ECE 321 - Homework #4

Filters. Due Monday, November 21th

1) Assume you have a filter

$$Y = \left(\frac{2s+10}{s^2+20s+100}\right)X$$

a) What is the differential equation relating X and Y?

Cross Multiply

$$(s^2 + 20s + 100)Y = (2s + 10)X$$

which means

$$\boxed{\frac{d^2y}{dt^2} + 20\frac{dy}{dt} + 100y = 2\frac{dx}{dt} + 10x}$$

b) Determine y(t) assuming

$$x(t) = 2 + 3\cos(4t)$$

Use superposition:

x(t) = 2

s = 0

$$\left(\frac{2s+10}{s^2+20s+100}\right)_{s=0} = 0.1$$

$$y = (0.1) \cdot 2$$

$$y = 0.2$$

x(t) = 3 cos(4t)

s = j4

$$\left(\frac{2s+10}{s^2+20s+100}\right)_{s=j4} = 0.1104\angle -4.94^0$$

$$y = (0.1104\angle -4.94^0) \cdot 3\cos(4t)$$

$$y = 0.3312\cos(4t - 4.94^0)$$

Add the two inputs to get x(t)

Add the two outputs to get y(t)

$$\boxed{y = 0.2 + 0.3312\cos(4t - 4.94^0)}$$

Design a filter for your light to sound circuit.

2) Requirements: Specify the requirements. For example,

Input: -10V to +10V analog, capable of 20mA, 0 to 10kHz

Output: -10V to +10V analog, capable of 20mA

Relationship: (Low-Pass Filter)

- 0.9 < Gain < 1.1 f < 250 Hz
- Gain < 0.2 f > 750 Hz

3) Analysis: Design a filter (i.e. give the transfer function) for a filter which meets your design specs.

The number of poles you need are:

$$\left(\frac{250Hz}{750Hz}\right)^n = 0.2$$

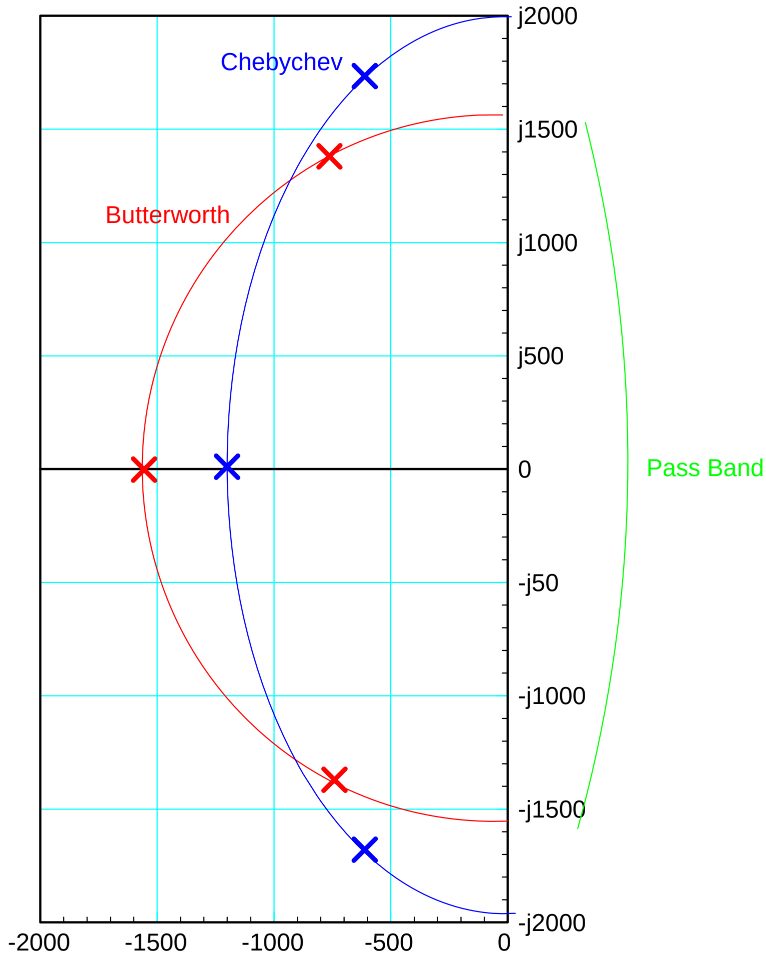
$$n = 1.46$$

You need at least 1.46 poles to meet this requirement. Let n = 3

Choosing a Chebychev filter with a corner at 250Hz:

```
-->p1 = 250*2*pi*  1.21*exp(j*69.5*pi/180)

    665.62641 + 1780.2987i

-->p2 = conj(p1)

    665.62641 - 1780.2987i

-->p3 = 250*2*pi * 0.85

    1335.1769

-->k = p1*p2*p3

    4.823D+09

-->Gs = k ./ ( (s + p1) .* (s + p2) .* (s + p3) );
-->plot(f,abs(Gs))
```

$$G(s) = \left(\frac{4.823 \cdot 10^9}{(s+1335)\left(s+1900\angle 69.5^0\right)\left(s+1900\angle -69.5^0\right)}\right)$$



