# ECE 321 - Homework #2

Phasors - Poles, Zeros, & Frequency Response. Due Monday, November 16th

Problem 1) For the following circuit,

- Determine the impedances of each component at 10 rad/sec
- Write the voltage node equations, and
- Determine the node voltages

Replace with impedances

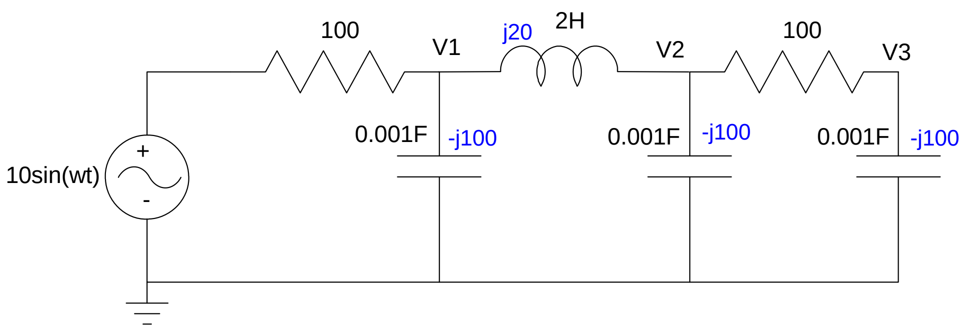
$$R \to R = 100$$

$$C \to \frac{1}{j\omega C} = -j100$$

$$L \to j\omega L = j20$$

$$V_{in} = -j10$$

..




Write the voltage node equations:

$$\left(\frac{V_1 - V_{in}}{100}\right) + \left(\frac{V_1 - V_2}{j20}\right) + \left(\frac{V_1}{-j100}\right) = 0$$

$$\left(\frac{V_2 - V_1}{j20}\right) + \left(\frac{V_2 - V_3}{100}\right) + \left(\frac{V_2}{-j100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{-j100}\right) = 0$$

Group terms

$$\left(\frac{1}{100} + \frac{1}{j20} + \frac{1}{-j100}\right)V_1 - \left(\frac{1}{j20}\right)V_2 = \left(\frac{1}{100}\right)V_{in}$$

$$\left(\frac{-1}{j20}\right)V_1 + \left(\frac{1}{j20} + \frac{1}{100} + \frac{1}{-j100}\right)V_2 + \left(\frac{-1}{100}\right)V_3 = 0$$

$$\left(\frac{-1}{100}\right)V_2 + \left(\frac{1}{100} + \frac{1}{-j100}\right)V_3 = 0$$

Put in matrix form

$$\begin{bmatrix} \left(\frac{1}{100}+\frac{1}{j20}+\frac{1}{-j100}\right) & \left(\frac{-1}{j20}\right) & 0 \\ \left(\frac{-1}{j20}\right) & \left(\frac{1}{j20}+\frac{1}{100}+\frac{1}{-j100}\right) & \left(\frac{-1}{100}\right) \\ 0 & \left(\frac{-1}{100}\right) & \left(\frac{1}{100}+\frac{1}{-j100}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{100}\right) \\ 0 \\ 0 \end{bmatrix} V_{in}$$

Solve in MATLAB

```
-->w = 10;
-->R = 100;
-->L = j*w*2

    20.i

-->C = 1/(j*w*0.001)

  - 100.i

-->A = [1/R+1/L+1/C,-1/L,0;-1/L,1/R+1/L+1/C,-1/R;0,-1/R,1/R+1/C]

    0.01 - 0.04i    0.05i            0
    0.05i           0.01 - 0.04i  - 0.01
    0             - 0.01             0.01 + 0.01i

-->B = [1/R;0;0]

    0.01
    0.
    0.


-->V = inv(A)*B*Vin

  - 2.3076923 - 1.5384615i
  - 3.5384615 - 1.6923077i
  - 2.6153846 + 0.9230769i
```

