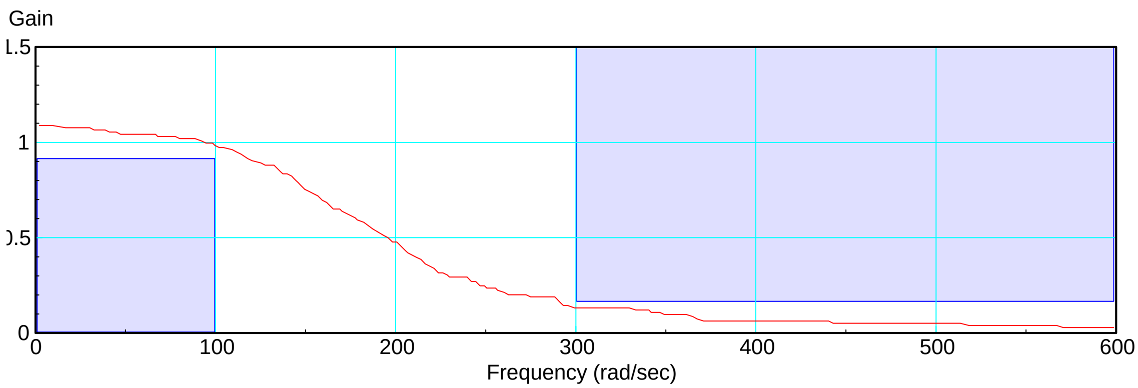
ECE 321 - Homework #3

Filters, 2-port models. Due Monday, November 23rd

1) Give the transfer function for a filter which meets the following requirements

- w < 100 rad/sec   Gain > 0.9
- w > 500 rad/sec   Gain < 0.1



There are multiple solutions. One what works:

Filter Order:

$$\left(\frac{100}{300}\right)^n < 0.1$$

$$n > 2.09$$

Assume a 3rd order filter.

Filter Type: Choose a Butterworth low pass filter (corner = 100 as a first guess)

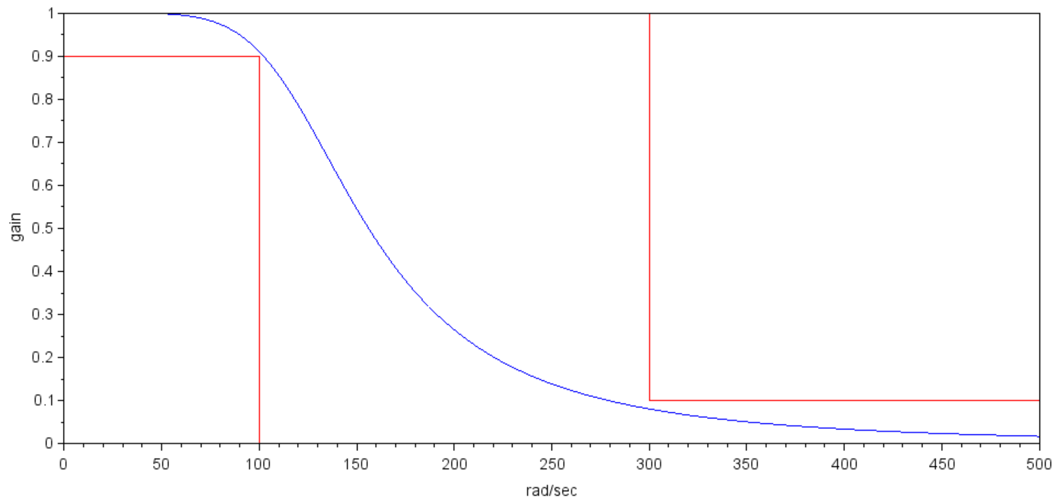
$$G(s) = \left(\frac{100^3}{(s+100)\left(s+100\angle 60^0\right)\left(s+100\angle -60^0\right)}\right)$$

