ECE 321 - Homework #2

Active Filters. Poles, Zeros, Freqency Response. Due Monday, November 13th, 2017

1) For the following filter

$$Y = \left(\frac{100}{(s+5)(s+10)}\right)X$$

a) What is the differential equation relating X and Y?

Cross multiply

$$(s+5)(s+10)Y = (100)X$$

$$(s^2+15s+50)Y = 100X$$

sY means *the derivative of Y*

$$\boxed{\frac{d^2y}{dt^2} + 15\frac{dy}{dt} + 50y = 100x}$$

b) Determine y(t) assuming

$$x(t) = 2 + 3\sin(5t) + 4\sin(100t)$$

Use superposition

x(t) = 2	x(t) = 3 sin(5t)	x(t) = 4 sin(100t)
s = 0	s = j5	s = j100
$\left(\frac{100}{(s+5)(s+10)}\right)_{s=0} = 2$	$\left(\frac{100}{(s+5)(s+10)}\right)_{s=j5} = 1.26\angle -71^0$	$\left(\frac{100}{(s+5)(s+10)}\right)_{s=j100} = 0.01\angle -171^0$
$y = (2)\cdot 2$	$y = (1.26\angle -71^0)\cdot 3\sin(5t)$	$y = (0.01\angle -171^0)\cdot 4\sin(100t)$
$y = 4$	$y = 3.79\sin(5t - 71^0)$	$y = 0.04\sin(100t - 171^0)$

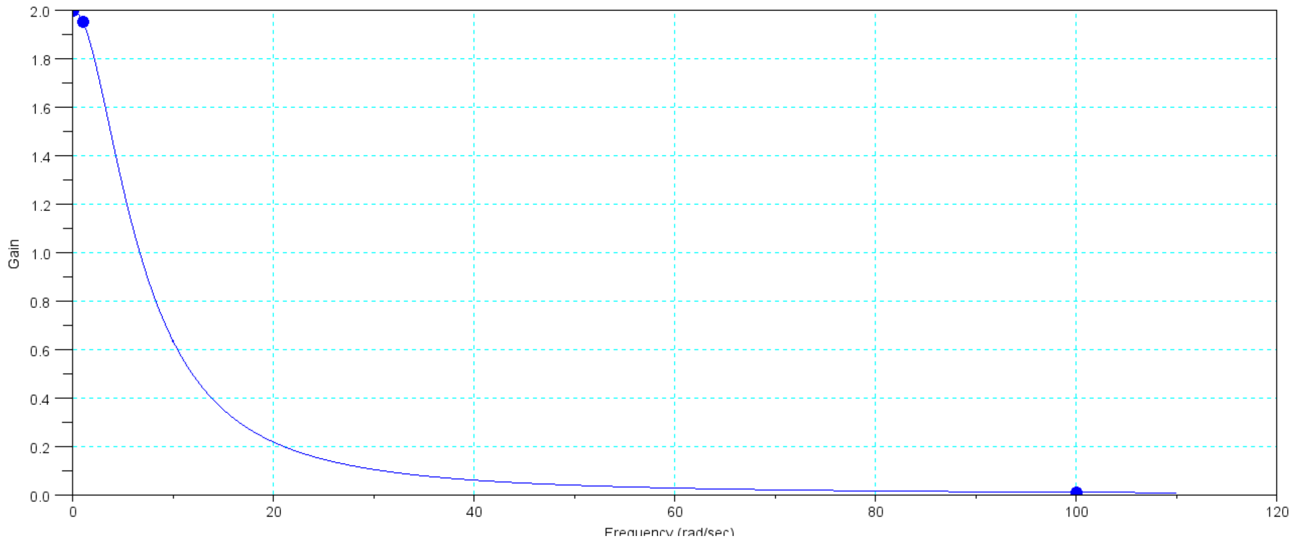
Add up the three inputs to get the total input.