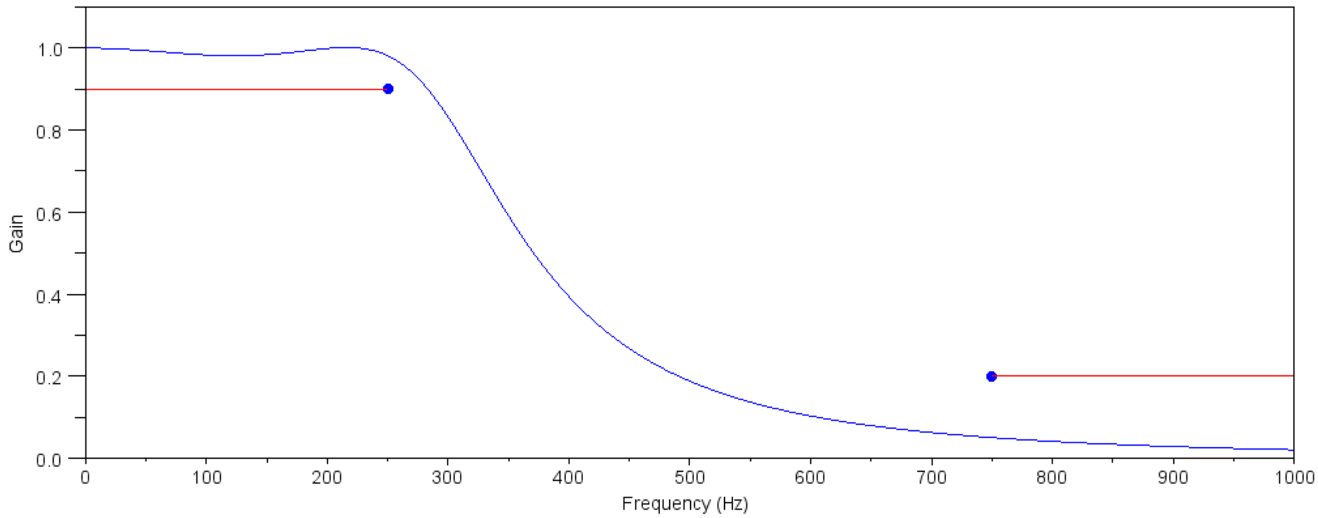
Pole Location for the Chebychev filter: 250Hz Passband

a) Compute the gain of your filter at several points. For example, for the low-pass filter, compute the gain at

{ 0Hz, 250Hz, 750Hz, 2kHz }

```
-->G = zpk([],[-p1,-p2,-p3],k)
-->evalfr(G,0)

    1.

-->abs(evalfr(G,j*250*%pi*2))

    0.9813375

-->abs(evalfr(G,j*750*%pi*2))

    0.0501830

-->abs(evalfr(G,j*1000*%pi*2))

    0.0203890
```

b) Plot the gain of your filter from 0Hz to 1kHz (or so) to verify it meets your requirements.