Filter Corner: Adjust in SciLab until it meets the specs

Corner	Gain at 100	Gain at 300
100	0.71	0.037
120	0.865	0.063
130	0.910	0.081

2) Verify that your filter meets these reqirements in MATLAB (or like program)

```
-->j = sqrt(-1);
-->s1 = -130;
-->s2 = -130*exp(j*60*%pi/180)
-->s3 = -130*exp(-j*60*%pi/180)

-->w = [0:500]';
-->s  = j*w;

-->G = 130^3 ./ ( (s-s1) .* (s-s2) .* (s-s3) );
-->plot(w,abs(G),'b');

-->xlabel('rad/sec');
-->ylabel('gain');

-->plot([0,100,100],[0.9,0.9,0],'r');
-->plot([300,300,500],[1,0.1,0.1],'r');
```



3) Design an op-amp circuit to impliment your filter.

$$\frac{1}{RC} = 130$$

Let

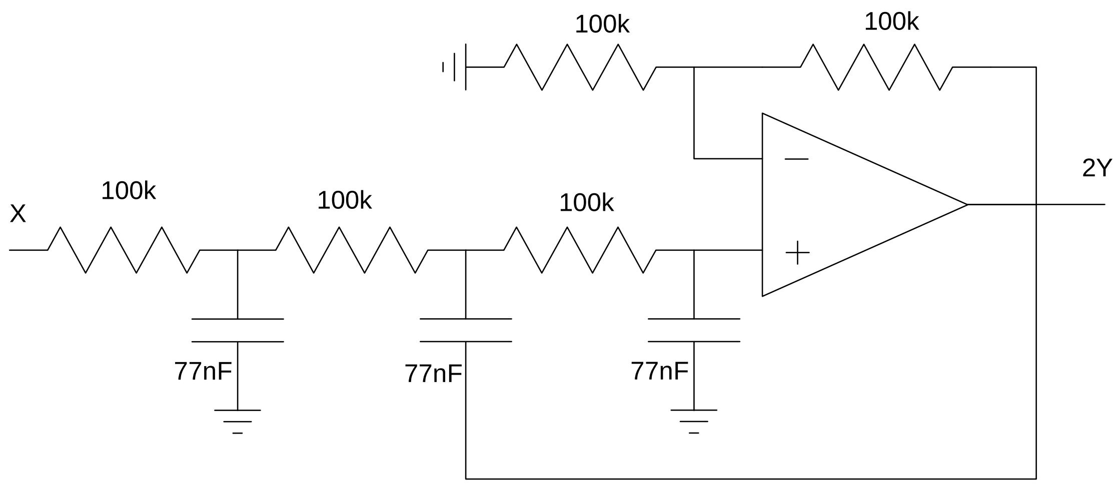
R = 100k,   C = 76.9nF

$$3 - k = 2\cos(60^0)$$

k = 2 = 1 + R1/R2

Let

R1 = R2 = 100k



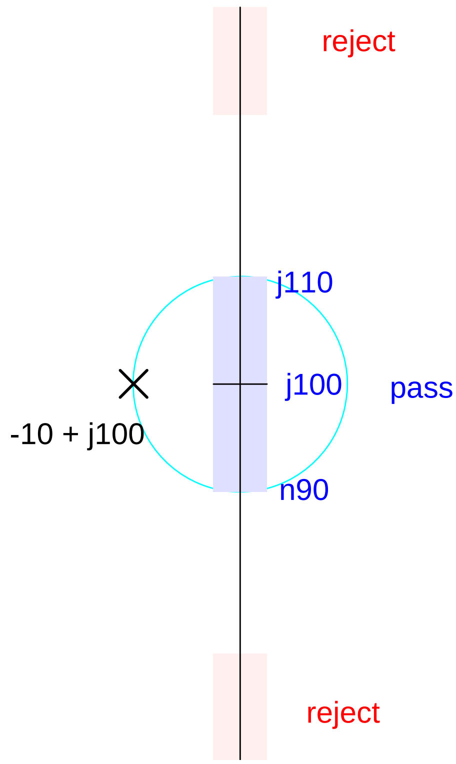
note: This filter has a DC gain of 2 (vs 1). You can reduce the gain with a voltage divider - or just incorporate that extra gain of 2 elsewhere.