Add up the tree outputs to get the total ouput.

$$\boxed{y(t) = 4 + 3.79\sin(5t - 71^0) + 0.04\sin(100t - 171^0)}$$

c) Plot the gain vs. frequency for this filter from 0 to 20 rad/sec

```
-->w = [0:0.01:20]';
-->j = sqrt(-1);
-->s = j*w;
-->G = 100 ./ ( (s+5).*(s+10) );
-->plot(w,abs(G))
-->xlabel('Frequency (rad/sec)');
-->ylabel('Gain');
```



Gain vs. frequency with the gain at {0, 5, 100} shown. Note that this is a low-pass filter:

- Low frequencies are passed   (0, 5)
- High frequencies are rejected (100)

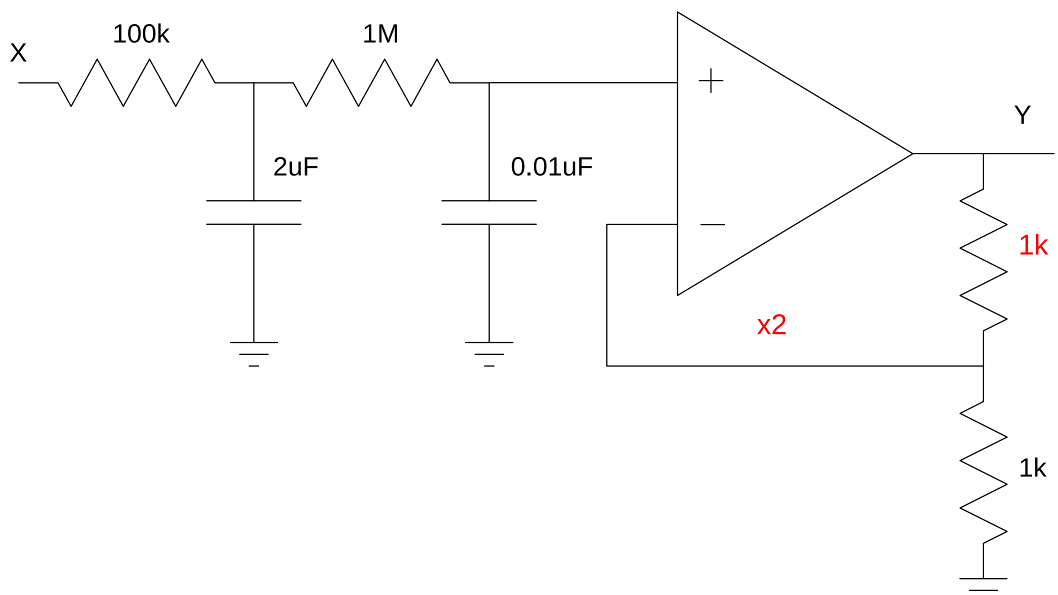
d) Design a circuit to implement this filter.

Since you have real poles, use a 2-stage RC filter

$\frac{1}{R_1 C_1} = 5$ R1 = 100k C1 = 2uF

$\frac{1}{R_2 C_2} = 100$ R2 = 1M C2 = 0.01F

To add a gain of 4 (DC gain is 4), add a x4 amplifier at the end:



2) For the following filter:

$$Y = \left(\frac{200}{s^2+4s+25}\right)X$$

a) What is the differential equation relating X and Y?

Cross multiply

$$(s^2 + 4s + 25)Y = (200)X$$

sY means *the derivative of Y*

$$\boxed{\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 25y = 200x}$$

b) Determine y(t) assuming

$$x(t) = 2 + 3\sin(5t) + 4\sin(100t)$$

x(t) = 2

s = 0

$$\left(\frac{200}{s^2+4s+25}\right)_{s=0} = 8$$

$$y = (8)\cdot 2$$

$$y = 16$$

x(t) = 3 sin(5t)