meaning

**V1 = -2.3076 cos(10t) + 1.5384 sin(10t)**

**V2 = -3.5384 cos(10t) + 1.6923 sin(10t)**

**V3 = -2.6153 cos(10t) - 0.9230 sin(10t)**

Problem 2) Repeat at 100 rad/sec

```
-->w = 100;
-->R = 100;
-->L = j*w*2
```

$L \rightarrow j\omega L$

```
     200.i

-->C = 1/(j*w*0.001)
```

$C \rightarrow \frac{1}{j\omega C}$

```
   - 10.i

-->A = [1/R+1/L+1/C,-1/L,0;-1/L,1/R+1/L+1/C,-1/R;0,-1/R,1/R+1/C]

     0.01 + 0.095i    0.005i           0
     0.005i           0.01 + 0.095i  - 0.01
     0              - 0.01             0.01 + 0.1i

-->B = [1/R;0;0]

     0.01
     0.
     0.

-->V = inv(A)*B*Vin

   - 1.0438042 - 0.1104658i
     0.0532108 + 0.0112425i
     0.0016400 - 0.0051571i
```

Meaning

**V1 = -1.0438 cos(100t) + 0.1104 sin(100t)**

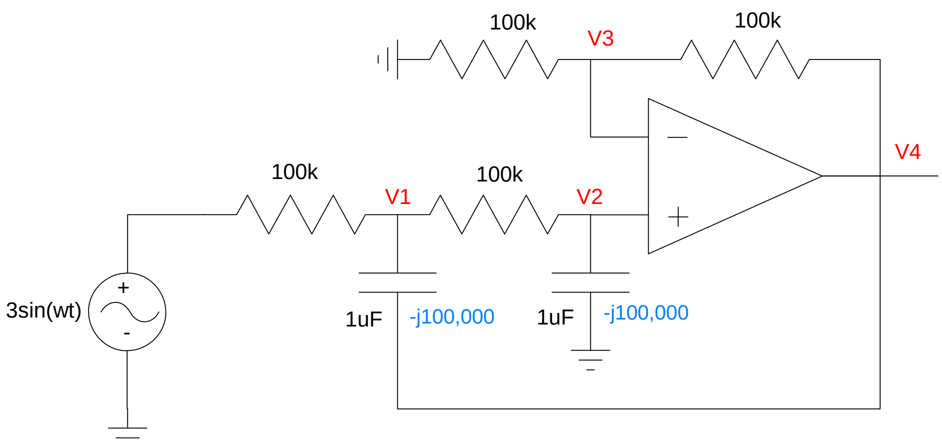
**V2 = 0.0532 cos(100t) - 0.0112 sin(100t)**

**V3 = 0.0016 cos(100t) + 0.0051 sin (100t)**

Problem 3) For the following op-amp circuit

- Determine the impedances of each component at 10 rad/sec
- Write the voltage node equations, and
- Determine the node voltages

Replace R / L / C with their complex impedances




Write the node equations

V2 = V3

$$\left(\frac{V_1 - V_{in}}{100k}\right) + \left(\frac{V_1 - V_2}{100k}\right) + \left(\frac{V_1 - V_4}{-j100k}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100k}\right) + \left(\frac{V_2}{-j100k}\right) = 0$$

$$\left(\frac{V_3}{100k}\right) + \left(\frac{V_3 - V_4}{100k}\right) = 0$$

Group terms and multiply each equation by 100k

$$(1)V_2 - (1)V_3 = 0$$

$$(2 + j)V_1 + (-1)V_2 + (-j)V_4 = (1)V_{in}$$

$$(-1)V_1 + (1+j)V_2 = 0$$

$$(2)V_3 + (-1)V_4 = 0$$

Place in matrix form

$$\begin{bmatrix} 0 & 1 & -1 & 0 \\ 2+j & -1 & 0 & -j \\ -1 & 1+j & 0 & 0 \\ 0 & 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

Solve in MATLAB

```
-->A = [0,1,-1,0;2+j,-1,0,-j;-1,1+j,0,0;0,0,2,-1]

    0            1.          - 1.     0
    2. + i     - 1.            0    - i
  - 1.           1. + i        0      0
    0            0             2.   - 1.

-->B = [0;1;0;0]

    0.
    1.
    0.
    0.

-->V = inv(A)*B*Vin

  - 10. - 10.i
  - 10.
  - 10.
  - 20.
```