4) Give the transfer function for a filter which meets the following requirements

- 90 < w < 110 rad/sec   Gain > 0.9
- w < 50 rad/sec   Gain < 0.1
- w > 150 rad/sec   Gain < 0.1

The center frequency = 100 rad/sec, meaning the complex part of the pole should be 100 rad/sec

The bandwidth is 10 rad/sec, so let the real part be 10



1st Attempt: Single Pole

Try a single pole at -10 + j100:

$s = -10 + j100$

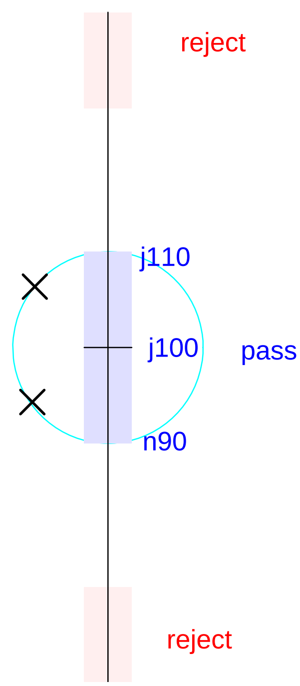
$$G(s) = \left(\frac{20s}{(s+10+j100)(s+10-j100)}\right)$$

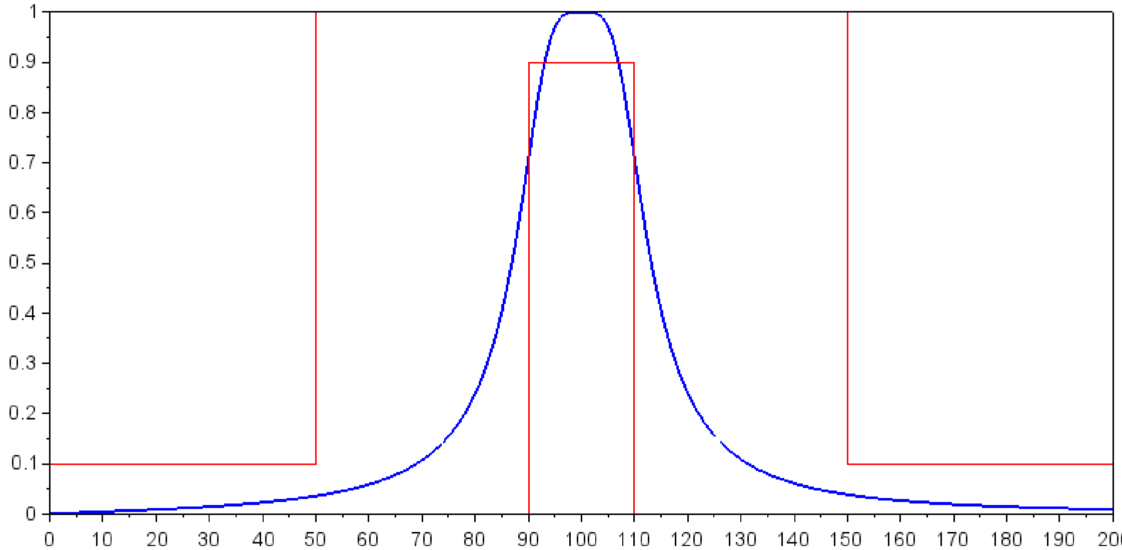
w	50	90	110	150
G(jw)	0.13	0.67	0.74	0.23

A single pole won't work.

2st Attempt: Try two poles (plus their conjugate)

```
-->s1 = -10*exp(j*45*%pi/180) + j*100
-->s2 = -10*exp(-j*45*%pi/180) + j*100
-->s3 = conj(s1)
-->s4 = conj(s2)

-->w = [0:0.1:200]';
-->s = j*w;



-->G = s ./ ( (s-s1) .* (s-s2) .* (s-s3) .* (s-s4) );
-->max(abs(G))

    0.0000250

-->k = 1/ans

    40000.125

-->G = k*G;

-->plot(w,abs(G))
-->plot([0,50,50],[0.1,0.1,1],'r');
-->plot([150,150,300],[1,0.1,0.1],'r');
-->plot([150,150,200],[1,0.1,0.1],'r');
-->plot([90,90,110,110],[0,0.9,0.9,0],'r');
```