s = j5

$$\left(\frac{200}{s^2+4s+25}\right)_{s=j5} = 10\angle -90^0$$

$$y = (10\angle -90^0)\cdot 3\sin(5t)$$

$$y = 30\sin(5t - 90^0)$$

x(t) = 4 sin(100t)

s = j100

$$\left(\frac{200}{s^2+4s+25}\right)_{s=j100} = 0.02\angle -178^0$$

$$y = (0.02\angle -178^0)\cdot 4\sin(100t)$$

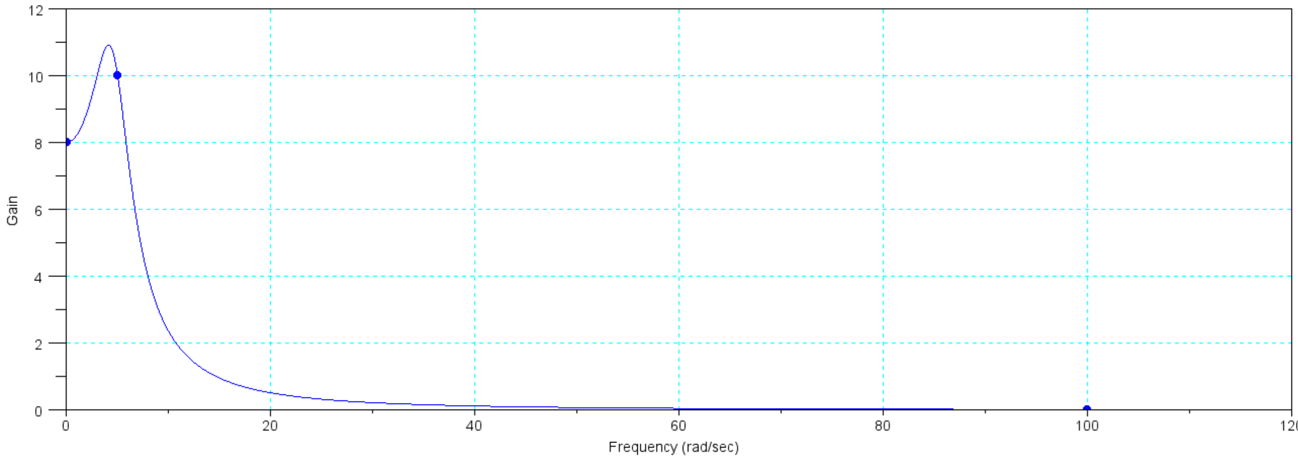
$$y = 0.08\sin(100t - 178^0)$$

Putting it all together:

$$\boxed{y(t) = 16 + 30\sin(5t - 90^0) + 0.08\sin(100t - 178^0)}$$

c) Plot the gain vs. frequency for this filter from 0 to 20 rad/sec

```
-->w = [0:0.01:110]';
-->s = j*w;
-->G = 200 ./ (s.^2 + 4*s + 25);
-->plot(w,abs(G))
-->xlabel('Frequency (rad/sec)');
-->ylabel('Gain');
```



d) Design a circuit to implement this filter.

$$\left(\frac{200}{s^2+4s+25}\right) = \left(\frac{k\left(\frac{1}{RC}\right)^2}{s^2+\left(\frac{3-k}{RC}\right)s+\left(\frac{1}{RC}\right)^2}\right)$$