meaning

**V1 = -10. cos(10t) + 10. sin(10t)**

**V2 = -10. cos(10t)**

**V3 = -10. cos(10t)**

**V4 = -20. cos(10t)**

Problem 4) Releat at 100 rad/sec

R stays 100k

C becomes -j10,000

Scaling 1/R and 1/Zc by 100,000 results in

R => 1

C => j10

$$\begin{bmatrix} 0 & 1 & -1 & 0 \\ 2+j10 & -1 & 0 & -j10 \\ -1 & 1+j10 & 0 & 0 \\ 0 & 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

```
-->A = [0,1,-1,0;2+j*10,-1,0,-j*10;-1,1+j*10,0,0;0,0,2,-1]

    0             1.          - 1.    0
    2. + 10.i   - 1.            0   - 10.i
  - 1.            1. + 10.i     0     0
    0             0             2.  - 1.

-->V = inv(A)*B*Vin

  - 1.009999 - 0.0010100i
  - 0.0101000 + 0.0999899i
  - 0.0101000 + 0.0999899i
  - 0.0202000 + 0.1999798i
```

Meaning

**V1 = -1.0099 cos(100t) + 0.0010 sin(100t)**

**V2 = -0.0101 cos(100t) - 0.0999 sin(100t)**

**V3 = -0.0101 cos(100t) - 0.0999 sin(100t)**

**V4 = -0.0202 cos(100t) - 0.1999 sin(100t)**

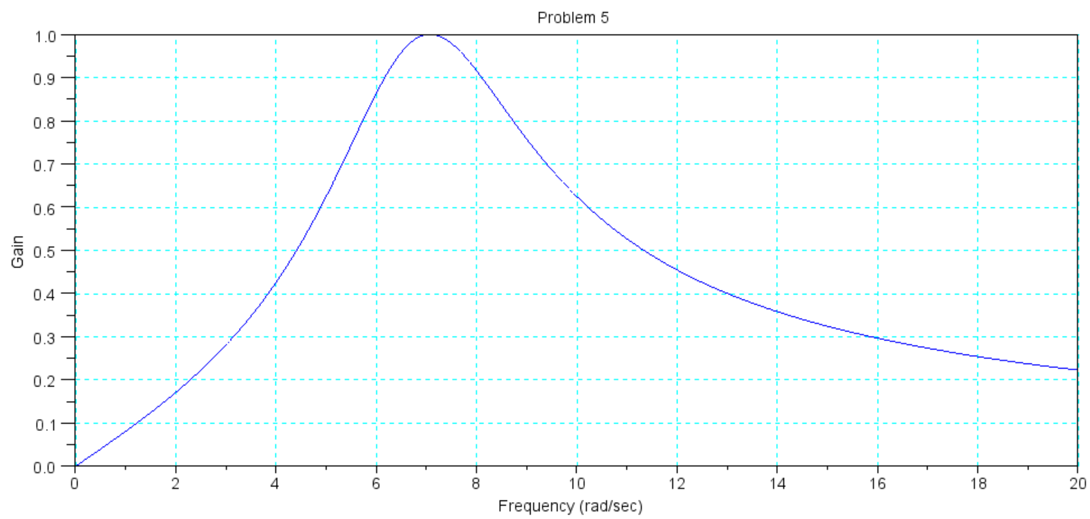
Problem 5 - 7)

- Plot the gain vs. frequency for the following filter for 0 < w < 20 rad/sec
- What kind of filter is it: low pass - band pass - high pass

5) $Y = \left(\frac{4s}{s^2+4s+50}\right) X$

```
-->w = [0:0.01:20]';
-->s = j*w;
-->G5 = 4*s ./ (s.^2 + 4*s + 50);
-->plot(w,abs(G5));
-->xlabel('Frequency (rad/sec)');
-->ylabel('Gain');
-->title('Problem 5')
```

