in isn't obvious. In that case, apply a 1V test votlage and compute the current. The Thevenin resistance is then

Rth = 1V / Iin

For example with this circuit

Iin = 6.136mA

meaning

Rth = 1V / 6.136mA = 163 Ohms