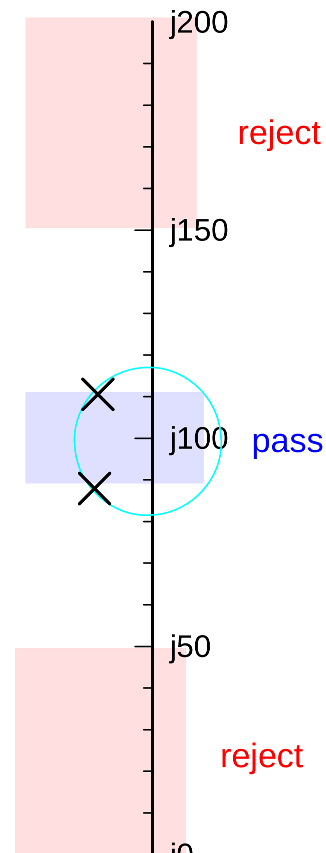


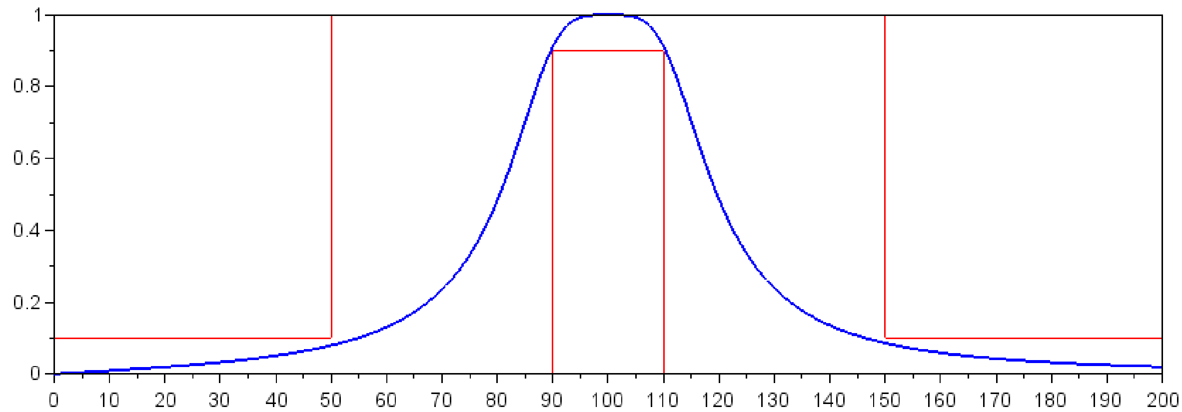


$$G(s) = \left(\frac{40{,}000s}{(s+10\angle 45^0+j100)(s+10\angle -45^0+j100)(s+10\angle 45^0-j100)(s+10\angle -45^0-j100)} \right)$$

3rd Attempt: Try a wider bandwitch with poles at 15

```
-->s1 = -15*exp(j*45*%pi/180) + j*100;
-->s2 = -15*exp(-j*45*%pi/180) + j*100;
-->s3 = conj(s1)
-->s4 = conj(s2);

-->G = s ./ ( (s-s1) .* (s-s2) .* (s-s3) .*
(s-s4) );

-->k = 1 / max(abs(G))

    90001.424

-->G = G*k;
-->plot(w,abs(G))
-->plot([90,90,110,110],[0,0.9,0.9,0],'r');
-->plot([150,150,200],[1,0.1,0.1],'r');
-->plot([0,50,50],[0.1,0.1,1],'r');
```





$$G(s) = \left(\frac{90{,}000s}{(s+15\angle 45^0 + j100)(s+15\angle -45^0 + j100)(s+15\angle 45^0 - j100)(s+15\angle -45^0 - j100)} \right)$$

5) Verify that your filter meets these reqirements in MATLAB (or like program)

see above

6) Design an op-amp circuit to impliment your filter.