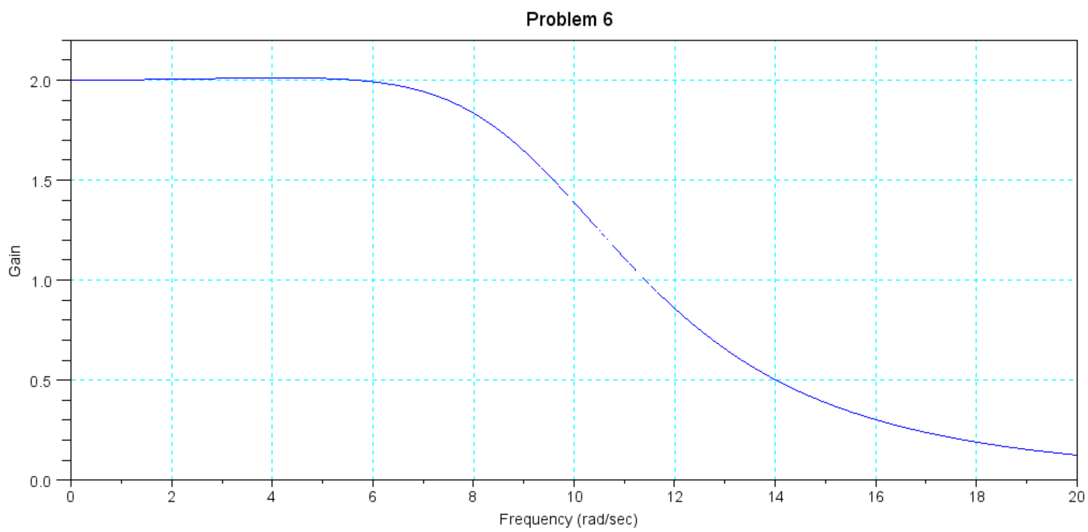
This is a band-pass filter, passing frequencies $\sqrt{50} \pm 2$ rad/sec



6) $Y = \left( \frac{20{,}000}{(s^2+18s+100)(s^2+8s+100)} \right) X$

```
-->w = [0:0.01:20]';
-->s = j*w;
-->G6 = 20000 ./ ( (s.^2  + 18*s + 100).*(s.^2 + 8*s+100) );
-->plot(w,abs(G6));
-->xgrid(4)
-->xlabel('Frequency (rad/sec)');
-->ylabel('Gain');
-->title('Problem 6');
```

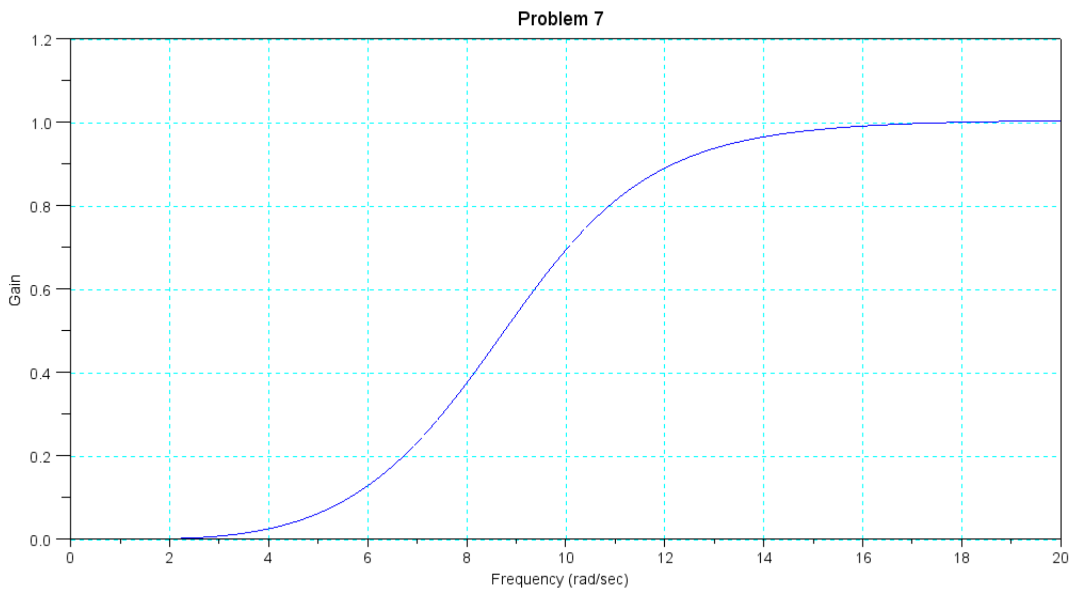
This is a low-pass filter. Frequencies below approximately 10 rad/sec are passed $\left(\sqrt{100}\right)$



7) $Y = \left(\frac{s^4}{(s^2+18s+100)(s^2+8s+100)}\right)X$

```
-->w = [0:0.01:20]';
-->s = j*w;
-->G7 = s.^4 ./ ( (s.^2  + 18*s + 100).*(s.^2 + 8*s+100) );
-->plot(w,abs(G7));
-->xlabel('Frequency (rad/sec)');
-->ylabel('Gain');
-->xgrid(4)
-->title('Problem 7');
```

This is a high-pass filter. It allows frequencies above 10 rad/sec to pass and blocks frequencies below 10.