This filter has poles at

$$s = -2 \pm j4.58 = -5\angle \pm 66.42^0$$

1/RC is the amplitude of the poles

$$\left(\frac{1}{RC}\right)^2 = 25$$

$$\left(\frac{1}{RC}\right) = 5$$

Let R = 100k, C = 2uF

k sets the angle of the pole

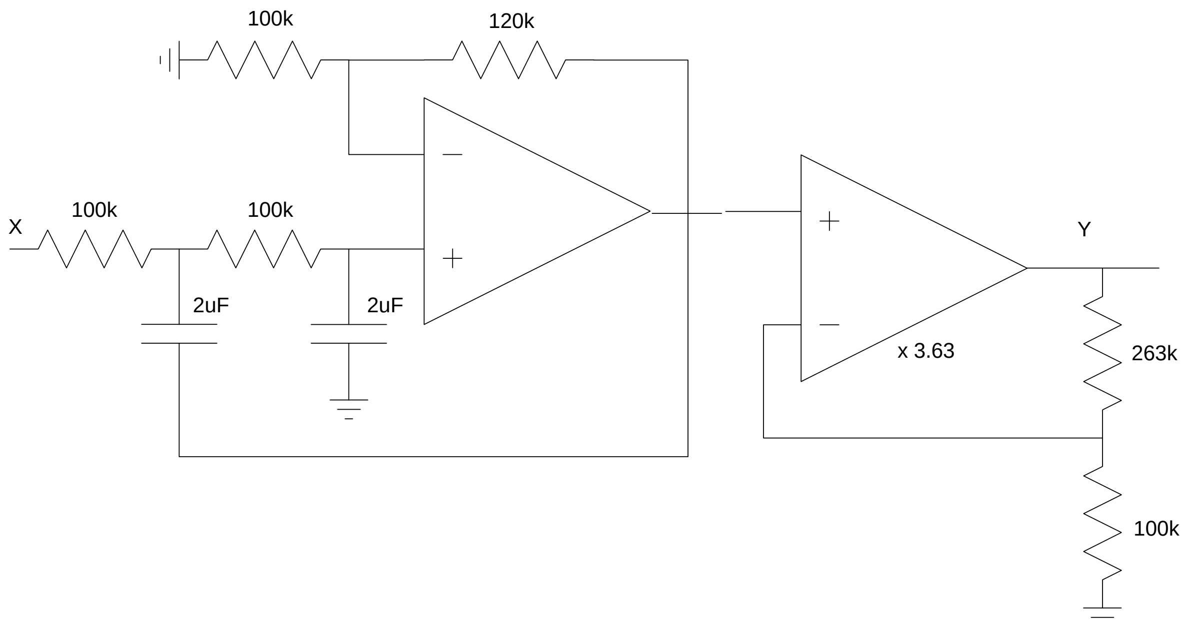
$$\left(\frac{3-k}{RC}\right) = 4$$

$$3 - k = 2\cos(66.42^0)$$

$$k = 2.2$$

Let R1 = 100k, R2 = 120k

This gives a DC gain of 2.2. For a DC gain of 8, add another gain of 3.63



100k
120k
X
100k
100k
2uF
2uF
Y
x 3.63
263k
100k

3) For the following filter:

$$Y = \left(\frac{2s}{s^2+2s+25}\right) X$$

a) What is the differential equation relating X and Y?

Cross multiply

$$(s^2 + 2s + 25)Y = (2s)X$$

sY means *the derivative of Y*

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 25y = 2\frac{dx}{dt}$$

b) Determine y(t) assuming

$$x(t) = 2 + 3\sin(5t) + 4\sin(100t)$$

x(t) = 2

s = 0

$$\left(\frac{2s}{s^2+2s+25}\right)_{s=0} = 0$$

$$y = (0)\cdot 2$$

$$y = 0$$

x(t) = 3 sin(5t)

s = j5

$$\left(\frac{2s}{s^2+2s+25}\right)_{s=j5} = 1$$

$$y = (1)\cdot 3\sin(5t)$$

$$y = 3\sin(5t)$$

x(t) = 4 sin(100t)