4) Design an op-amp circuit to implement your filter. Let R = 100k

$$\left(\frac{1}{RC}\right) = 1335$$

$$C = 0.0075\mu F$$

$$\left(\frac{1}{RC}\right) = 1900$$

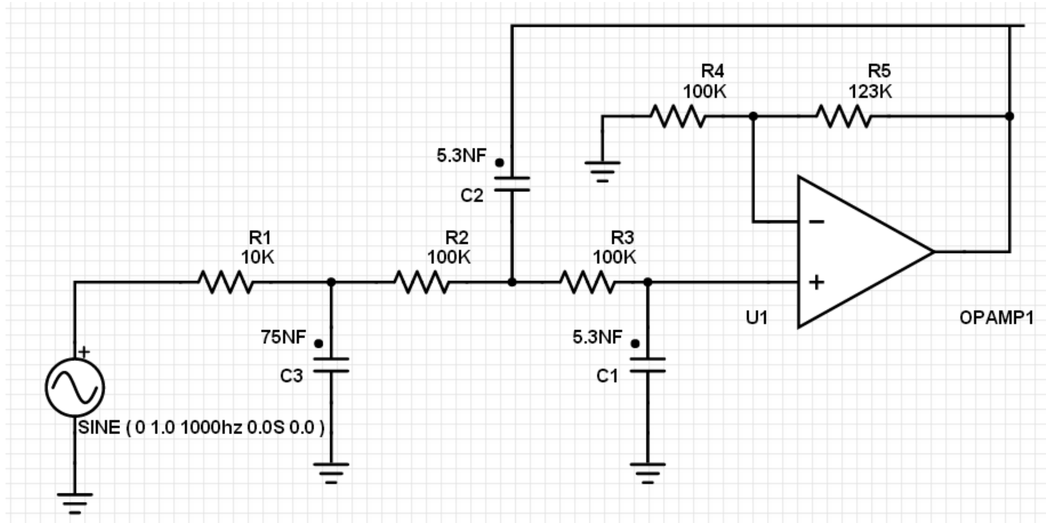
$$C = 0.0053\mu F$$

$$3 - k = 2\cos(69.5^0)$$

$$k = 2.23 = 1 + \frac{R_1}{R_2}$$

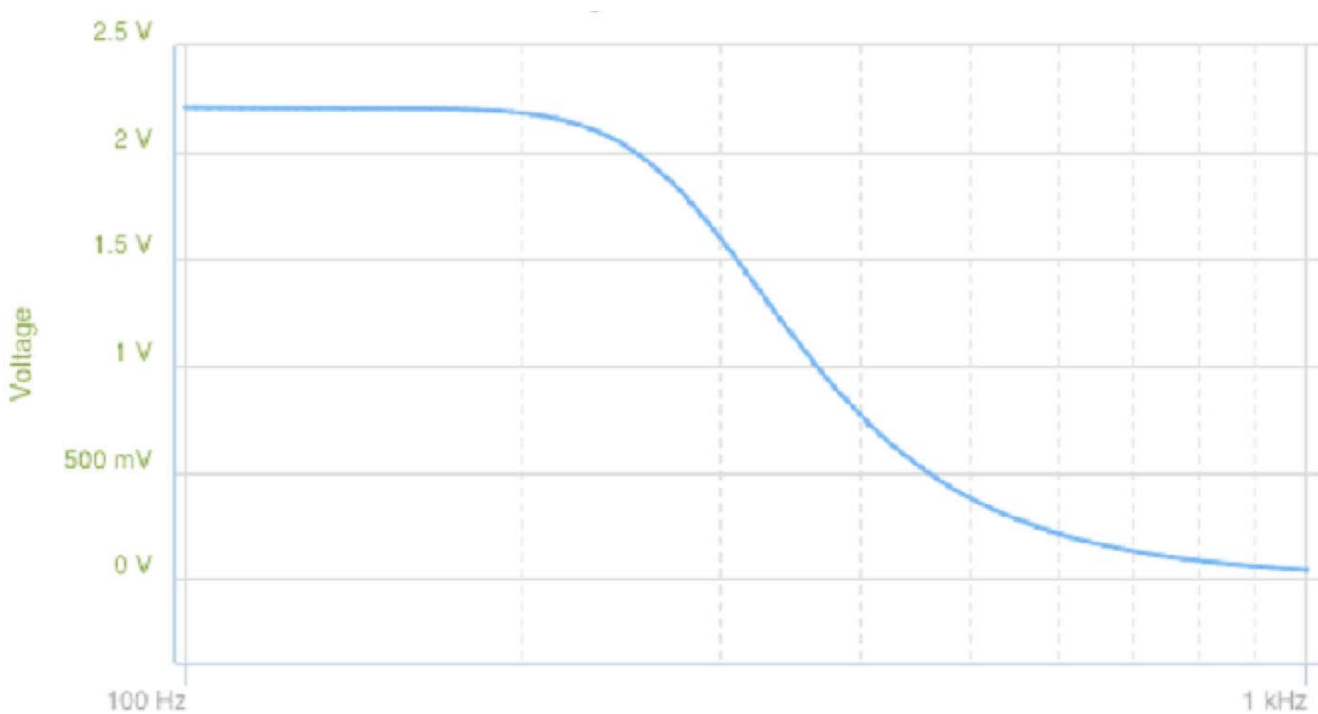
R2 = 100k

R1 = 123k



5) Simulation: Check your design in PartSim or similar programs at the same frequencies you computed in part 3.

	Calculated	Simulated	Measured
100 Hz	2.1924	2.209	-
250 Hz	2.1884	2.005	-
750 Hz	0.1119	0.106	-
1000 Hz	0.0455	0.044	-



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6) Validation: Build a the op-amp circuit and measure the gain at the frequencies computed in part 3.