8) Give the transfer function for a filter which passes frequencies between 0 and 100 rad/sec (16Hz). Plot the gain of your filter from 0 to 300 rad/sec to check your design.

This filter is centered at j0 rad/sec (DC). The complex part of the poles should be zero.

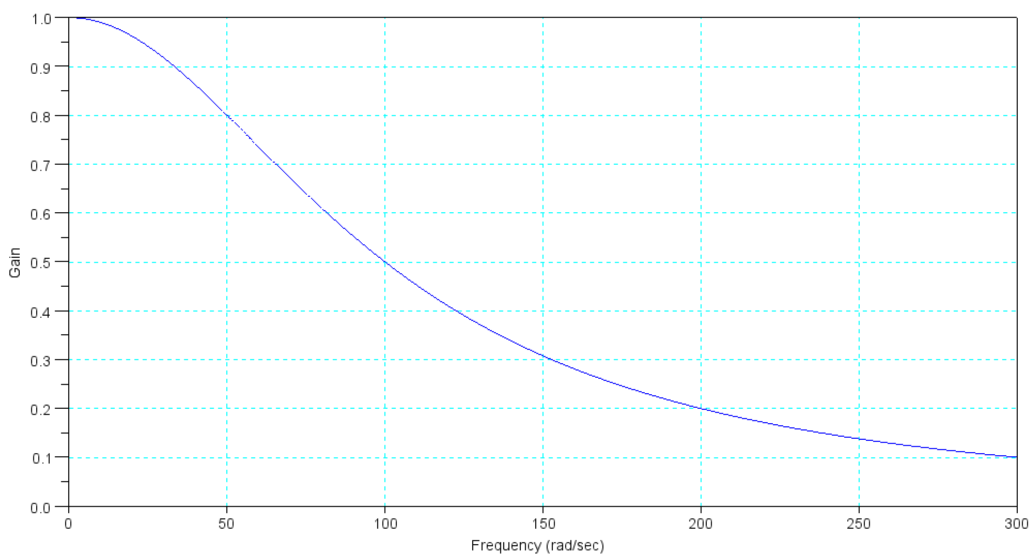
This filter passes 0 +/- 100 rad/sec. The amplitude of the poles is 100

Pick a 2nd-order filter (order is arbitrary with these requirements)

Pick the numerator so that the DC gain is one (max gain = 1)

$$G_8(s) = \left(\frac{100}{s+100}\right)^2$$

```
-->w = [0:0.1:300]';
-->s = j*w;
-->G8 = (100 ./ (s + 100) ) .^ 2;
-->plot(w,abs(G8));
-->xgrid(4)
-->xlabel('Frequency (rad/sec)');
-->ylabel('Gain');
```




Note: This isn't a very good filter. We can do better, but that requires complex poles - which we'll get to next week.

9) Give the transfer function for a filter which passes frequencies between 25 and 33 rad/sec. Plot the gain of your filter from 15 to 40 rad/sec to check your design.

The center frequency = 29 rad/sec. The complex part of the pole is j29

The filter passes 29 rad/sec +/- 4 rad/sec. The real part of the pole is 4

Add a zero at DC to make the gain here zero

Add a gain on top to make the max gain one

$$G_9 = \left( \frac{8s}{(s+4+j29)(s+4-j29)} \right)$$

```
-->w = [15:0.01:40]';
-->s = j*w;
-->G9 = s ./ ( (s+4+j*29).*(s+4-j*29) );
-->max(abs(G9))

    0.1249975

-->k = 1/ans

    8.0001616

-->G9 = k * s ./ ( (s+4+j*29).*(s+4-j*29) );
-->plot(w,abs(G9));
-->xlabel('Frequency (rad/sec)');
-->ylabel('Gain');
-->xgrid(4)
```

Again, it's not great and we can do better. We'll cover how next week with Butterworth and Chebychev filters.