s = j100

$$\left(\frac{2s}{s^2+2s+25}\right)_{s=j100} = 0.02\angle -88^0$$

$$y = (0.02\angle -88^0)\cdot 4\sin(100t)$$

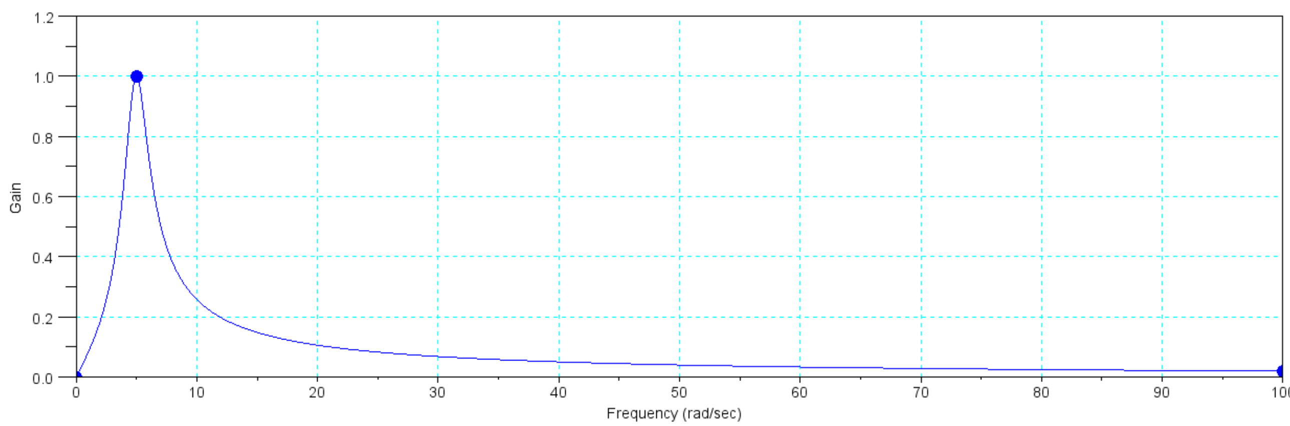
$$y = 0.08\sin(100t - 88^0)$$

Putting it all together:

$$y(t) = 0 + 3\sin(5t) + 0.08\sin(100t - 88^0)$$

c) Plot the gain vs. frequency for this filter from 0 to 20 rad/sec

```
-->w = [0:0.01:100]';
-->s = j*w;
-->G = 2*s ./ (s.^2 + 2*s + 25);
-->plot(w,abs(G))
-->xlabel('Frequency (rad/sec)');
-->ylabel('Gain');
```

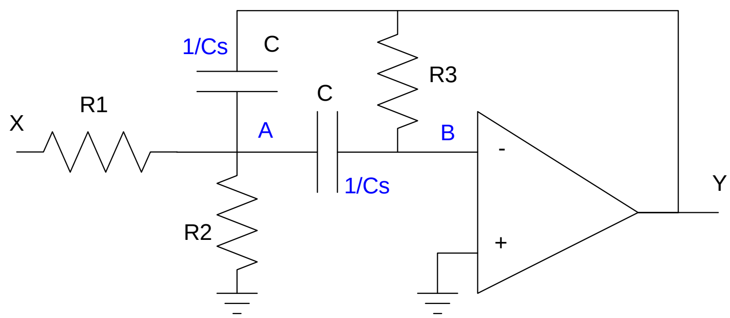


This is a band-pass filter: frequencys over the band of 3 to 7 rad/sec are passed.

d) Design a circuit to implement this filter.

$$Y = \left(\frac{2s}{s^2+2s+25}\right)X$$

From the lecture notes, this form has the following circuit:



$$Y = \left(\frac{-\left(\frac{1}{R_1C}\right)s}{s^2+\left(\frac{2}{R_3C}\right)s+\left(\frac{R_1+R_2}{R_1R_2}\right)\left(\frac{1}{R_3C^2}\right)}\right)X$$

Matching terms: Let C = 1uF

$\left(\frac{1}{R_1C}\right) = 2$ R1 = 500k

$\left(\frac{2}{R_3C}\right) = 2$ R3 = 1M

$\left(\frac{R_1+R_2}{R_1R_2}\right)\left(\frac{1}{R_3C^2}\right) = 25$ R2 = 43.5k