$$G(s) = \left(\frac{90{,}000s}{(s+111.11\angle \pm 84.522^0)(s+90.02\angle \pm 83.23^0)} \right)$$

$$G(s) = \left(\frac{4.018s}{s+111.11\angle \pm 84.522^0} \right)\left(\frac{2.764 \cdot 90.02^2}{s+90.02\angle \pm 83.23^0} \right)$$

$$G(s) = \left(\frac{4.018s}{s^2+21.21s+12{,}345} \right)\left(\frac{2.764 \cdot 90.02^2}{s+90.02\angle \pm 83.23^0} \right)$$

Stage 1:

$$\left(\frac{1}{R_1 C} \right) = 4.018$$

C = 1uF R1 = 248k

$$\left(\frac{2}{R_3 C} \right) = 21.21$$

C = 1uF, R3 = 94.3k

$$\left(\frac{R_1+R_2}{R_1 R_2} \right)\left(\frac{1}{R_3 C^2} \right) = 12,345$$

R2 = 861

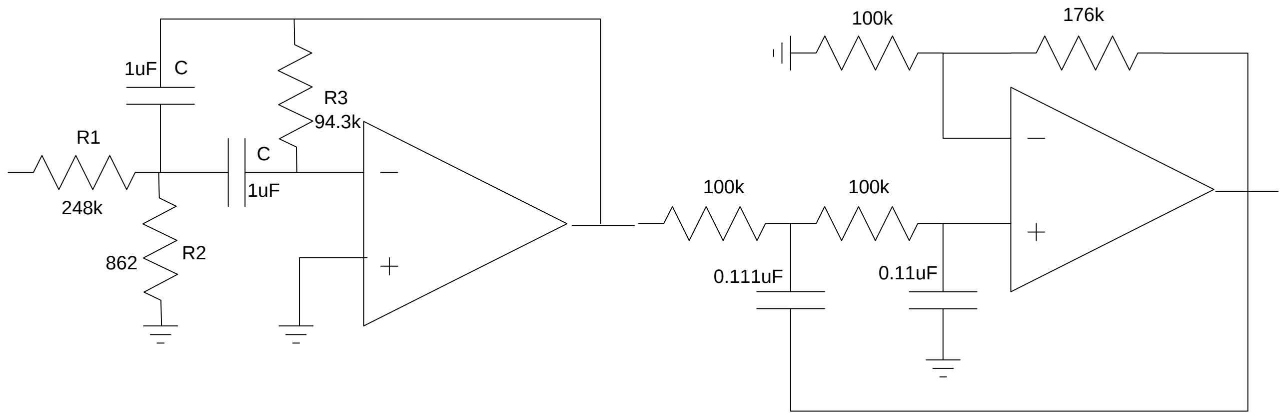
Stage 2:

$$\left(\frac{1}{RC} \right) = 90.02$$

R = 100k C = 0.111uF

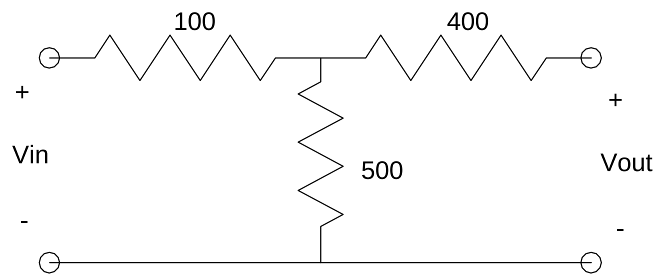
$$3 - k = 2\cos(83.23^0)$$

$k = 2.76 = 1 + R1/R2$



Problem 4-6

7) Give a 2-port model for the following circuit



Rin: Short Vout, measure the resistance looking in

$$R_{in} = 100 + 400||500$$

$$R_{in} = 222$$

Ai Set Vout = 1V, measure Vin

$$A_i = \left(\frac{500}{500+400}\right) = 0.555$$

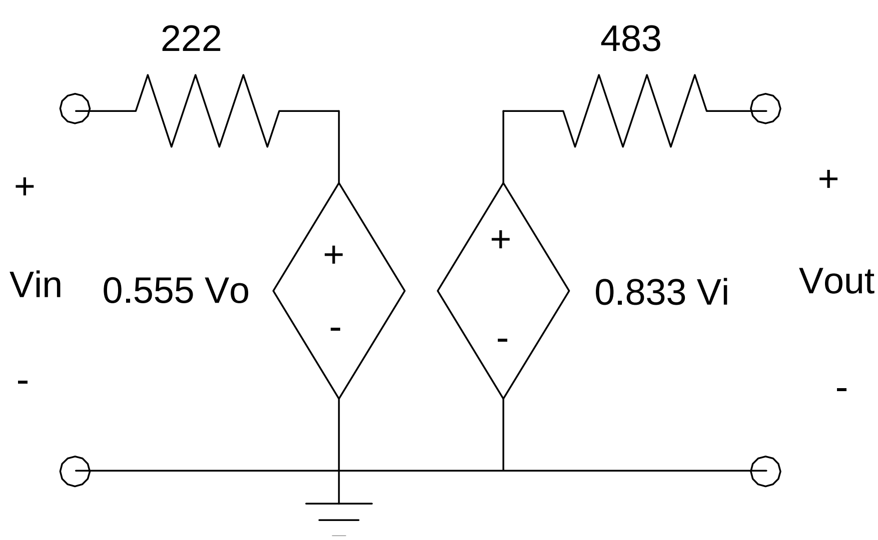
Rout: Short Vin, measure the resistance at the output

$$R_{out} = 400 + 100||500$$

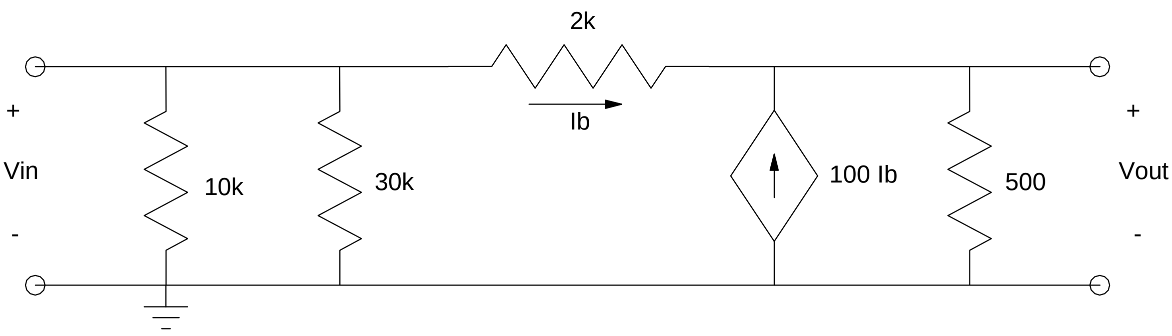
$$R_{out} = 483$$

Ao: Set Vin = 1V, measure Vout

$$A_o = \left(\frac{500}{500=100}\right) = 0.833$$



8) Give a 2-port model for the following circuit



Rin: Set Vout = 0V, measure the resistance at the input

$$R_{in} = 10k||30k||2k$$

$$R_{in} = 1579\Omega$$

Ai: Set Vout = 1V, measure the voltage at the input

$$A_i = \left(\frac{10k||30k}{10k||30k+2k}\right) = 0.7895$$

Rout: Set Vin = 0V, measure the resistance at the output. This isn't obvious, so apply a 1V test voltage and measure the current

$$I = \frac{1}{500} + \frac{1}{2k} + 100\left(\frac{1}{2k}\right) = 52.5mA$$

$$R_{out} = \frac{V}{I} = \frac{1V}{52.5mA} = 19.04\Omega$$

Ao: Set Vin = 1V, measure Vout. This isn't obvious so use voltage nodes

$$\left(\frac{V_{out}-1}{2k}\right) - 100\left(\frac{1-V_{out}}{2k}\right) + \left(\frac{V_{out}}{500}\right) = 0$$

$$V_{out} = A_o = 0.9619$$